Introduction to QCD and Loop Calculations

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Exercise 5.1: DIS

Consider deeply inelastic scattering (DIS) in the parton model, $e(k) + q(p) \to e(k') + q'(p')$, where the quark q struck by the electron has momentum $p^\mu = \xi P^\mu$, so carries a fraction ξ of the proton's momentum. The momentum transfer between the electrons is $q^\mu = k^\mu - k'^\mu$, with $q^2 = -Q^2$. In the lectures we also introduced the *scaling variable* $x = Q^2/(2P\cdot q)$ and the relative energy loss $y = (P\cdot q)/(P\cdot k) = Q^2/\hat{s}$, where $\hat{s} = (p+k)^2$. We have shown that

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e_q^2 e^4}{Q^4} L^{\mu\nu} Q_{\mu\nu} = 2 e_q^2 e^4 \frac{\hat{s}^2}{Q^4}\left(1 + (1-y)^2\right) . \tag{1}$$

Starting from this expression, calculate the partonic differential cross section

$$\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}x\,\mathrm{d}y} = \frac{4\pi\alpha^2}{yQ^2}\left[1 + (1-y)^2\right] \frac{1}{2} e_q^2 \delta(\xi - x) .$$

Hint:

$$\text{Use} \qquad \mathrm{d}\hat{\sigma} = \frac{1}{2\hat{s}}\,\mathrm{d}\Phi_2\,\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 \qquad \text{and derive}$$

$$\mathrm{d}\Phi_2 = \frac{1}{(2\pi)^3}\frac{\mathrm{d}^3k'}{2E'}\frac{\mathrm{d}^4p'}{(2\pi)^4}\,2\pi\delta\left(p'^2\right)(2\pi)^4\,\delta^4(k + p - k' - p') = \frac{\mathrm{d}\phi}{(4\pi)^2}\,\mathrm{d}x\,\mathrm{d}y\,\delta(\xi - x) .$$

Exercise 5.2: Splitting functions

The so-called "plus-prescription" for a function $g(x)$ which is divergent at $x = 1$ is defined by

$$\int_0^1 \mathrm{d}x\, f(x)\,[g(x)]_+ = \int_0^1 \mathrm{d}x\,\Big(f(x) - f(1)\Big)\,g(x) .$$

where $f(x)$ is an arbitrary (smooth) function. Example:

$$\int_0^1 \mathrm{d}x\,\frac{f(x)}{[1-x]_+} = \int_0^1 \mathrm{d}x\,\frac{f(x) - f(1)}{1-x} .$$

The regularised splitting function $P_{q\to qg}(x)$ is given by

$$P_{q\to qg}(x) = C_F\left(\frac{1+x^2}{[1-x]_+} + K\,\delta(1-x)\right) . \tag{2}$$

(a) Calculate K using the fact that

$$\int_0^1 \mathrm{d}x\,P_{q\to qg}(x) = 0 . \tag{3}$$

(b) What is the physical meaning of eq. (3) ?