June 16, 2005

SOME MODULI SUPERPOTENTIALS FROM SUPERGRAVITY GAUGINGS

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Superstring phenomenology,
München, June 13-18, 2005

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hep-th/0411276/ Nucl.Phys. B715 (2005) 211,
(hep-th/0503229), and to appear

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- Most string compactifications generate massless moduli.
- These states are welcome to: generate scales required by phenomenology, participate in supersymmetry breaking, adjust the vacuum energy.
- The number of these states should be minimal, they should all acquire masses once the theory has settled in its "low-energy" ground state.
- In any case, the fate of moduli is related to the generation of a scalar potential, a superpotential (and D-terms) in N=1 theories

- The analysis of configurations with fluxes is commonly performed using two different approaches:

  1) Solving D=10 (D=11) string equations ("top-down")

  2) Using an effective four-dimensional Lagrangian approach ("bottom-up")

- The effective supergravity analysis has an obvious advantage: simplicity, it mostly relies upon identifying the symmetries of the compactified theory.
  It can suggest interesting models to study in D=10, 11.
  On the basis of our study: an accurate and reliable method to study string solutions.

Contents:

- The compactification setup;
- Basics of N=4, D=4 supergravity;
- N=4 gaugings and N=1 superpotentials;
- Geometric fluxes;
- Gaugings and flux parameters: a typical example;
- The case of the IIA orientifold theory.

i: THE COMPACTIFICATION SETUP

To compactify to four dimensions, use an orbifold breaking 3/4 supercharges (same as Calabi-Yau).

Several possible choices, leading to different moduli structures:

$Z_2 \times Z_2$ (six geometric moduli)

$Z_3, Z_4, Z_6, Z_6', Z_7, \ldots$.. (9, 6, 4, 3, 3 geometric moduli)

In heterotic strings, leads to N=1, D=4 supersymmetry
In type II strings, needs an orientifold projection to get to N=1, D=4 supersymmetry.

The $Z_2 \times Z_2$ orbifold splits the six-dimensional internal spaces into three complex planes, with a natural plane-interchange symmetry [an SO(3) symmetry of the N=4 supergravity].

Closely similar to Calabi-Yau [for which however symmetry is SU(3)].

Type II superstrings:

Choose an orientifold compatible with the orbifold point group:

Coordinate	$(Z_2)_1$	$(Z_2)_2$	IIA/D_6	IIB/D_9	IIB/D_3
x^5	$-$	$+$	$+$	$+$	$-$
x^6	$-$	$+$	$-$	$+$	$-$
x^7	$-$	$-$	$+$	$+$	$-$
x^8	$-$	$-$	$-$	$+$	$-$
x^9	$+$	$-$	$+$	$+$	$-$
x^{10}	$+$	$-$	$-$	$+$	$-$

Fluxes may or may not respect the plane-interchange symmetry.

In this talk, I'll consider in examples this orbifold only and in general fluxes respecting the plane-interchange symmetry because:

- a simple, complete analysis of combined fluxes feasible for all strings;
- a quite general set of moduli (Kähler and complex classes) treated in a symmetric way, useful to study stabilization.

ii: BASICS OF N=4, D=4 SUPERGRAVITY

- 16 supercharges, maximal supersymmetry with gauge/matter multiplets.
- A unique scalar sigma-model structure:

$$\frac{SU(1,1)}{U(1)} \times \frac{SO(6,n)}{SO(6)\times SO(n)}$$

- Supergravity multiplet: 6 vector fields, supergravity dilaton in SU(1,1)/U(1).
- n vector multiplets, each with 6 real scalars.
- Explicit S-duality from SU(1,1)/U(1).
- Its parameters are the structure constants of the algebra gauged by the 6+*n* vector fields:

the choice of the GAUGING

- The gauging encodes the 16 supercharge BPS mass formula

The orbifold/orientifold breaks N=4, D=4 to N=1, D=4 supersymmetry with sigma-model:

$$M_{Z_2\times Z_2} = \frac{SU(1,1)}{U(1)} \times \prod_{A=1}^{3} \frac{SO(2,2+n_A)}{SO(2)\times SO(2+n_A)}$$

and, with moduli only (n_A=0), since

$$\frac{SO(2,2)}{SO(2)\times SO(2)} = \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)}$$

the N=1 scalar manifold becomes simply:

$$\left(\frac{SU(1,1)}{U(1)}\right)_S \times \prod_{A=1}^{3} \left(\frac{SU(1,1)}{U(1)}\right)_{T_A} \times \prod_{A=1}^{3} \left(\frac{SU(1,1)}{U(1)}\right)_{U_A}$$

