

Compactifications on Generalized Geometries

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Introduction

String Theory: strings moving in 10-dimensional space-time background M_{10}
 contact with particle physics: compactification

$$M_{10} = M_4 \times Y_6$$

M_4 : 4 dim. space-time, Y_6 : Calabi-Yau threefold

Problems:

- mechanism of supersymmetry breaking
- stabilization of moduli

Recently:

- Standard Model on D-branes
- supersymmetry breaking/moduli stabilization with background fluxes
alternatively: compactifications on generalized class of manifolds

Purpose of this talk:

- ⇒ discuss string compactifications on manifolds with $SU(3)$ -structure
- ⇒ compute Kähler potential and potential

Compactification: determine Y_6

Lorentz group on space-time background $M_{10} = M_4 \times Y_6$ decomposes

$$SO(1,9) \rightarrow SO(1,3) \times SO(6)$$

spinor decomposes accordingly: $\mathbf{16} \rightarrow (\mathbf{2}, \mathbf{4}) \oplus (\bar{\mathbf{2}}, \bar{\mathbf{4}})$

phenomenologically most interesting case: minimal amount of supersymmetry

\Rightarrow impose two conditions:

1. demand that supercharge Q exist \Rightarrow structure group of Y_6 has to be reduced

$$SO(6) \rightarrow SU(3) \quad \text{s.t.} \quad \mathbf{4} \rightarrow \mathbf{3} + \mathbf{1}$$

\Rightarrow invariant spinor η exists $\Rightarrow Y_6$ has $SU(3)$ -structure

2. background preserves supersymmetry

$$\delta\Psi_M = \nabla_M \eta + (\gamma \cdot F)_M = 0, \quad F = \text{background flux}$$

- for $F = 0$: $\nabla_M \eta = 0 \Rightarrow Y_6$ is Calabi-Yau manifold Y
- here: study $\nabla_M \eta \neq 0$

Manifolds with $SU(3)$ structure: characterized by two invariant tensors J, Ω
 (follows from existence of invariant spinor η)

⇨ two-form

$$J_{mn} = \eta^\dagger \gamma_{[m} \gamma_{n]} \eta, \quad dJ \neq 0$$

⇒ almost complex structure

$$I_m{}^n = J_{mp} g^{pn}, \quad I^2 = -1, \quad N(I) \neq 0$$

J is (1, 1)-form with respect to I

⇨ (3, 0)-form

$$\Omega_{mnp} = \eta^\dagger \gamma_{[m} \gamma_n \gamma_p] \eta, \quad d\Omega \neq 0$$

Remarks:

- $dJ, d\Omega \sim$ torsion of Y_6
- Calabi-Yau: $\nabla\eta = 0 \Rightarrow dJ = d\Omega = N(I) = 0$

Low energy effective action: [Strominger; Bodner,Cadavid,Ferrara; Candelas,de la Ossa]

$$\begin{aligned}
 S_{\text{NS}} &= \int d^{10}x \sqrt{g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12}H^2 \right] \\
 &= \int d^{10}x \sqrt{g^{(4)}} \left[R^{(4)} - 2(\partial\phi^{(4)})^2 - \frac{1}{12}e^{-4\phi^{(4)}} H_{(4)}^2 \right. \\
 &\quad \left. - \frac{1}{4}G^{mp}G^{nq} (\partial_\mu G_{mn} \partial^\mu G_{pq} + \partial_\mu B_{mn} \partial^\mu B_{pq}) + \dots \right]
 \end{aligned}$$

where $G_{\mu\nu}^{(4)} = e^{-2\phi_4} G_{\mu\nu}$, $\phi^{(4)} = \phi - \frac{1}{4} \ln \det G_{mn}$

interpret **last term** as metric on the space of metric/ B -field-deformations and decompose

$$\delta G_{mn} = [\delta G_{mn}]_{\mathbf{8}} + [\delta G_{mn}]_{\mathbf{6}+\bar{\mathbf{6}}} = \delta J + \delta\Omega + \delta\bar{\Omega}$$

put S into σ -model form

$$S = - \int_{M_4} d^4x \sqrt{g^{(4)}} g_{ab}(z) \partial_\mu z^a \partial^\mu z^b + \dots$$

Scalar fields of compactification: deformations of metric/B-field
 \simeq deformations of J, Ω

