Exercise Sheet 1 April 13, 2018

Introduction to QCD and Loop Calculations

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Exercises:
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Exercise 1.1: Gauge transformations

Consider the Lagrangian for the quark fields

$$\mathcal{L}_q(q_f,\, m_f) = \sum_{j,k=1}^{N_c} \bar{q}_f^j(x)\; (i\,\gamma_\mu \boldsymbol{D}^\mu[\boldsymbol{A}] - m_f)_{jk}\; q_f^k(x)\;, \tag{1}$$

where the covariant derivative $\boldsymbol{D}^\mu$ is given by

$$\boldsymbol{D}^\mu[A] = \partial^\mu + i\,g_s\,\boldsymbol{A}_\mu\;, \tag{2}$$

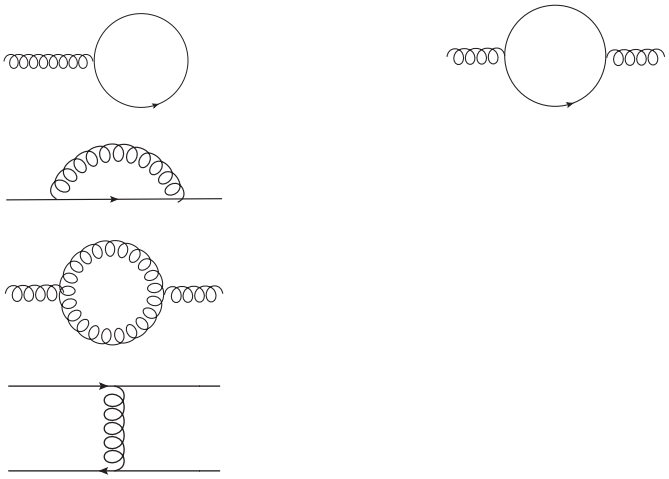
with $\boldsymbol{A}^\mu = t^a\,A_a^\mu\,,\ a = 1\,\ldots\,N_c^2 - 1$.
The quark and gluon fields transform under local gauge transformations $U(x)$ as

$$\begin{aligned} q_j'(x) &= U_{jk}\,q^k(x) \\ \boldsymbol{A}_\mu'(x) &= U(x)\boldsymbol{A}_\mu(x)U^{-1}(x) + \frac{i}{g_s}\big(\partial_\mu U(x)\big)U^{-1}(x)\;. \end{aligned} \tag{3}$$

(a) Show explicitly that $\boldsymbol{D}^\mu[\boldsymbol{A}']\,q'(x) = U(x)\Big(\boldsymbol{D}^\mu[\boldsymbol{A}]\,q(x)\Big)$ and therefore the Lagrangian $\mathcal{L}_q$ in Eq. (1) is invariant under local gauge transformations.
(b) Why would a mass term for the gluons break local gauge invariance?

Exercise 1.2: Colour factors

(a) Calculate the colour factors for the following Feynman diagrams:



(b) Derive the explicit values for C_F and C_A by considering pictorial identities like