The Kähler potential is fully defined by solutions to the N=4 scalar field constraints.

iii: N=4 GAUGINGS AND N=1 SUPERPOTENTIALS

The standard construction of N=4 supergravity starts from the superconformal theory, gauge-fixed to the Poincaré theory by imposing appropriate *Poincaré constraints*.

(de Roo; Bergshoeff, Koh, Sezgin; de Roo, Wagemans; ...)

In the standard construction, the gauging is defined at the superconformal level, *before* solving the Poincaré constraints.

The solution of the Poincaré constraints, which defines the fields of N=4 Poincaré supersymmetry, is then chosen in agreement with the gauging.

We need the opposite: our moduli are defined from a specific solution of the Poincaré constraints (from our compactification setup), *independent* of the choice of a gauging which will correspond to a choice of fluxes.

We must then fit the gauging to a predefined solution to the Poincaré constraints.

Retaining geometric moduli only, the N=4 theory has 12 vector fields, and a global SO(6,6) symmetry broken in general by the gauging of a twelve-dimensional algebra.

The gauging introduces structure constants verifying Jacobi identities:

$$[T_A, T_B] = f_{AB}{}^C T_C$$

Consistency of the N=4 theory requires antisymmetry of

$$\tilde{f}_{ABC} = f_{AB}{}^D \, \eta_{DC}$$

where η_{DC} is the *SO(6,6)* metric (vector representation).
(This is in general not the Cartan metric of the gauge algebra).

A consistent N=4 gauging verifies then two kinds of conditions, Jacobi and antisymmetry.
These are conditions on admissible flux values.

But there is another important ingredient, *duality phases*.

Gauge kinetic terms of N=4 supergravity read:

$$\mathcal{L}_{gauge} = -\frac{1}{4}\sum_{R,S} \eta_{RS}\, S_{\delta_R} F^{R-}_{\mu\nu} F^{\mu\nu\, S-} + \text{h.c.} + \ldots$$

$$F^{R\pm}_{\mu\nu} = F^{R}_{\mu\nu} \pm i\widetilde{F}^{R}_{\mu\nu}$$

$$S_{\delta_R} = \frac{\cos\delta_R\, S - i\sin\delta_R}{-i\sin\delta_R\, S + \cos\delta_R}$$

These duality phases are free parameters provided they respect the (non-abelian) gauge invariance of the theory.
Their introduction follows from the embedding of SU(1,1) duality in the full duality algebra of the theory.
Discretization leads then to:

$$\delta_R = 0 \;\leftrightarrow\; S_{\delta_R} = S \qquad \text{and} \qquad \delta_R = \pi/2 \;\leftrightarrow\; S_{\delta_R} = 1/S$$

There are then two sectors, "perturbative" in S, and "non-perturbative" in S.

The scalar sector:

(Will be used to find the N=1 Kähler potential and the superpotential).

Supergravity dilaton Kähler potential:

$$K(S, \overline{S}) = -\ln\,(S + \overline{S})$$

Vector multiplet scalar fields [*6* of *SU(4)*]:

$$\phi^R_{ij} = -\phi^R_{ji} = \frac{1}{2}\epsilon_{ijkl}\phi^{kl\,R}\,, \quad \phi^{ij\,R} = (\phi^R_{ij})^* \quad (i, j, \ldots = 1, \ldots, 4,\ R = 1, \ldots, 6{+}n)$$

N=4 auxiliary field equation:

$$\eta_{RS}\,\phi^R_{ij}\phi^{kl\,S} = \frac{1}{12}\left(\delta^k_i\delta^l_j - \delta^l_i\delta^k_j\right)\eta_{RS}\,\phi^R_{mn}\phi^{mn\,S}$$

Poincaré constraint:

$$\eta_{RS}\,\phi^R_{ij}\phi^{ij\,S} \equiv \phi^R_{ij}\phi^{ij}_R = -6$$

Orbifold truncation to N=1 supergravity and multiplets:

