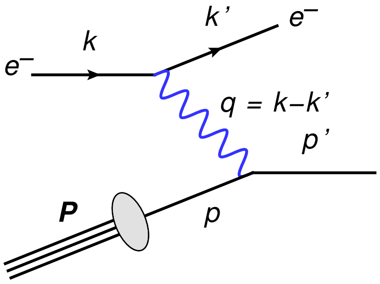
Introduction to QCD and Loop Calculations

Exercise 5.1 DIS



momentum conservation:
$k + p = k' + p'$; $\hat{s} = (k+p)^2 = 2k \cdot p$
$y = \frac{P\cdot q}{P\cdot k} = \frac{p\cdot q}{p\cdot k} = \frac{Q^2}{\hat{s}} = \frac{1-E'/E}{\hat{s}}$; $x = \frac{Q^2}{2qP}$
$p'^2 = 0 = (q+p)^2 = -Q + 2qp \rightarrow 2qp = Q^2$

$$\frac{1}{4}\sum_{spins}|\mathcal{M}|^2 = e_q^2 e^4 \frac{1}{Q^4} L^{\mu\nu} Q_{\mu\nu} = e_q^2 e^4 \frac{2\hat{s}^2}{Q^4}(1 + (1-y)^2)$$

Partonic cross section:

$$d\hat{\sigma} = \frac{1}{2\hat{s}} d\Phi_2 \frac{1}{4}\sum_{spins}|\mathcal{M}|^2$$

Two-particle phase space in 4 dimensions:

$$\begin{aligned} d\Phi_2 &= \frac{d^4k'}{(2\pi)^3}\,\delta(k'^2)\,\frac{d^4p'}{(2\pi)^3}\,\delta(p'^2)(2\pi)^4\,\delta^4(k+p-k'-p') \\ &= \frac{d^3k'}{(2\pi)^2\cdot 2E'}\,d^4p'\,\delta(p'^2)\,\delta(k+p-k'-p') \\ &= \frac{d^3k'}{(2\pi)^2\cdot 2E'}\,\delta((q+p)^2) \\ &= \frac{1}{8\pi^2}d\varphi\, d\cos(\vartheta) d|k'||k'|^2\delta(Q^2(\frac{\xi}{x}-1)) \\ &= \frac{1}{4\pi}\frac{d\varphi}{2\pi}d\cos(\vartheta)E'\,dE'\frac{x}{Q^2}\delta(\xi - x) \qquad (*) \end{aligned}$$

$q = k - k'$
$p = \xi P$

$Use:$

$$\begin{aligned} (q+p)^2 &= -Q^2 + 2p\cdot q \\ &= -Q^2 + 2qP\cdot\xi \\ &= -Q^2(1-\xi/x) \end{aligned}$$

$|k'| = E'$

$$\text{Now use } E' = E(1-y) = \frac{\sqrt{\hat{s}}}{2}(1-y) \Rightarrow dE' = Edy$$

$$\text{and } q^2 = (k-k')^2 = -2EE'(1-cos(\vartheta)) = -2E^2(1-y)(1-cos(\vartheta))$$

$$\Rightarrow 1 - cos(\vartheta) = \frac{q^2}{-2E^2(1-y)} = \frac{-2MExy}{-2E^2(1-y)} = \frac{Mxy}{E(1-y)}$$

$$\Rightarrow dcos(\vartheta) = \frac{My}{E(1-y)}dx$$

$$dE'dcos(\vartheta) = \frac{My}{1-y}dxdy \text{ insert into } (*):$$

$$d\Phi_2 = \frac{1}{4\pi}\frac{d\Phi}{2\pi}E(1-y)\frac{x}{Q^2}\frac{My}{1-y}dxdy\delta(\xi - x)$$

$$\text{Use } EMxy = \frac{Q^2}{2} \ \Rightarrow\ d\Phi_2 = \frac{1}{8\pi}\frac{d\Phi}{2\pi}dxdy$$

$$\Rightarrow \frac{d^2\hat{\sigma}}{dxdy} = \frac{1}{16\pi}\int \frac{d\Phi}{2\pi}\frac{1}{\hat{s}}\delta(\xi - x)\frac{1}{4}\sum_{spins}|\mathcal{M}|^2 \qquad \text{Use } \alpha = \frac{e^2}{4\pi}$$

$$= \alpha^2 e_q^2 \frac{\hat{s}}{Q^4}\int d\Phi\delta(\xi - x)(1 + (1-y)^2) \qquad \text{Use } \int d\Phi = 2\pi$$

$$= 4\pi\alpha^2 \frac{1}{yQ^2}(1 + (1-y)^2)\frac{1}{2}e_q^2\delta(\xi - x) \qquad Q^2 = \hat{s}y$$

Exercise 5.2 Splitting functions
Plus prescription (divergence @ x=1):

$$\int_0^1 dx f(x)[g(x)]_+ = \int_0^1 dx\,(f(x) - f(1))\,g(x)$$

a)

$$\int_0^1 dx \left(\frac{1+x^2}{[1-x]_+} + K\delta(1-x)\right) \stackrel{!}{=} 0 \qquad \text{Choose } f(x) = 1 + x^2$$

$$\int_0^1 dx \left(\frac{1}{1-x}(1 + x^2 - 2) + K\delta(1-x)\right) =$$

$$\int_0^1 dx \left(\frac{-(1-x)(1+x)}{1-x} + K\delta(1-x)\right) =$$

$$-\int_0^1 dx(1+x) + K = -\frac{3}{2} + K \stackrel{!}{=} 0 \to k = \frac{3}{2}$$

b)
The parton distribution function that is giving the probability to find a quark in a quark can be written as:

$$f_{q/q}(x, \mu_f) = \delta(1-x) + \frac{\alpha_s}{2\pi}P_{q\to qg}(x)\cdot log\left(\frac{Q^2}{{\mu_f}^2}\right) + ...$$

so we must have

$$\int_0^1 dx f_{q/q}(x, \mu_f) = 1$$

and therefore

$$\int_0^1 P_{q\to qg}(x) = 0$$