σ -model metric in 4d \mathcal{L}_{eff} : product of special geometries [Grana,Waldram,JL]

$$\mathcal{M} = \mathcal{M}_J \times \mathcal{M}_\Omega$$

with

$$g_{ab} = \partial_a \partial_b (K_J + K_\Omega), \quad e^{-K_J} = \int_{Y_6} J \wedge J \wedge J, \quad e^{-K_\Omega} = \int_{Y_6} \Omega \wedge \bar{\Omega}$$

Remarks:

- ⇨ same expression as in Calabi-Yau compactifications
 but with non-harmonic modes contributing \Rightarrow Hitchin functional
- ⇨ geometry can already be discovered in $D = 10$
 (i.e. including full tower of KK-states [de Wit, Nicolai])
- ⇨ type II: add RR-sector [Cecotti,Ferrara,Girardello; Ferrara,Sabharwal]

$$\mathcal{M}_{J/\Omega} \subset \mathcal{M}_H^{QK} \quad (\text{c-map})$$

Compute potential from supersymmetry transformation of gravitino [Grana,Waldram,JL]

type II: $N = 2$ in $D = 4$

$$\delta\psi_{A\mu} = D_\mu\varepsilon_A + i\gamma_\mu S_{AB}\varepsilon^B + \dots, \quad S_{AB} = \frac{i}{2} e^{\frac{1}{2}K_V} \vec{\sigma}_{AB} \vec{P}, \quad A = 1, 2$$

IIA:

\vec{P} = Killing prepotentials

$$P^1 + iP^2 = e^{\frac{1}{2}K_\Omega + \phi^{(4)}} \int_{Y_6} e^{-(B+iJ)} \wedge d\Omega, \quad P^3 = e^{2\phi^{(4)}} \int_{Y_6} e^{-(B+iJ)} \wedge F_A$$

IIB

$F \equiv \sum_{\text{RR-forms}} F^{\text{RR}}$

$$P^1 - iP^2 = e^{\frac{1}{2}K_J + \phi^{(4)}} \int_{Y_6} \Omega \wedge de^{-(B+iJ)}, \quad P^3 = -e^{2\phi^{(4)}} \int_{Y_6} \Omega \wedge F_B$$

Remarks:

- potential: $V = V(\vec{P}, D\vec{P})$
- torsion $d\Omega$, dJ appear in \vec{P} but not in K
- compute F , D -terms of $N = 1$ by further truncation

[Andrianopoli,D'Auria,Ferrara,Trigiante,Vaula; Grimm,JL]

or directly from the heterotic string

Heterotic string: $N = 1$ in $D = 4$

$$\delta\psi_\mu = D_\mu\varepsilon + i\gamma_\mu e^{K/2} W \varepsilon + \dots, \quad W = \text{superpotential}$$

$$W = \int_{Y_6} \Omega \wedge de^{-(B+iJ)},$$

[Cardoso, Curio, Dall'Agata, Lüst, Manousselis, Zoupanos; Becker, Becker, Dasgupta, Green; Gurrieri, Lukas, Micu; Benmachiche, JL]

Expand in basis of forms

[D'Auria, Ferrara, Trigiante, Vaula; Grana, Waldram, JL]

$$B + iJ = t^i \omega_2^i, \quad \Omega = z^A \alpha_3^A - F_A(z) \beta_3^A$$

ω_2^i basis of 2-forms, (α_3^A, β_3^A) basis of 3-forms

with

$$d\omega_2^i = m_A^i \alpha^A - e_A^i \beta^A, \quad H_3 = m_A^0 \alpha^A - e_A^0 \beta^A,$$

$$\text{torsion: } m_A^i, e_A^i, \quad \text{H-flux: } m_A^0, e_A^0$$

$$\Rightarrow W = z^A e_A^I t_I + t_I m_A^I F_A, \quad t_I = (t_0, t_i)$$

[cf. Berglund, Mayr]

Summary

- ⇨ compactifications on manifolds with $SU(3)$ structure
 - ⇒ product of special geometries
 - ⇒ $K = K_J + K_\Omega$ is independent on torsion
- ⇨ potential depends on torsion and background fluxes

- ⇨ phenomenological questions:
 - stabilization of moduli
 - inclusion of D-branes
 - supersymmetry breaking