4D Heterotic Models with U(1) Bundles

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Motivation

Mainly two kinds of semi-realistic string compactifications:

- Compactifications with intersecting D-branes

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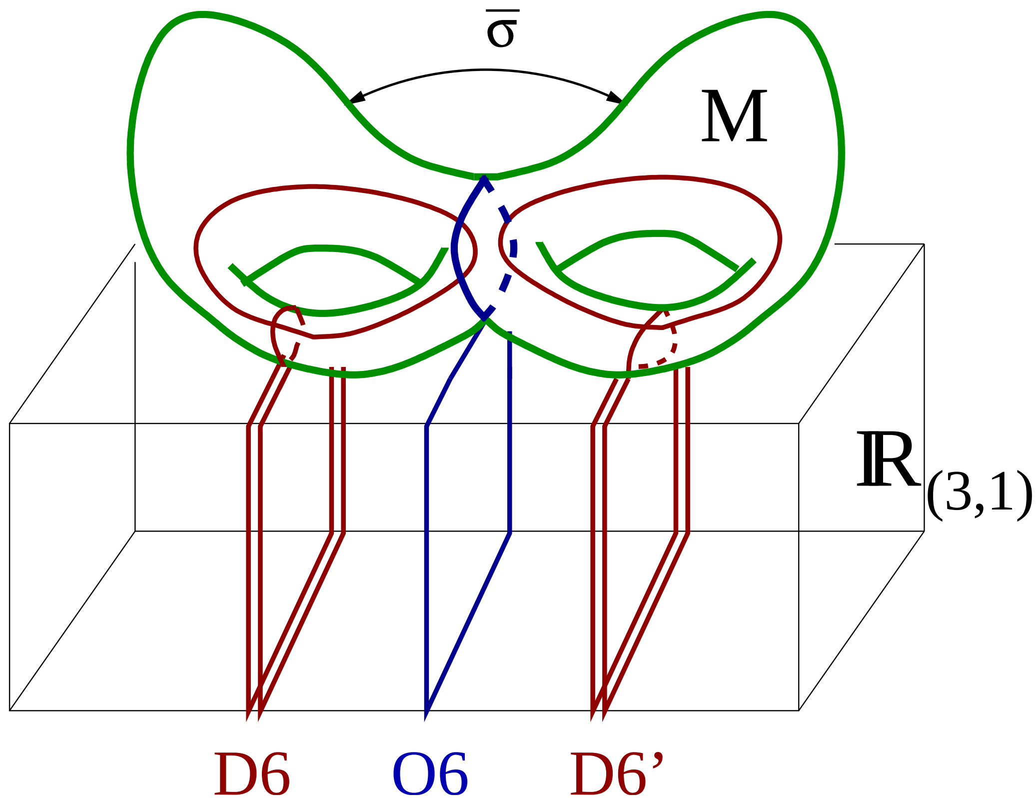

