Exercise Sheet 3 May 17, 2018

Introduction to QCD and Loop Calculations

Technical University Munich, Summer Term 2018

Exercises:
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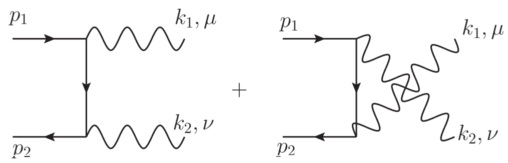
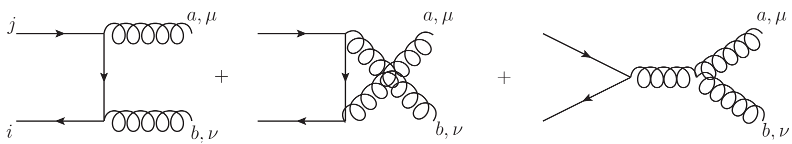
Exercise 3.1: QED Ward Identity



Calculate the tree level amplitude $\mathcal{M}_{\mu\nu}$ for $e^+e^- \to \gamma\gamma$ (assuming massless electrons) and show explicitly that $k_1^\mu M_{\mu\nu} = 0, k_2^\mu M_{\mu\nu} = 0$.

Exercise 3.2: Gauge invariance of QCD amplitudes



Calculate the tree level amplitude $\mathcal{M}^{\rm QCD}_{\mu\nu}$ for $q\,\bar{q} \to g\,g$ (with massless quarks) and show explicitly that $k_1^\mu M^{\rm QCD}_{\mu\nu}$ only vanishes if k_2 is the momentum of a physical gluon.

Exercise 3.3: One-loop tensor Integrals

In the representation of tensor integrals, for tensor ranks $r \geq 2$, higher dimensional integrals I_N^{D+2m} arise as coefficients of metric tensors $(g^{\mu\nu})^{\otimes m}$. To see how these integrals arise, start from the representation in eq. (3.26) of the script, in terms of Feynman parameters and quadratic forms in the loop momentum, but this time with loop momenta in the numerator:

$$L_N^{\mu_1\mu_2} \;=\; \Gamma(N) \int_0^\infty \prod_{i=1}^N dz_i\, \delta(1 - \sum_{l=1}^N z_l) \int_{-\infty}^\infty \frac{d^D l}{i\pi^{\frac{D}{2}}}\, l^{\mu_1} l^{\mu_2} \left[l^2 - R^2 + i\delta\right]^{-N}.$$

Then make the ansatz

$$L_N^{\mu_1\mu_2} = K\, g^{\mu_1\mu_2}$$

and determine K in terms of I_N^{D+2}, using the functional form given in eq. (3.18) of the script.