Introduce new truncated fields (one N=1 complex scalar for each N=4 multiplet):

$$\sigma_A^1,\ \sigma_A^2,\ \rho_A^1,\ \rho_A^2,\ \chi_A^I, \qquad\qquad A=1,2,3, \qquad I=1,\ldots,n_A$$

N=1 constraints:

$$\begin{aligned} |\sigma_A^1|^2+|\sigma_A^2|^2-|\rho_A^1|^2-|\rho_A^2|^2-\textstyle\sum_I|\chi_A^I|^2 &= 1/2\,, \\ (\sigma_A^1)^2+(\sigma_A^2)^2-(\rho_A^1)^2-(\rho_A^2)^2-\textstyle\sum_I(\chi_A^I)^2 &= 0\,, \end{aligned}$$

These conditions for SO(2,2)/SO(2)xSO(2) are SO(3)-invariant.

Rotation invariance on the 3 complex planes,
"Plane-interchange" symmetry.

Solution: Poincaré moduli are then:

$$\sigma_A^1 = \frac{1}{2}\,\frac{1 + T_A U_A - (Z_A^I)^2}{[Y(T_A, U_A, Z_A^I)]^{1/2}}\,, \qquad \sigma_A^2 = \frac{i}{2}\,\frac{T_A + U_A}{[Y(T_A, U_A, Z_A^I)]^{1/2}}\,,$$

$$\rho_A^1 = \frac{1}{2}\,\frac{1 - T_A U_A + (Z_A^I)^2}{[Y(T_A, U_A, Z_A^I)]^{1/2}}\,, \qquad \rho_A^2 = \frac{i}{2}\,\frac{T_A - U_A}{[Y(T_A, U_A, Z_A^I)]^{1/2}}\,,$$

$$\chi_A^I = i\,\frac{Z_A^I}{[Y(T_A, U_A, Z_A^I)]^{1/2}}\,.$$

The quantities Y will appear in the N=1 Kähler potential:

$$Y(T, U, Z^I) = (T + \overline{T})(U + \overline{U}) - \sum_I (Z^I + \overline{Z}^I)^2$$

$$K = -\ln(S + \overline{S}) - \sum_{A=1}^{3} \ln Y(T_A, U_A, Z_A^I)$$

With (geometric) moduli only (independent of the gauging !):

$$K = -\ln(S + \overline{S}) - \sum_{A=1}^{3} \ln(T_A + \overline{T}_A) - \sum_{A=1}^{3} \ln(U_A + \overline{U}_A)$$

Next step is to introduce the gauging and derive the N=1 superpotential.

As usual, the superpotential is most conveniently obtained from the gravitino mass terms in the N=4 supergravity Lagrangian.

After truncation of these terms to N=1, the superpotential reads:

$$W = \frac{4}{3}\sqrt{2}\,[\cos\delta_R - i\sin\delta_R S]\left[\prod_{A=1}^{3} Y(T_A, U_A, Z_A^I)\right]^{1/2} f_{RST}\,\epsilon_{ABC}\,\phi^{R\,A}\phi^{S\,B}\phi^{T\,C}$$

[the Y factors separate the (non-holomorphic) e^K contribution from the holomorphic superpotential]

Finally: once these N=1 formula established, solve the consistency constraints

(Jacobi + antisymmetry of gauging structure constants)

For twelve gauge fields, there are in principle 220 antisymmetric structure constants.

The truncation to N=1 leaves $4^3 = 64$ parameters in the superpotential. Plus the duality phases.

The superpotential is then a polynomial up to order seven in the moduli. Each term is of order 0 or 1 in each modulus.