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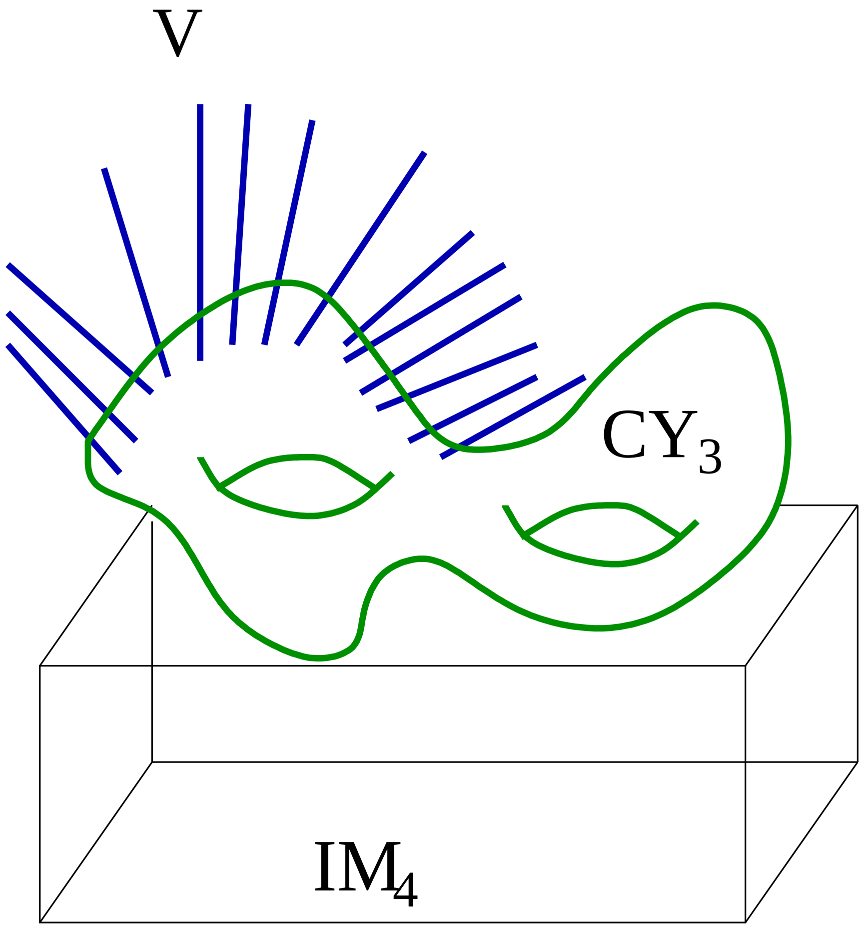

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- Conflict with S-duality $\rightarrow$ systematically study heterotic string with abelian (line) bundles and Type I vacua with non-abelian bundles.

Heterotic Compactifications:

M. Green/J. Schwarz/P. West '85, E. Witten '85, J.-P. Derendinger/L. Ibanez/H.-P. Nilles '85, K. Choi/J. Kim '85, J. Distler/B. Greene '88, A. Lukas/K. Stelle '99, B. Andreas, R. Blumenhagen, V. Braun, G. Cardoso, G. Curio, M. Cvetic, K. Dienes, J. Erler, A. Faraggi, S. Förste, J. Gray, M. Hillenbach, G.H., J. Kim, T. Kobayashi, J. Louis, D. Lüst, P. Mayr, S. Groot Nibbelink, B. Ovrut, J. Polchinski, F. Quevedo, E. Rabinovici, S. Raby, E. Sharpe, S. Stieberger, T. Weigand, A. Wingerter + others

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Intersecting/magnetised branes & fluxes:

S. Abel, B. Acharya, G. Aldazabal, C. Angelantonj, I. Antoniadis, C. Bachas, K. Behrndt, M. Bianchi, O. Loaiza Brito, M. Cardella, J. Conlon, D. Cremades, G. Dall'Agata, F. Denef, J. Erdmenger, S. Ferrara, A. Font, M. Garcia del Moral, D. Ghilencea, F. Gmeiner, T. Grimm, R. Helling, N. Irges, H. Jockers, R. Kallosh, G. Kane, E. Kiritsis, B. Körs, C. Kokorelis, C. Kounnas, P. Langacker, T. Maillard, F. Marchesano, J. Polchinski, G. Pradisi, R. Rabadan, S. Reffert, A. Sagnotti, G. Shiu, T. Taylor, M. Timirgaziu, M. Trapletti, S. Trivedi, A. Uranga, F. Zwirner + others

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Stability:

M. Douglas/B. Fiol/C. Römelsberger '00, H. Enger/C. Lutken '03, M. Marino/R. Minasian/G. Moore/A. Strominger '99, P. Aspinwall '04 + others

Compactifications of heterotic strin

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$E_8 \times E_8$ HS with vector bundles of the following form

$$W = \bigoplus_{i=1}^{K} V_{n_i} \oplus \bigoplus_{m=1}^{M} L_m,$$

where the V_{n_i} are $SU(n_i)$ or $U(n_i)$ bundles and the L_m denote some complex line bundles with structure group $U(1)_m$.

