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Introduction to Gauge/Gravity Duality

Examples XII

To hand in Friday 31st January in the examples class (before the exam)

I. **Embedding D7-branes in** $AdS_5 \times S^5$.

Let us consider a D7-brane in $AdS_5 \times S^5$, whose metric may be written as

$$ds^2 = \frac{r^2}{R^2} dx_\mu dx^\mu + \frac{R^2}{r^2} \sum_{i=1}^{6} dY^i dY^i \,.$$

x^μ are the spacetime coordinates. The D7-brane will wrap all four spacetime coordinates $x^\mu (\mu = 0, \ldots, 3)$ as well as four of the six coordinates Y^i, without loss of generality Y^1, Y^2, Y^3 and Y^4. Moreover, we will consider the following embedding

$$Y^5 = 2\pi\alpha' \chi, \qquad Y^6 = L + 2\pi\alpha' \phi \,,$$

where ϕ and χ are functions of the worldvolume coordinates x^μ and Y^a with $a = 1, \ldots, 4$.

The relevant part of the action for the D7-brane is given by

$$S_{D7} \sim \int d^4x d^4y \sqrt{-\det G_{ab}} \,,$$

where a, b enumerate the worldvolume coordinates and G_{ab} is the induced metric on the D7-brane.

a) Determine the Lagrangian up to second order in χ and ϕ. (3 points)

b) Derive the equations of motion for ϕ and χ for the induced metric

$$ds^2 = \frac{\rho^2 + L^2}{R^2} dx_\mu dx^\mu + \frac{R^2}{\rho^2 + L^2} \left(d\rho^2 + \rho^2 d\Omega_3^2 \right) \,.$$

Here we have introduced spherical coordinates for $Y^1, \ldots, Y^4$. (2 points)

c) Use the ansatz $\phi(x^\mu, \rho, \Omega_3) = \tilde{\phi}(\rho) e^{ikx} \mathcal{Y}^l(\Omega_3)$, where $\mathcal{Y}^l(\Omega_3)$ are the scalar spherical harmonics on S^3 which satisfy $\nabla^a \nabla_a \mathcal{Y}^l = -l(l+2)\mathcal{Y}^l$ on the S^3. Derive an ODE for $\tilde{\phi}(\rho)$! (2 points)

d) Solve the ODE for $\tilde{\phi}(\rho)$ in terms of a hypergeometric function. You may wish to use the redefinitions $\varrho = \frac{\rho}{L}$ and $M^2 = -\frac{k^2 R^4}{L^2}$. (3 points)

Hint: The solution to the exercise can be found in the paper arXiv: hep-th/0304032, in particular eq. (3.4) for exercise a), (3.5) and (3.6) for exercise b) as well as (3.10) for exercise c) and (3.15) for exercise d). We recommend to read the relevant sections of the paper.