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Introduction to Gauge/Gravity Duality

Examples XI

To hand in Friday 18th January in the examples class

I. **Shear viscosity over entropy density**

a) To determine its correlation functions, the energy-momentum tensor $T^{\mu\nu}$ has to be coupled to a source, the metric $g_{\mu\nu}$. In this exercise we consider the following small perturbations $h_{\mu\nu}$ around the flat metric

$$\begin{aligned} g_{ij}(t,\vec{x}) =& \delta_{ij} + h_{ij}(t)\,, \quad h_{ij} \ll 1 \\ g_{00}(t,\vec{x}) =& -1\,, \quad g_{0i}(t,\vec{x}) = 0\,, \end{aligned} \tag{1}$$

with i, j being spatial indices.
The dissipative part of the energy-momentum tensor $T^{\mu\nu}$ in curved space-time is

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left[\eta\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha\right) + \left(\zeta - \frac{2}{3}\eta\right) g_{\alpha\beta}\nabla\cdot u\right], \tag{2}$$

with the local fluid velocity $u^\mu = (1,0,0,0)$ and the projector $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, which projects onto the hyperplane transverse to the fluid velocity.
Compute σ^{xy} to leading order in h_{ij} by expanding the explicit expressions for the covariant derivatives in (2). Relate σ^{xy} to the retarded Green's function $G^R_{xy,xy}(\omega,\vec{0})$. State Kubo's formula for the shear viscosity η. (5 points)

b) Consider the black-brane solution of type IIB supergravity in $AdS_5 \times S^5$

$$ds^2 = \frac{(\pi T L)^2}{u^2}\left(-f(u)dt^2 + dx^2\right) + \frac{L^2}{4u^2 f(u)}du^2 + L^2 d\Omega_5^2\,, \tag{3}$$

with $f(u) = 1 - u^2$, T the temperature and L the AdS_5 radius. The linearized equation of motion of the metric fluctuation h_{xy} above this background is of the form $\partial_\mu\left(g^{\mu\nu}\partial_\nu\phi\right) = 0$, with $\phi = h^x{}_y$.

Show that this equation of motion corresponds to the following equation in momentum space ($' = \partial_u$)

$$\phi'' - \frac{1+u^2}{uf}\phi' + \frac{\tilde{\omega}^2 - \tilde{k}^2 f}{uf^2}\phi = 0\,, \tag{4}$$

with $\tilde{\omega} = \omega/(2\pi T)$ and $\tilde{k} = k/(2\pi T)$.

Using the recipe by Son and Starinets (see literature list below), we obtain the following retarded Green's function of energy-momentum tensor T_{xy},

$$G^R_{xy,xy}(\omega, 0) = -\frac{\pi^2 N^2 T^4}{4} i\tilde{\omega}. \tag{5}$$

Compute the ratio of the shear viscosity η and the entropy density s (For the entropy, note last weeks exercise sheet!). (5 points)

Recommended literature:

D. T. Son and A. O. Starinets, "Viscosity, Black Holes, and Quantum Field Theory," Ann. Rev. Nucl. Part. Sci. **57** (2007) 95 [arXiv:0704.0240 [hep-th]].

P. Kovtun, D. T. Son and A. O. Starinets, "Viscosity in strongly interacting quantum field theories from black hole physics," Phys. Rev. Lett. **94** (2005) 111601 [hep-th/0405231].

D. T. Son and A. O. Starinets, "Minkowski-space correlators in AdS/CFT correspondence: recipe and applications," JHEP **0209** (2002) 042 [hep-th/0205051].