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- The vector bundle W has to admit spinors

$$c_1(W) \in H^2(\mathcal{M}, 2\mathbb{Z}).$$

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- At string tree level, the field strength of the vector bundle has to satisfy the hermitian Yang-Mills equations

$$F_{ab} = F_{\overline{a}\overline{b}} = 0, \quad g^{a\bar{b}} F_{a\bar{b}} = 0.$$

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- A necessary condition is the so-called Donaldson-Uhlenbeck-Yau (DUY) condition,

$$\int_{\mathcal{M}} J \wedge J \wedge c_1(V_{n_i}) = 0, \qquad \int_{\mathcal{M}} J \wedge J \wedge c_1(L_m) = 0,$$

to be satisfied for all n_i, m. If so, a theorem by Uhlenbeck-Yau guarantees a unique solution provided each bundle is $stable$.

Tadpole cancellation

Tadpole cancellation

- The Bianchi identity for $H = dB - \frac{\alpha'}{4}(\omega_Y - \omega_L)$,

$$dH = \frac{\alpha'}{4}\left(\mathrm{tr}(R^2) - \mathrm{tr}(F_1^2) - \mathrm{tr}(F_2^2)\right),$$

imposes the so-called tadpole condition in cohomology

$$\sum_{i=1}^{K} \mathrm{ch}_2(V_{n_i}) + \sum_{m=1}^{M} a_m\, c_1^2(L_m) = -c_2(T).$$

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- The structure group G of the bundle W has to be embedded into $E_8 \times E_8$. The observable gauge group in four dimensions H is the commutant of G in $E_8 \times E_8$.

Massless spectrum

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- The massless spectrum is determined by various cohomology classes

$$H^*(\mathcal{M},\mathcal{W}) \qquad \mathcal{W} = \bigotimes_{i=1}^{K} \bigwedge^{p_i} V_{n_i} \otimes \bigotimes_{m=1}^{M} L_m^{q_m},$$

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  where the charges p_i and q_m can be derived from the explicit embedding of the structure group into $E_8 \times E_8$.
- The net-number of chiral matter multiplets is given by the Euler characteristic of the respective bundle $\mathcal{W}$

$$\begin{aligned} \chi(\mathcal{M},\mathcal{W}) &= \sum_{i=0}^{3} (-1)^i \dim(H^i(\mathcal{M},\mathcal{W})) \\ &= \int_{\mathcal{M}} \left[\mathrm{ch}_3(\mathcal{W}) + \frac{1}{12} c_2(T)\, c_1(\mathcal{W}) \right]. \end{aligned}$$

The Green-Schwarz mechanism

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- All non-abelian cubic gauge anomalies do cancel, whereas the mixed abelian-nonabelian, the mixed abelian-gravitational and the cubic abelian ones do not:

$$
\begin{aligned}
A_{U(1)_m - SU(N)^2} &\sim f_m \mathrm{tr} F_1^2 \int_{\mathcal{M}} \overline{f}^m \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2 \right), \\
A_{U(1)_m - G^2_{\mu\nu}} &\sim f_m \mathrm{tr} R^2 \int_{\mathcal{M}} \overline{f}^m \left(\mathrm{tr}\overline{F}_1^2 - \frac{5}{12}\mathrm{tr}\overline{R}^2 \right) \\
A_{U(1)_{mnp}} &\sim f_m f_n f_p \left[\int_{\mathcal{M}} \delta_{np} \overline{f}^m \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2 \right) \right. \\
&\qquad \left. + c_{mnp} \overline{f}^m \overline{f}^n \overline{f}^p \right]
\end{aligned}
$$

Need to be cancelled by a generalized Green-Schwarz mechanism.

The Green-Schwarz mechanism

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Dimensionally reducing the 10D 1-loop counter term

$$S_{GS} = \frac{1}{48\,(2\pi)^5\,\alpha'} \int B \wedge X_8, \qquad X_8 = \frac{1}{4}\left(\mathrm{tr} F_1^2\right)^2 + \cdots$$

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leads to vertex couplings for the internal axions

$$S_{GS} \;=\; \frac{1}{64\,(2\pi)} \int_{\mathbb{R}_{1,3}} \sum_{k=1}^{h_{11}} \left(b_k^{(0)}\,\mathrm{tr} F_1^2\right)\ \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right)_k$$

