

Introduction to Gauge/Gravity Duality

Examples VIII

To hand in Friday 20th December in the examples class

I. Penrose-Brown-Henneaux Transformation

Show that, when transforming the metric

$$ds^2 = L^2 \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{\mu\nu} dx^\mu dx^\nu \quad (1)$$

according to the Penrose-Brown-Henneaux transformation

$$\rho = \rho'(1 - 2\sigma(x')) \quad x^\mu = (x')^\mu + a^\mu(x', \rho') \quad (2)$$

and demanding that

$$g'_{55} = g_{55} \quad \text{and} \quad g'_{\mu 5} = g_{\mu 5}, \quad (3)$$

one obtains

$$\partial_5 a^\mu = \frac{L^2}{2} g^{\mu\nu} \partial_\nu \sigma \quad (4)$$

and

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + 2\sigma \left(1 - \rho \frac{\partial}{\partial \rho} \right) g_{\mu\nu} + \nabla_\mu a_\nu + \nabla_\nu a_\mu. \quad (5)$$

Note that the index 5 stands for the ρ direction.

5 points

II. Operators and Fixed Points

a) Explain what irrelevant, relevant and marginal operators are.

3 points

b) Comment on the stability conditions of fixed points.

2 points

References:

“Quantum Field Theory and Critical Phenomena” by Jean Zinn-Justin
“An Introduction to Quantum Field Theory” by Peskin and Schroeder