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Moduli stabilization
from
magnetic fluxes

with T. Maillard hep-th/0412008

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Outline

- Motivations

- Framework

Type I string theory with magnetized D9 branes

- T^6 example with O9 planes I.A-Maillard '04

- SUSY conditions

- tadpole cancellation

- positivity requirements

- explicit example

- infinite discrete class with large volume(s)

- T^6/\mathbb{Z}_2 orientifold with O3 planes

combined with 3-form fluxes

I.A-Kumar-Maillard '05

fix also the dilaton

Moduli stabilization with 3-form fluxes:
significant progress but

- no exact string description
low energy SUGRA approximation
- fix only complex structure

Type I with internal magnetic fluxes:
alternative/complementary approach

- exact string description
- Kähler class stabilization
 T^6 : all geometric moduli fixed
- natural implementation in intersecting
D-brane models

General framework

- Type I string theory compactified in 4d on 6d Calabi-Yau

⇒ $N = 2$ SUSY in the bulk, $N = 1$ on branes

- Magnetic fluxes on 2-cycles

⇒ SUSY breaking

Dirac quantization: $H = \frac{m}{nA} \equiv \frac{p}{A}$

H : constant magnetic field

m : units of magnetic flux / 1st Chern class

n : brane wrapping

A : area of the 2-cycle

Spin-dependent mass shifts for all charged states

$$[p_i, p_j] = iqH\epsilon_{ij} \quad q: \text{charge}$$

⇒ Landau spectrum

Exact open string description:

$$q \rightarrow q_L + q_R \quad \text{endpoint charges}$$

$$qH \rightarrow \theta_L + \theta_R \quad ; \quad \theta_{L,R} = \arctan q_{L,R} H \alpha'$$

weak field limit \Rightarrow field theory

T-dual representation:

$$R_2 \quad H = \frac{m}{n} \frac{1}{R_1 R_2} \quad R_2 \rightarrow \alpha' / R_2 \equiv \tilde{R}_2 \Rightarrow$$

$$R_1$$

$$\tilde{R}_2 \quad m \quad H = \frac{m}{n} \frac{\tilde{R}_2}{R_1} = \tan \theta$$

$$R_1 \quad n$$

m, n : wrapping numbers

Magnetic fluxes can be used to stabilize moduli

I.A.-Maillard '04

e.g. T^6 : 36 moduli (geometric deformations)

internal metric: $6 \times 7/2 = 21 = 9 + 2 \times 6$

type IIB RR 2-form: $6 \times 5/2 = 15 = 9 + 2 \times 3$

complexification \Rightarrow $\left\{ \begin{array}{l} \text{Kähler class } J \\ \text{complex structure } \tau \end{array} \right.$

9 complex moduli for each

magnetic flux: 6×6 antisymmetric matrix F

complexification \Rightarrow

$F_{(2,0)}$ on holomorphic 2-cycles: potential for τ

$F_{(1,1)}$ on mixed (1,1)-cycles: potential for J

T^6 parametrization/complexification

$$x^i \equiv x^i + 1 \quad y_i \equiv y_i + 1 \quad i = 1, 2, 3$$

orientation choice: $\int_{T^6} \prod_{i=1}^3 (dx^i \wedge dy_i) = 1$

$$z^i = x^i + \tau^{ij} y_j \quad \tau: \text{ complex structure matrix}$$

$$\delta g_{i\bar{j}} : \text{ Kähler deformations} \rightarrow J = \delta g_{i\bar{j}} i dz_i \wedge d\bar{z}_j$$

ω_r : real basis of 2-cycles

$H^2(T^6)$ cohomology

$$\Rightarrow \text{ signs: } \mathcal{K}_{rst} = \int_{T^6} \omega_r \wedge \omega_s \wedge \omega_t$$

$N = 1$ SUSY conditions:

$$(1) F_{(2,0)} = 0 \Rightarrow \tau \tau^\top p_{xx} \tau - (\tau^\top p_{xy} + p_{yx} \tau) + p_{yy} = 0$$

$$(2) J \wedge J \wedge F_{(1,1)} = F_{(1,1)} \wedge F_{(1,1)} \wedge F_{(1,1)} \Rightarrow J$$

Appropriate choice of magnetic fluxes F^a

in several abelian directions $U(1)_a \Rightarrow$

all moduli vanish except the 6 radii of T^6

which are fixed in terms of the quantized fluxes

$$T^6 = \prod_{I=1}^3 T_I^2 \leftarrow \text{orthogonal 2-torus}$$

$$\tau_I = \frac{R_I}{R'_I} \quad J_I = R_I R'_I \quad H_I^a = \frac{F_I^a}{J_I}$$