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and mass terms for the external (universal) axions:

$$S^0_{mass} \;=\; \sum_{m=1}^{M} \frac{\mathcal{Q}_0^m}{2\pi\alpha'} \int_{\mathbb{R}_{1,3}} \left(b_0^{(2)} \wedge f_m\right),$$

$$\mathcal{Q}_0^m \;=\; \frac{\mathrm{tr}_{E_8}(Q_m^2)}{32\,(2\pi)^4} \int_{\mathcal{M}} \overline{f}^m \wedge \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right),$$

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$$S_{\mathrm{mass}} = \sum_{k=1}^{h_{11}} \sum_{m=1}^{M} \frac{\mathcal{Q}_k^m}{2\pi\alpha'} \int_{\mathbb{R}_{1,3}} \left(b_k^{(2)} \wedge f_m \right),$$

$$\mathcal{Q}_k^m = \frac{\mathrm{tr}_{E_8}(Q_m^2)}{4\pi} \overline{f}_k^m$$

Massive gauge fields + axions

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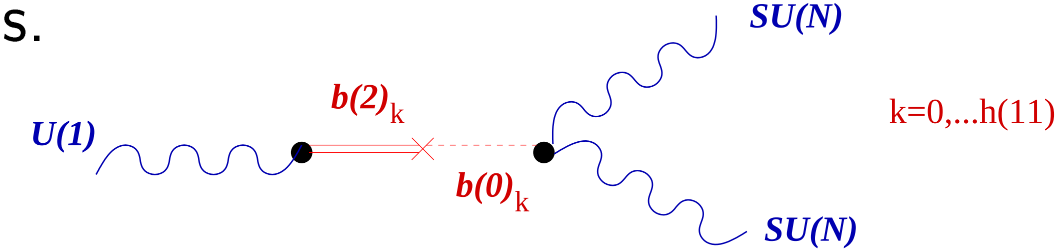
- GS diagrams precisely cancel the mixed abelian anomalies.



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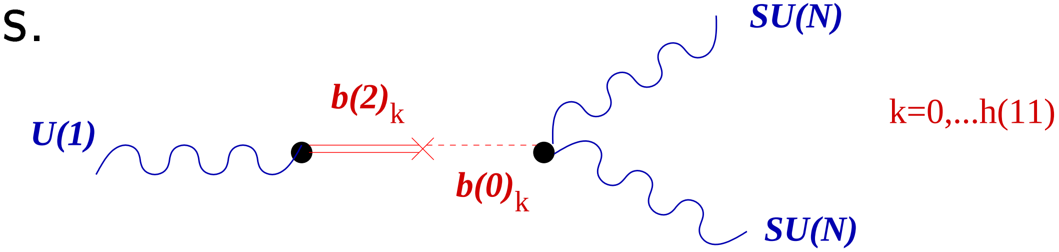

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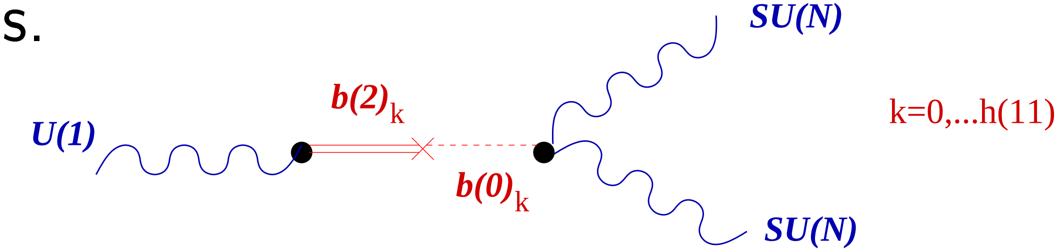

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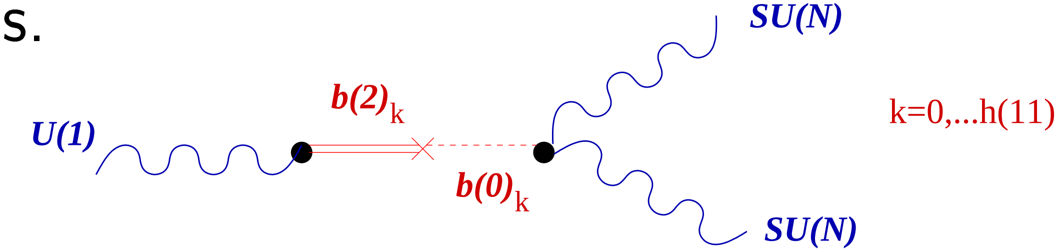

