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Introduction to Gauge/Gravity Duality

Examples X

To hand in Friday 19th December in the examples class

I. The Wilson loop Operator

In this exercise we determine the quark-antiquark potential for $\mathcal{N} = 4$ SYM at strong coupling using AdS/CFT. The quark-antiquark potential can be calculated from the expectation value of the Wilson loop operator $\langle \mathcal{W}(\mathcal{C}) \rangle$, where $\mathcal{C}$ is a rectangular loop with sides of length $T \to \infty$ and L in euclidean space. The quark-antiquark potential $V(L)$ (where L is the distance between the quark and the antiquark) may be read off from the exponential behaviour

$$\langle \mathcal{W}(\mathcal{C}) \rangle \sim e^{-T\,V(L)}\,.$$

Using AdS/CFT a natural proposal for the expectation value of the Wilson loop is

$$\langle \mathcal{W}(\mathcal{C}) \rangle \sim e^{-S},$$

where S is the regularized on-shell value of the Nambu-Goto action for the fundamental string

$$S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{\det G_{MN} \partial_\alpha X^M \partial_\beta X^N}\,.$$

The worldsheet of the fundamental string has to end on the loop $\mathcal{C}$.

Hint for the exercise: You find the calculation in the paper hep-th/9803002.

a) Simplify the Nambu Goto action for the rectangular loop which has length $T \gg L$ in t-direction and length L in x-direction!

Hints: You may use the following metric for euclidean AdS_5

$$ds^2 = \alpha' \left[\frac{U^2}{R^2} \left(dt^2 + dx^i dx^i \right) + R^2 \frac{dU^2}{U^2} \right],$$

where $R^4 = 4\pi g_s N$. Moreover the string can be embedded into AdS_5 by $U = U(x)$. You may also take the static gauge $\tau = t, \sigma = x$. (3 points)

b) Write down the Euler-Lagrange equations of the action S! (2 points)

c) Solve the equations of motion!

Hint: Determine x as a function of U/U_0, where U_0 is the minimum value of U for the embedding of the string. Express U_0 in terms of L! (2 points)

d) Calculate the on-shell action for the fundamental string S. How can we regularize the action? (2 points)

e) Compute the quark-antiquark potential using the above Wilson loop calculation! Are the probe quarks confined or deconfined? (1 points)