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Introduction to Gauge/Gravity Duality

Examples VI

To hand in Friday 30th November in the examples class

I. Relation between propagators in AdS

Let us consider euclidean AdS in the Poincaré patch.

a) Derive the equations of motion for a scalar field with mass m in euclidean AdS_{d+1}.

(2 points)

b) Use the ansatz $\phi(z) = z^{\Delta}$ near the boundary $z \to 0$ and determine the two possible values of $\Delta_{\pm}$, where $\Delta_+ > \Delta_-$.

(2 points)

c) The bulk-to-boundary propagator $K(x, z; x')$ is the solution of the equations of motion which is regular in the interior and diverges like

$$\lim_{z \to \epsilon} K(z, x; x') = \epsilon^{\Delta_-} \delta(x - x')$$

near the boundary, i.e. for $\epsilon \ll 1$. The bulk-to-bulk propagator is given by the solution of the equation of motion with a pointlike source term,

$$\left(\Box_{x,z} - m^2\right) G(z, x; z', x') = \frac{1}{\sqrt{g}} \delta(x - x') \delta(z - z')\,,$$

where $\Box_{x,z}$ is the scalar Lapacian in euclidean AdS_{d+1} acting only on x and z. Moreover the bulk-to-bulk propagator is regular in the interior.

Show that the bulk-to-boundary propagator $K(x, z; x')$ can be calculated from the bulk-to-bulk propagator $G(z, x; z', x')$ by

$$K(z, x; x') = \lim_{z' \to \epsilon} \frac{\Delta_+ - \Delta_-}{\epsilon^{\Delta}} G(z, x; z', x').$$

Hint: Do not use the explicit solution for $G(z, x; z', x')$, but Green's second identity!

(6 points)