- The # massive $U(1)$ gauge fields is given by $\mathrm{rank}(\mathcal{Q})$.
- All mass terms are of the same order in both string and sigma model perturbation theory.
- Massive axions $b_k^{(0)}$ are longitudinal modes of U(1)s. Since axions complexify the Kähler moduli and the dilaton, supersymmetry dictates that the same # is massive.
  Only source for mass terms is the DUY equation.

Gauge kinetic function

Gauge kinetic function

- Defining the complexified dilaton

$$S = \frac{1}{2\pi}\left[e^{-2\phi_{10}}\frac{\mathrm{Vol}(\mathcal{M})}{\ell_s^6} + i\, b_0^{(0)}\right].$$

and the complexified Kähler moduli

$$T_k = \frac{1}{2\pi}\left[-\alpha_k + i b_k^{(0)}\right],$$

the GS terms induce the axionic part of the gauge kinetic function, e.g.

$$S_{kin} = \frac{1}{8\pi}\int_{\mathbb{R}_{1,3}} b_0^{(0)} \wedge \mathrm{tr} F_1^2$$

Gauge kinetic function

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- The 1-loop corrected gauge kinetic function for the non-abelian gauge fields are

$$f = S + \frac{1}{16} \sum_k T_k \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2} \mathrm{tr}\overline{R}^2 \right)_k .$$

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- For the abelian ones we find

$$\begin{aligned} f_{mn} &= \mathrm{tr}_{E_8}(Q_m^2)\,\delta_{mn} \times \\ &\left(S + \frac{1}{16}\sum_k T_k\left[\left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right)_k\right) \\ &+ \frac{\mathrm{tr}_{E_8}(Q_m^2)\mathrm{tr}_{E_8}(Q_n^2)}{12}\sum_{i,j} T_k\, d_{ijk}\overline{f}_i^m\overline{f}_j^n \end{aligned}$$

Fayet-Iliopoulos terms

Fayet-Iliopoulos terms

- Since we are dealing with anomalous $U(1)$ gauge factors, there are potential Fayet-Iliopoulos (FI) terms generated. Employing the standard supersymmetric field theory formula

$$\frac{\xi_m}{g_m^2} = \frac{\partial \mathcal{K}}{\partial V_m}\bigg|_{V=0},$$

  the FI parameters ξ_m can be computed from the Kähler potential $\mathcal{K}$, which in our case takes the following gauge invariant form.
  V_m denotes the vector superfields.

Fayet-Iliopoulos terms

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$$\mathcal{K} \sim -\ln\Big(S + S^* - \sum_m \mathcal{Q}_0^m V_m\Big) - \ln\Big(\sum_{i,j,k=1}^{h_{11}} \frac{d_{ijk}}{6}\big(T_i + T_i^* - \sum_m \mathcal{Q}_i^m V_m\big)\big(T_j + \cdots\big)\big(T_k + \cdots\big)\Big).$$

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For the FI term one gets

$$\frac{\xi_m}{g_m^2} = -\frac{1}{2\ell_s^6}\,\mathrm{tr}_{E_8}(Q_m^2)\Big[e^{-2\,\phi_{10}}\frac{1}{2}\int_{\mathcal{M}} J\wedge J\wedge \overline{f}_m - \frac{(2\pi\alpha')^2}{16}\int_{\mathcal{M}} \overline{f}_m \wedge \Big(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\Big)\Big].$$

Apparently, the first term arises at string tree-level, whereas the second term is a 1-loop term.

1-loop DUY equation

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- Interpretation: $\xi_m = 0$ contains a 1-loop correction to the DUY condition. In contrast to earlier claims that abelian gauge fluxes freeze some combinations of the Kähler moduli, we now realize that actually combinations of the dilaton and the Kähler moduli are frozen.