With plane interchange symmetry: twenty parameters, plus duality phases. And Jacobi identities become "CYCLICITY EQUATIONS":

$$f_{Aa_1\,Bb_2\,Cc_3} = \Lambda_{a_1 b_2 c_3}\,\epsilon_{ABC}\,, \qquad (a_1, b_2, c_3 = 1, \ldots, 4)$$

$$f_{Aa_1\,Bb_2}{}^{Cc_3} = \Lambda_{a_1 b_2}{}^{c_3}\,\epsilon_{ABC}\,, \qquad \Lambda_{a_1 b_2}{}^{c_3} = \eta^{c_3 d_3}\Lambda_{a_1 b_2 d_3}$$

$$\eta^{df}\Lambda_{abd}\Lambda_{cfe} = \eta^{df}\Lambda_{bcd}\Lambda_{afe} = \eta^{df}\Lambda_{cad}\Lambda_{bfe}\,, \qquad \forall a, b, c, e$$

iv: GEOMETRIC FLUXES

Besides the familiar NS-NS (or heterotic) 3-form H3 and the R-R forms F3, F0, F2, F4, F6 flux sources leading to moduli scalar potentials, we may as well use the Scherk-Schwarz mechanism.

The Scherk-Schwarz mechanism also produces fluxes, the GEOMETRIC FLUXES.

Schematically, it corresponds to introduce a gauging (flux parameters) in the (internal) spin connection:

$$\begin{aligned}
\omega_M{}^{AB} \;=\; & -\tfrac{1}{2}\left(\partial_M e_N^A - \partial_N e_M^A - f^P{}_{MN} e_P^A\right) e^{NB} \\
& +\tfrac{1}{2}\left(\partial_M e_N^B - \partial_N e_M^B - f^P{}_{MN} e_P^B\right) e^{NA} \\
& -\tfrac{1}{2} e^{PA} e^{QB} \left(\partial_P e_Q^C - \partial_Q e_P^C - f^R{}_{PQ} e_R^C\right) e_{MC}
\end{aligned}$$

This introduces (internal) torsion in the theory:

$$S^P_{MN} = \frac{1}{2} f^P{}_{MN}$$

[See also: Andrianopoli, Lledo, Trigiante, 0502083; Ferrara's talk]

Introducing geometric fluxes is necessary in a general study.
Superpotentials terms with more moduli do appear in general.
[It may be also necessary to undertand string duality correspondances with non-trivial fluxes.]

Consistency conditions apply. They are very plausibly less restrictive than in the original treatment (global conditions on all fluxes).

Some work has to be done ...

In the case of the orbifold Z2 x Z2:

Heterotic theory: geometric fluxes generate a superpotential

[linear in T-moduli] x [a polynomial in U-moduli (up to order 3)]

All the T's and U's may be stabilized.

IIB / D3 theory: no geometric flux allowed, the superpotential does not depend on T-moduli.

IIB / D9 (type I) theory: the same as heterotic strings.

IIA / D6 orientifold: geometric fluxes generate a quadratic superpotential with all terms of the form:

[one of the T's] x [S or one of the U's]

[more on this later on]

v: GAUGING AND FLUX PARAMETERS: A TYPICAL EXAMPLE

Consider the six-dimensional algebra *SO(4)* and *SO(1,3)*:

$$SO(4) \sim SU(2) \times SU(2) \qquad SO(1,3) \sim Sl(2,C)$$

In the standard basis, the Lie algebras are:

$$[M_i, M_j] = \epsilon_{ijk} M_k, \quad [M_i, \hat{M}_j] = \epsilon_{ijk} \hat{M}_k, \quad [\hat{M}_i, \hat{M}_j] = \pm \epsilon_{ijk} M_k$$

Since we have solved the Poincaré constraints in a specific basis (which preserves the *SO(3)* structure), we may want to rewrite the algebra in a general basis:

$$\begin{aligned} N_i &= aM_i + b\hat{M}_i, & M_i &= \delta^{-1}(dN_i - b\hat{N}_i), \\ \hat{N}_i &= cM_i + d\hat{M}_i, & \hat{M}_i &= \delta^{-1}(-cN_i + a\hat{N}_i), \end{aligned} \qquad \delta = ad - bc$$

This introduces four parameters.

To use this algebra as a N=4 gauging, the "*gauging structure constants*" should be antisymmetric:

$$\tilde{f}_{IJL} = f_{IJ}{}^{K}\eta_{KL} \qquad \eta = \frac{1}{2}\begin{pmatrix} 0_6 & I_6 \\ I_6 & 0_6 \end{pmatrix}$$

Result: one condition on the four parameters: $\quad ac = \pm bd$

$$f_{i_S j_S k_S} = \pm\epsilon_{ijk} b^2/c, \qquad f_{i_S j_S k_A} = \epsilon_{ijk} a,$$

$$f_{i_S j_A k_A} = \epsilon_{ijk} c, \qquad f_{i_A j_A k_A} = \pm\epsilon_{ijk} d^2/a.$$

We have then introduced three independent gauging parameters / fluxes with:

$$f_{i_S j_S k_S} f_{i_A j_A k_A} = f_{i_S j_S k_A} f_{i_S j_A k_A}$$

$f_{i_S j_S k_S}$ and $f_{i_S j_A k_A}$: have identical / opposite signs in the case of SO(4) / SO(1,3).

