Introduction to QCD and Loop Calculations

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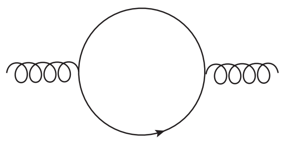
Exercise 4.1: One-loop beta-function



Calculate the fermion loop contribution to the coefficient b_0 of the QCD β-function.

To this aim, calculate first the fermion loop contribution to the gluon selfenergy (shown above) and extract the renormalisation constant Z_3 (more precisely only the fermionic contribution to $Z_3, Z_3^{(F)}$). Then use the Slavnov-Taylor identities to relate $Z_3^{(F)}$ to Z_g and use $Z_g = 1 - \frac{1}{\epsilon}\frac{b_0}{2}\alpha_s + \mathcal{O}(\alpha_s^2)$.

Useful formulas for Traces, 1-point and 2-point integrals:

$$
\begin{aligned}
\text{Trace}[\gamma_\mu \not{p}\, \gamma_\nu \not{k}\,] &= 4\,(k_\mu p_\nu + k_\nu p_\mu - g_{\mu\nu} k\cdot p)\ ,\ \text{Trace}[\gamma_\mu\gamma_\nu\,] = 4\, g_{\mu\nu} \\
I_1^D(m) &= \frac{1}{\epsilon}\, m^2 + \mathcal{O}(\epsilon^0) \\
I_2^D(p^2,m,m) &= \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0) \\
I_2^{D,\mu}(p^2,m,m) &= -\frac{1}{2}\, p^\mu\, I_2^D(p^2,m,m) = -\frac{1}{2}\, p^\mu\, \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0) \\
I_2^{D,\mu\nu}(p^2,m,m) &= g^{\mu\nu}\,\frac{1}{\epsilon}\left(-\frac{1}{12}\,p^2 + \frac{1}{2}\,m^2\right) + p^\mu p^\nu\,\frac{1}{\epsilon}\,\frac{1}{3} + \mathcal{O}(\epsilon^0)\ . \qquad (1)
\end{aligned}
$$

Exercise 4.2: D-dimensional phase space integrals

Show that the D-dimensional three-particle phase space for $Q \to p_1 + p_2 + p_3$, with $p_i^2 = 0$, can be expressed in terms of the kinematic invariants $s_{ij} = (p_i + p_j)^2$ as

$$
\mathrm{d}\Phi_3 = \frac{(2\pi)^{3-2D}}{2^{D+1}}\,(Q^2)^{\frac{2-D}{2}}\,\mathrm{d}\Omega_{D-2}\mathrm{d}\Omega_{D-3}\,\mathrm{d}s_{12}\,\mathrm{d}s_{13}\,\mathrm{d}s_{23}\,\left(s_{12}s_{13}s_{23}\right)^{\frac{D-4}{2}}\,\delta\left(Q^2 - s_{12} - s_{13} - s_{23}\right)\ .
$$