1-loop DUY equation

- Interpretation: $\xi_m = 0$ contains a 1-loop correction to the DUY condition. In contrast to earlier claims that abelian gauge fluxes freeze some combinations of the Kähler moduli, we now realize that actually combinations of the dilaton and the Kähler moduli are frozen.
- Support from Type I - HS duality. It is known that a D9-brane wrapping the Calabi-Yau $\mathcal{M}$ with Kähler form J^I and a $U(1)$ bundle with field strength F is supersymmetric if it satisfies the MMMS condition

$$\frac{1}{2}\int_{\mathcal{M}} J^I \wedge J^I \wedge F - \frac{(2\pi\alpha')^2}{3!}\int_{\mathcal{M}} F \wedge F \wedge F = 0$$

1-loop DUY equation

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The MMMS condition is at string tree level and the F^3 term is an α' correction. Applying the heterotic-Type I string duality relations

$$\begin{aligned} e^{\phi^I_{10}} &= e^{-\phi^H_{10}}, \\ J^I &= J^H \, e^{-\phi^H_{10}}, \end{aligned}$$

precisely leads to the form of the HS-DUY equation.

Non-abelian MMMS equation

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Similar analysis for the $SO(32)$ string leads after applying S-duality to the non-abelian generalization of the integrability condition for perturbative Π stability for $\varphi = 0$

$$\cos\varphi \left[\frac{1}{2}\int_{\mathcal{M}} J^2 \, \mathrm{tr}\overline{\mathcal{F}}_x - \frac{1}{3!}\int_{\mathcal{M}} \left(\mathrm{tr}\overline{\mathcal{F}}_x^3 - \frac{1}{16}\mathrm{tr}\overline{\mathcal{F}}_x \, \mathrm{tr}\overline{R}^2\right)\right]$$
$$+ \sin\varphi \left[\frac{N_x}{3!}\int_{\mathcal{M}} J^3 - \frac{1}{2}\int_{\mathcal{M}} J \, \left(\mathrm{tr}\overline{\mathcal{F}}_x^2 - \frac{N_x}{48}\mathrm{tr}\overline{R}^2\right)\right] = 0$$

This equation also includes the case of curved backgrounds.

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This equation also includes the case of curved backgrounds. The second line is obtained from gauge kinetic function.

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This equation also includes the case of curved backgrounds. The second line is obtained from gauge kinetic function. It can be compactly written as

$$\int_{\mathcal{M}} \mathrm{tr}_{U(N)} \left[\mathrm{Im}\left(e^{i\varphi}\, e^{J+i\mathcal{F}} \sqrt{\hat{A}(\mathcal{M})}\right)\right] = 0$$

Vector bundles via exact sequences

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- CY manifold $\mathcal{M}$ given by a complete intersection in some toric variety, e.g. $\mathbb{P}_4[5]$, and has $k = h_{11}$ Kähler parameters.
- A line bundle L on $\mathcal{M}$ is specified completely by its first Chern class which takes values in $H^2(\mathcal{M}, \mathbb{Z})$ and can be expanded in terms of a basis ω_i with $n_i \in \mathbb{Z}$

$$c_1(L) = \sum_{i=1}^{h_{11}} n_i \, \omega_i .$$

One also denotes such a line bundle as $\mathcal{O}(n_1, \ldots, n_k)$.

Vector bundles via exact sequences

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- Then a vector bundle V of rank r is defined by the cohomology of the monad $(i = 1, \ldots, h_{11})$

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- For $SU(N)$ bundles one has $c_1(V) = 0$, whereas for $U(N)$ bundles at first sight no condition.

Example: SU(4)× U(1) bundle

Example: SU(4)$\times$ U(1) bundle

- $E_8 \times E_8$ heterotic string compactified on a Calabi-Yau manifold $\mathcal{M}$ equipped with the specific class of bundles

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  with structure group $G = SU(4) \times U(1)$.
- Embedding this structure group into one of the E_8 factors leads to the breaking to gauge group $H = SU(5) \times U(1)$, where the adjoint of E_8 decomposes as follows into $G \times H$ representations.

