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Introduction to Gauge/Gravity Duality

Examples V

To hand in Friday 23rd November in the examples class

I. The AdS_5/CFT_4 duality

a) State the precise AdS_5/CFT_4 duality in the strongest form, i.e. for general ranks N of the gauge group and for arbitrary 't Hooft coupling constants λ! What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g_{YM}^2 = 4\pi g_s$, $\lambda = N g_{YM}^2$ and $R^4 = 4\pi g_s N \alpha'^2$. (3 points)

b) What is the operator–field map? What are normalizable and non–normalizable modes and what is their meaning on the field theory side? (2 points)

II. Comparison of gravity and gauge theory absorption cross-sections

a) Compute the decay rate of a scalar ϕ (dilaton) into two $SU(N)$ gauge bosons using the relevant part of the worldvolume theory action of D3-branes

$$S = \int d^4x \left[-\frac{1}{4} e^{-\phi} \mathrm{tr}(F^2) \right] \tag{1}$$

and the bulk action

$$S_{bulk} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + ... \right] . \tag{2}$$

This corresponds to the absorption cross-section of a dilaton by N coincident D3-branes. (4 point)

b) The gravitational background of extremal black 3-brane is given by the metric

$$ds^2 = H(r)^{-1/2}(-dt^2 + dx_i dx^i) + H(r)^{1/2}(dr^2 + r^2 d\Omega_5^2) , \tag{3}$$

with $H(r) = 1 + R^4/r^4$ and $i \in \{1, 2, 3\}$.
The absorption cross-section for a minimally coupled s-wave scalar with energy ω in this background is given by

$$\sigma_{3-brane} = \frac{\pi^4}{8} \omega^3 R^8 . \tag{4}$$

Rewrite this absorption cross-section for N coincident microscopic D3-branes by using the following definition for R

$$R^4 = \frac{\kappa_{10} N}{2\pi^{5/2}} . \tag{5}$$

(1 point)

c) Compare the two cross-sections obtained in a) and b)! Why is this surprising and what is the relation to gauge-string duality? (3 points)