In addition, there are duality phases, one for each independent gauge-invariant kinetic term (two in this example)

- Introducing different gauge coupling constants effectively modifies the SO(6,n) metric, not the algebra.

- It then affects the gauging structure constants (dependence on the gauge coupling)

The Frey-Polchinski example: IIB/D3 with H3 and F3 fluxes:

$$W = (f_1+ih_1S)[1+U_1U_2+U_2U_3+U_3U_1]-(if_2-h_2S)[U_1+U_2+U_3+U_1U_2U_3].$$

Gauging constants verify the above conditions:

$$\Lambda_{111} = f_1 + ih_1S, \qquad \Lambda_{114} = -f_2 - ih_2S,$$
$$\Lambda_{144} = -f_1 - ih_1S, \qquad \Lambda_{444} = f_2 + ih_2S.$$

vi: THE CASE OF THE IIA ORIENTIFOLD THEORY

Type IIA strings has the following antisymmetric tensors (curvatures):

- NS-NS three-form H_3
- R-R even forms: F_0, F_2, F_4, F_6

All of them have components surviving the orientifold and orbifold projections. The same is true for components of:

- Geometric fluxes (spin connection)

These terms lead to the following superpotential contributions:

Geometric fluxes:

$$
\begin{array}{ccc}
C_{679} = C_{895} = C_{1057} \equiv \Lambda'_{112} & \leftrightarrow & -S\ (T_1 + T_2 + T_3) \\
C_{6810} = C_{8106} = C_{1068} \equiv \Lambda_{113} & \leftrightarrow & (T_1\, U_1 + T_2\, U_2 + T_3\, U_3) \\
C_{896} = C_{1067} = C_{689} = & & -(T_1 U_2 + T_1 U_3 + T_2 U_1 \\
C_{1058} = C_{6710} = C_{8105} \equiv \Lambda_{124} & \leftrightarrow & +T_2 U_3 + T_3 U_1 + T_3 U_2)
\end{array}
$$

H3 (NS-NS) fluxes:

$$H_{579} = \Lambda'_{111} \quad \leftrightarrow \quad i\ S$$
$$H_{689} = H_{6710} = H_{5810} = \Lambda_{114} \quad \leftrightarrow \quad i\ (U_1 + U_2 + U_3)$$

F0 (R-R) fluxes: (the mass parameter of massive IIA supergravity, dual to the flux of a nine-form potential)

$$F_0 = \Lambda_{222} \leftrightarrow -i\ (T_1\, T_2\, T_3)$$

F2 (R-R) fluxes:

$$F_{56} = F_{78} = F_{910} = \Lambda_{122} \leftrightarrow -(T_1\, T_2 + T_1\, T_3 + T_2\, T_3)$$

F4 (R-R) fluxes:

$$F_{5678} = F_{78010} = F_{56910} = \Lambda_{112} \leftrightarrow i\ (T_1 + T_2 + T_3)$$

F6 (R-R) fluxes:

$$F_6 = \Lambda_{111} \leftrightarrow 1$$

Combined IIA fluxes are then:

$$(\Lambda'_{111}, \Lambda'_{112}) \quad \text{with } SU(1,1) \text{ phase factor } iS$$
$$(\Lambda_{111}, \Lambda_{112}, \Lambda_{122}, \Lambda_{222}, \Lambda_{113}, \Lambda_{114}, \Lambda_{124}) \quad \text{with } SU(1,1) \text{ phase } 1$$

And the consistency cyclicity equations reduce to:

$$\Lambda_{222}\,\Lambda_{114} + \Lambda_{113}\,\Lambda_{122} = 2\,\Lambda_{122}\,\Lambda_{124}\,, \qquad \Lambda_{113}\Lambda_{124} = \Lambda_{124}^2$$