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$SU(5)\times U(1)$	Cohomology
$\mathbf{10}_{-1}$	$H^*(\mathcal{M}, V\otimes L^{-1})$
$\mathbf{10}_4$	$H^*(\mathcal{M}, L^4)$
$\overline{\mathbf{5}}_3$	$H^*(\mathcal{M}, V\otimes L^3)$
$\overline{\mathbf{5}}_{-2}$	$H^*(\mathcal{M}, \bigwedge^2 V\otimes L^{-2})$
$\mathbf{1}_{-5}$	$H^*(\mathcal{M}, V\otimes L^{-5})$

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- Intersection form
$$I_3 = 5\,\eta^3$$
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$$c_2(T) = 10\,\eta^2.$$
- Choose a bundle of the form
$$W = V_1 \oplus V_2 \oplus L,$$
where the $SU(4)$ bundle V_1 and the line bundle L are embedded into the visible E_8 factor and the $SU(4)$ bundle V_2 into the hidden E_8 bundle.

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- Concretely, we define both vector bundles as the cohomology of the monad

$$0 \to \mathcal{O}|_{\mathcal{M}} \to \mathcal{O}(1)^{\oplus 5} \oplus O(3)|_{\mathcal{M}} \to \mathcal{O}(8)|_{\mathcal{M}} \to 0$$

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- Defining the Kähler form $J = \ell_s^2\, r\, \eta$, the DUY equation reads

$$r^2 = 10\, g_s^2\,,$$

  Choosing for instance $g_s = 0.3$ yields $r = 0.95$.

Massless spectrum

Massless spectrum

The chiral $SU(5) \times U(1)$ spectrum reads, where the anomalous $U(1)$ only survives as a global symmetry:

reps.	χ
$\mathbf{10}_{-1}$	290
$\mathbf{10}_{4}$	460
$\overline{\mathbf{5}}_{3}$	170
$\overline{\mathbf{5}}_{-2}$	580
$\mathbf{1}_{-5}$	-2150

Gauge symmetry enhancement

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- $(\mathbf{1}, \mathbf{10})_4$ is counted by $H^*(M, L^4)$. It enhances $SU(5) \times U(1) \to SO(10)$ for $L = 0$.

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- Whenever cohomology classes

$$H^*(M, \otimes_i L_i^{q_i})$$

are to be computed, non-abelian gauge symmetry enhancement occurs when

$$\sum_i q_i \, c_1(L_i) = 0.$$

Gauge symmetry enhancement

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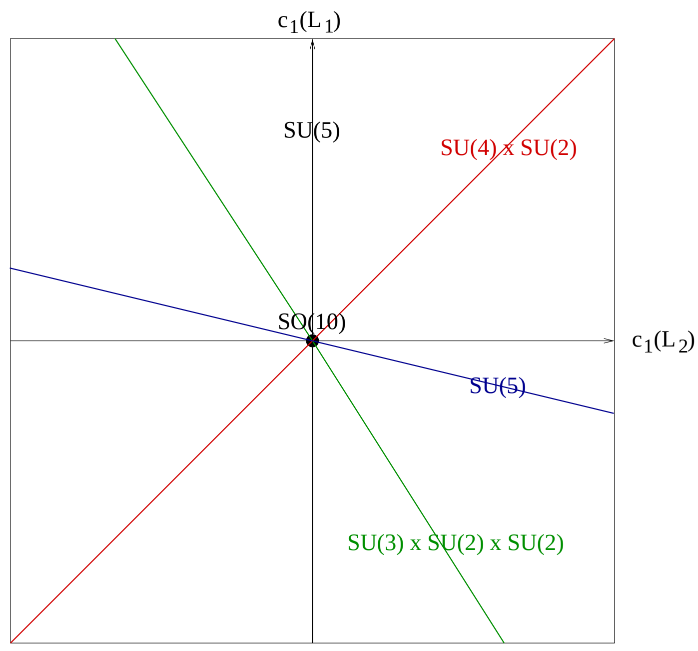
- For an $SU(4) \times U(1) \times U(1)$ bundle in E_8

$$W = V \oplus L_1 \oplus L_2$$

we obtained the following enhancements of $SU(3) \times SU(2)(\times U(1))$



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- As on the Type I side, one finds non-universal gauge threshold corrections.
- Does the non-abelian version of the MMMS equation help to uncover the non-abelian Born-Infeld action?
- What about the heterotic string landscape?