(1) fixes the ratios τ_I

(2) fixes the sizes J_I

$$H_1 + H_2 + H_3 = H_1 H_2 H_3 \Leftrightarrow \theta_1 + \theta_2 + \theta_3 = 0$$

DBI action:

$$\{[J \wedge J \wedge J - J \wedge F \wedge F]^2 + [J \wedge J \wedge F - F \wedge F \wedge F]^2\}^{1/2}$$

becomes polynomial

$$\mathcal{S}_{\text{DBI}} = -T_9 \int \sqrt{g} (1 - |H_1 H_2| + |H_2 H_3| + |H_1 H_3|)$$

or permutation of the minus sign

I.A.-Maillard, Bianchi-Trevigne

Tadpole conditions

$$Q_9 = \sum_a N_a \sum_{rst} \mathcal{K}_{rst} n_r^a n_s^a n_t^a = 16 \leftarrow \text{O9 charge}$$

sign due to orientation choice

$$Q_5 = \sum_a N_a \sum_{st} \mathcal{K}_{rst} n_r^a m_s^a m_t^a = 0 \quad \forall \text{ 2-cycle } r$$

- volume positivity: $J \wedge J \wedge J - J \wedge F^a \wedge F^a > 0$
- no antibranes: $\mathcal{K}_{rst} n_r^a n_s^a n_t^a > 0$

Main ingredients for moduli stabilization

1. “oblique” magnetic fields \Rightarrow
fix off-diagonal components of the metric
2. magnetized D9’s \Rightarrow -ve 5-brane tension
3. Non linear DBI action \Rightarrow fix overall volume

(1)+(2) : necessary for Q_5 cancellation

(2)+(3) : not valid in six dimensions

Stabilization of RR moduli

- Kähler class: absorbed by massive $U(1)$'s

$$dC_2 \wedge \star(A^a \wedge F^a) \Rightarrow$$

4d G-S kinetic mixing with magnetized $U(1)$'s

\Rightarrow need at least 9 brane stacks

- Complex structure: get potential through mixing with NS moduli

Bianchi-Trevigne '05

Stack #	Fluxes	Fixed complex structure	5 – brane local.	Fixed Kähler class
#1	$(F_{x_1 y_2}^1, F_{x_2 y_1}^1)$	$\tau_{31} = \tau_{32} = 0$ $p_{x_1 y_2}^1 \tau_{11} = \tau_{22} p_{x_2 y_1}^1$	$[x_3, y_3]$	$V_{1\bar{2}} - V_{2\bar{1}} = 0$
#2	$(F_{x_1 y_3}^2, F_{x_3 y_1}^2)$	$\tau_{21} = \tau_{23} = 0$ $p_{x_1 y_3}^2 \tau_{11} = \tau_{33} p_{x_3 y_1}^2$	$[x_2, y_2]$	$V_{1\bar{3}} - V_{3\bar{1}} = 0$
#3	$(F_{x_1 x_2}^3, F_{y_1 y_2}^3)$	$\tau_{13} = 0$ $\tau_{11} \tau_{22} = -\frac{p_{y_1 y_2}^3}{p_{x_1 x_2}^3}$	$[x_3, y_3]$	$V_{1\bar{2}} + V_{2\bar{1}} = 0$
#4	$(F_{x_2 x_3}^4, F_{y_2 y_3}^4)$	$\tau_{12} = 0$	$[x_1, y_1]$	$V_{2\bar{3}} + V_{3\bar{2}} = 0$
#5	$(F_{x_1 x_3}^5, F_{y_1 y_3}^5)$		$[x_2, y_2]$	$V_{1\bar{3}} + V_{3\bar{1}} = 0$
#6	$(F_{x_2 y_3}^6, F_{x_3 y_2}^6)$		$[x_1, y_1]$	$V_{2\bar{3}} - V_{3\bar{2}} = 0$

$$V_{i\bar{j}} = (J \wedge J)_{i\bar{j}} = 0 \Rightarrow J_{i\bar{j}} = 0 \quad \forall i \neq j$$

4th column: 5-brane charge localization on the 2-cycles $[x_i, y_i]$

Compatibility condition: p^4, p^5, p^6 constrained by p^1, p^2, p^3 , so that τ_{ii} remain unchanged

