Introduction to QCD and Loop Calculations

Technical University Munich, Summer Term 2018

Exercises:
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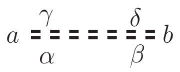
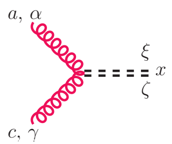
Exercise 2.1: Four-gluon vertex

The 4-gluon vertex can be written as in a form where colour and kinematic parts of the Feynman rules factorise by introducing an auxiliary field with propagator

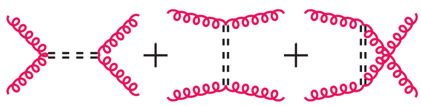


$= -\frac{i}{2}\delta^{ab}(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\delta}g^{\beta\gamma})$ which couples only to the gluon, via



$= i\sqrt{2}g_s\, f^{xac}g^{\alpha\xi}g^{\gamma\zeta}.$

Show that a single four-gluon vertex can be written as a sum of the three graphs shown below, where colour and Lorentz indices factorise.



Exercise 2.2: Renormalisation schemes

The QCD β-function is of the form

$$\beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = -b_0\alpha_s^2 \left[1 + b_1\,\alpha_s + b_2\,\alpha_s^2 + \mathcal{O}(\alpha_s^3) \right] \ . \tag{1}$$

Consider two renormalisation schemes A and B, where the couplings are related by

$$\alpha_s^B = \alpha_s^A \left[1 + c_1\,\alpha_s^A + c_2\,(\alpha_s^A)^2 + \mathcal{O}((\alpha_s^A)^3) \right] \ . \tag{2}$$

Show that the first two coefficients, b_0 and b_1, are scheme-independent, while the third one in scheme B fulfills the relation

$$b_2^B = b_2^A + c_2 - b_1\,c_1 - c_1^2 \ . \tag{3}$$