Superpotential:

$$\begin{aligned} W \;=\;& \Lambda_{111} + i\,\Lambda'_{111}\,S + i\,\Lambda_{112}\,(T_1+T_2+T_3) - \Lambda'_{112}\,S\,(T_1+T_2+T_3) \\ & + i\,\Lambda_{114}\,(U_1+U_2+U_3) + \Lambda_{113}\,(T_1U_1+T_2U_2+T_3U_3) \\ & - \Lambda_{122}\,(T_1T_2+T_1T_3+T_2T_3) \\ & - \Lambda_{124}\,(T_1U_2+T_1U_3+T_2U_1+T_2U_3+T_3U_1+T_3U_2) - i\,\Lambda_{222}\,T_1T_2T_3 \end{aligned}$$

All seven moduli appear, contrary to heterotic or IIB strings:
the richest phenomenology !

SOME SELECTED EXAMPLES

The N=1 supergravity potential for the seven complex moduli $z_i = S, T_A, U_A$ reads in general:

$$e^{-K} V = \sum_i |W - W_i (z_i + \overline{z}_i)|^2 - 3|W|^2$$

The first seven terms are the supergravity auxiliary fields contributions.

In case the superpotential does not depend on a modulus, its auxiliary field may cancel one of the three negative contributions.

In IIB/D3 superstrings, the potential is always semi-positive definite since it depends at most on four moduli.

In heterotic or IIB/D9 superstrings, up to six moduli can be stabilized.

In IIA superstrings, many physical situations occur.

Gaugings with V < 0, stabilization of all moduli:

A non trivial example with all possible IIA fluxes switched on.

$$-\frac{1}{9}\Lambda_{111} = -\frac{1}{2}\Lambda'_{112} = \frac{1}{6}\Lambda_{113} = \Lambda_{122} = A$$

$$\frac{1}{2}\Lambda'_{111} = -\frac{1}{3}\Lambda_{112} = \frac{1}{2}\Lambda_{114} = -\frac{1}{5}\Lambda_{222} = B$$

Consistency (cyclicity conditions) requires: $6\,A^2 = 10\,B^2$

Superpotential:

$$\begin{aligned} W &= A\,[2\,S\,(T_1+T_2+T_3) - (T_1T_2+T_2T_3+T_3T_1) + 6\,(T_1U_1+T_2U_2+T_3U_3) - 9] \\ &\quad + i\,B\,[2\,S + 5\,T_1T_2T_3 + 2\,(U_1+U_2+U_3) - 3\,(T_1+T_2+T_3)] \end{aligned}$$

Cancelling the seven auxiliary fields stabilizes all seven moduli.

The superpotential is not zero at this point.

We then have a vacuum with AdS_4, N=1 supersymmetry.

All axions except one (used as a Stückelberg field) are stabilized as well. [Camara, Font, Ibanez, 0506066]

Flat gaugings, no-scale models: stabilization of four moduli

Switch on geometric, NS-NS three-form, F0 and F2 fluxes:

(no consistency condition on A, B, C, D)

$$(\omega_3, H_3, F_0, F_2): \qquad \Lambda'_{112} = -A\,, \qquad \Lambda_{122} = -AB\,,$$

$$\Lambda'_{111} = C\,, \qquad \Lambda_{222} = -CD\,,$$

Superpotential:

$$W = A\Big[S(T_1+T_2+T_3) + B(T_1T_2+T_2T_3+T_1T_3)\Big] + iC\Big[S + D\,T_1T_2T_3\Big]$$

Scalar potential, is semi-positive definite:

$$e^{-K}V = |W - (S+\overline{S})W_S|^2 + \sum_{A=1}^{3} |W - (T_A + \overline{T}_A)W_{T_A}|^2$$

Four moduli stabilize, V vanishes at this point, supersymmetry breaks, a no-scale model.

$$\langle S\rangle = B\,T\,, \qquad \langle T_1\rangle = \langle T_2\rangle = \langle T_3\rangle = T\,, \qquad T = \sqrt{\frac{B}{D}}$$

The supergravity gauging approach to string and moduli superpotentials proves efficient and accurate.

The sixteen supercharge N=4 algebra knows quite a lot of string theory . . .