Fix areas of the 3 T^2 's \Rightarrow add 3 more stacks:

Stack #	Fluxes	5 – brane localization
#7	$(F_{x_1y_1}^7, F_{x_2y_2}^7, F_{x_3y_3}^7)$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$
#8	$(F_{x_1y_1}^8, F_{x_2y_2}^7, F_{x_3y_3}^8)$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$
#9	$(F_{x_1y_1}^9, F_{x_2y_2}^9, F_{x_3y_3}^9)$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$

$$\Rightarrow \begin{pmatrix} F_1^7 & F_2^7 & F_3^7 \\ F_1^8 & F_2^8 & F_3^8 \\ F_1^9 & F_2^9 & F_3^9 \end{pmatrix} \begin{pmatrix} J_2 J_3 \\ J_1 J_3 \\ J_1 J_2 \end{pmatrix} = \begin{pmatrix} F_1^7 F_2^7 F_3^7 \\ F_1^8 F_2^8 F_3^8 \\ F_1^9 F_2^9 F_3^9 \end{pmatrix}$$

Consistency conditions and tadpole cancellation are ok

• large volume:

- rescale $F_1^{7,8,9} \rightarrow \Lambda F_1^{7,8,9} \Rightarrow J_1 \rightarrow \Lambda J_1$: one large T^2

tadpoles are satisfied by appropriate rescaling of F^{1-6}

- rescale all fluxes and all J_I : three large T^2

Geometric moduli: fixed without breaking
gauge symmetries

- more magnetic fields:

in general incompatible unless special fluxes or

turn on open string moduli

charged states modifying the D-terms

⇒ - fixed at non trivial values

- break gauge symmetries

- fixing the dilaton?

combine magnetic and 3-form fluxes

lose the exact string description

3-brane charge ⇒ T^6/\mathbb{Z}_2 with O3 planes

I.A.-Kumar-Maillard '05

T-duality to T^6/\mathbb{Z}_2

magnetic flux $m \leftrightarrow$ winding number n

- non-trivial because of 0-eigenvalues ($m = 0$)
- weak string coupling \Rightarrow minimize
the 3-brane charge of magnetized branes
- new infinite discrete series
with the same values for all moduli

Models

- use 3-form fluxes to fix only the dilaton
compatible with the values of T^6 moduli
- part of moduli are fixed by magnetic
and part by 3-form fluxes

$N = 1$ SUSY condition for Kähler moduli:


$$J \wedge J \wedge J = J \wedge F_{(1,1)} \wedge F_{(1,1)} \Rightarrow$$

$$\mathcal{S}_{\text{DBI}} = -T_9 \int \sqrt{g} (|H_1 H_2 H_3| - |H_1| + |H_2| + |H_3|)$$

or permutation of the minus sign

Tadpole conditions

$$Q_3 = \sum_a N_a \sum_{rst} \mathcal{K}_{rst} m_r^a m_s^a m_t^a = 16 - N_3$$

3-form fluxes 

$$Q_7 = \sum_a N_a \sum_{st} \mathcal{K}_{rst} m_r^a n_s^a n_t^a = 0 \quad \forall \text{ 2-cycle } r$$

- volume positivity: $F^a \wedge F^a \wedge F^a - J \wedge J \wedge F^a > 0$
- no antibranes: $\mathcal{K}_{rst} m_r^a m_s^a m_t^a > 0$

$$\text{0-eigenvalue} \Rightarrow \mathcal{K}_{rst} n_r^a m_s^a m_t^a > 0$$

Example

- branes 1-6: - add 2 diagonal fluxes in each
- min 3-brane charge = 6

⇒ fix the complex structure in a diagonal form

- branes 7-9: take out 1 diag flux from each
→ vanishing 3-brane charge

⇒ Kähler form diagonal and fixed

- particular rescaling of diag. fluxes in all branes
rescaled 7-brane charge of 1-6 cancel with 7-9

⇒ new ∞ series with the same moduli values

- 3-form flux compatible with above values \Rightarrow

$$N_3 = hf \qquad \frac{1}{g_s} = (\text{Im}\tau_{33}) \frac{f}{h}$$

↗ ↖
NS-NS R-R

$g_s \text{ min} \Rightarrow h \text{ min} \quad f \text{ max allowed by } N_3$

non-trivial $B \Rightarrow h_{\text{min}} = 1 \quad g_s = \frac{1}{4\text{Im}\tau_{33}}$

- special rescalings \Rightarrow large radii + small g_s