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Introduction to Gauge/Gravity Duality

Examples VII

To hand in Friday 7th December in the examples class

I. **Saturation of unitarity bound**

Consider a free scalar field with mass m in the Poincare patch of euclidean AdS_{d+1} with radius R.

a) Show that the usual bulk action

$$S_1 = -\frac{1}{2}\int d^dx\, dz\sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)\,,$$

evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z=0$, is finite if $\Delta > d/2$, i.e. the solution with $\Delta = \Delta_+$ (Δ_+ being the larger root of $\Delta(\Delta-d) = m^2R^2$) are normalizable with respect to the action S_1.

(4 points)

b) Consider the bulk action

$$S_2 = -\frac{1}{2}\int d^dx\, dz\sqrt{g}\phi\left(-\Box_g + m^2\right)\phi\,.$$

Show that S_2 can be obtained by adding a boundary term to the action S_1 (hint: partial integration) and that the equations of motion are the same as for the action S_1.

(2 points)

c) Show, that the action S_2 evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z=0$ is finite if $\Delta > (d-2)/2$. Conclude that for

$$-\frac{d^2}{4} < m^2R^2 < -\frac{d^2}{4} + 1$$

both solutions, i.e. $\Delta = \Delta_+$ and $\Delta = \Delta_-$ are normalizable with respect to the action S_2.

(4 points)

For more additional information see arXiv: hep-th/9905104.

II. **Fefferman-Graham expansion**

Consider the five-dimensional metric

$$ds^2 = L^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j \right) . \tag{1}$$

According to the Fefferman-Graham-Theorem $g_{ij}(x,\rho)$ can be expanded in the form

$$g_{ij}(x,\rho) = \bar{g}_{ij}(x) - \rho g_{(2)ij}(x) + \rho^2 g_{(4)ij}(x) + \rho^2 \ln(\rho)\, h_{(4)ij}(x) + ... \tag{2}$$

if the five-dimensional Einstein equations $R_{\mu\nu} = -4g_{\mu\nu}$ are satisfied. The functions $g_{(l)ij}(x)$ and $h_{(l)ij}(x)$ can be expressed in terms of local functions of $\bar{g}_{ij}(x)$ and curvature tensors of $\bar{g}_{ij}(x)$.

Calculate $g_{(2)ij}(x)$ by using Einstein equations! (5 points)