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Introduction to Gauge/Gravity Duality

Examples XI

To hand in Friday 9th January in the examples class

I. **Thermodynamics of the AdS Schwarzschild black hole**

Consider the following metric of a $(d+1)$ dimensional asymptotically AdS Schwarzschild black hole

$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^i dx_i\right) ,$$

with $f(r) = 1 - \left(\frac{r}{r_H}\right)^d$.

a) Determine the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R (It is not necessary to do a calculation, a good argument is enough)!

(2 points)

b)* Consider the (Euclidean) action

$$S_E = -\frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{g}\,(R - 2\Lambda) + \frac{1}{2\kappa^2}\int_{r\to 0} d^d x\sqrt{\gamma}\left(-2K + \frac{2(d-1)}{L}\right) ,$$

where $\Lambda = -d(d-1)/(2L^2)$ is the cosmological constant, $\gamma_{\mu\nu}$ is the induced metric on the boundary $r \to 0$, n^μ is an outward pointing normal vector to the boundary and $K = \gamma^{\mu\nu}\nabla_\mu n_\nu$ is the trace of the extrinsic curvature.

Compute the on-shell action by plugging the metric of the AdS Schwarzschild black hole into the euclidean action S_E.

[Result: the euclidean on-shell reads

$$S_E = -\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} V_{d-1} T^{d-1} ,$$

where V_{d-1} is the (spatial) volume of the corresponding field theory.] (4 points)

c) Compute the partition function

$$Z = \exp\left(-S_E\right)$$

as well as the free energy F and the entropy S

$$F = -T\log Z\,, \qquad S = -\frac{\partial F}{\partial T}.$$

(4 points)

*This is a very tedious exercise. Note, however, that the result is given at the end of b). Therefore if necessary you may solve exercise c) using the result given at the end of b).