

Flux Compactification of Type IIB Supergravity

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Based work done with: M. Cvetič and P. Gao

- 1) Introduction
- 2) Fluxes in type IIA supergravity
- 4) Fluxes in type IIB supergravity
- 4) Conclusion

Introduction

Major problem in string compactifications:

emergence of moduli space of string vacua

moduli appear in two guises:

- (i) closed string moduli: deformations of internal cycles
- (ii) open string moduli: wrapped branes not rigid

standard model of particle physics & (inflationary) cosmology

→ These moduli have to be fixed !

If supersymmetry is broken only at fairly low energies

need fixing mechanism that preserves
some supersymmetry

So far : only fluxes seem to provide mechanism
for lifting *both* moduli while preserving some susy

fluxes = field strength of RR and NS forms
that are non-zero in the vacuum

(i) fluxes: expand (shrink) parallel (perpendicular) cycles

→ fix closed string (geometrical) moduli

But often: run-away behavior

(ii) they couple to Born-Infeld action → generate potential

→ can also fix open string moduli

they are compact - potential has always extrema

But also : metric fluxes (twisted tori e.g.)

(...Hull, Ferrara, Dall'Agata, D'Auria, Derendinger, Kounas, Zwirner)

all fluxes :

related to generalized Scherk-Schwarz reductions

these reductions are especially interesting, because

Scherk-Schwarz reductions yield consistent truncation
on massless KK spectrum

In addition: → related to gauged supergravity

→ can calculate the potential explicitly

and hence:

- KK spectrum
- number of vacua
- de Sitter extrema

However: gauged supergravity does not yield the deformed (internal) geometry

lift to 10 dimension in most cases not clear !

To get new (internal) geometry

solve directly the 10-d equations

- uncovers embedding of the fluxes
- solution is classified by torsion classes

or the set of globally defined differential forms (G-structures)

remark: branes are charged \rightarrow fluxes give embedding of branes so that back reaction of the branes on geometry is under control

branes vs. fluxes? - branes = sources in eom or Bianchi identity
- fluxes “near horizon” limit of branes

unfortunately: explicit solutions available only for special cases

→ on IIB side for specific fluxes (3-form fluxes)

→ in massive IIA – nearly Kahler spaces

(→ next section)

BUT on type IIA: fluxes have severe back reaction on geometry !

on type IIB : back reaction often preserve CY-property of
internal space

after finding the spaces:

ask (the mathematicians) for moduli spaces
related to this geometry

AND : Is there still a landscape of vacua
coming from fluxes on non-CY spaces ?

Our results: - we solved the Killing spinor equations explicitly
for internal spinor: SU(3) singlet (ie. SU(3) structure)

(i) type IIA supergravity

- all (geometric) moduli fixed
- fixed dilaton : ratio of fluxes
- chiral matter (gauge group $U(k)^3$)
- internal space cannot be CY !

(ii) type IIB supergravity

- internal space complex
- solution is given by one free holomorphic function
- cosmological constant is zero (flat vacuum)
- axion/dilaton is non-holomorphic (not constant!)

known solutions : only 3-form flux or for flow type solutions

recall : from sugra we can expect to **fix all moduli** only
if **all (even and odd) fluxes are non-zero**

Fluxes in (massive) type IIA supergravity

Literature: Polchinski, Strominger, Minasian, Kaste, Tomasiello, Gauntlett, Martelli
Lüst/Tsimpis, Kachru, Kashani-Poor, Derendinger, Kounas, Petropoulos, Zwirner

bosonic fields :

common sector (e, B, ϕ)

RR-sector **type IIA** (C_1, C_3)

$$F^{(2)} = mB + dC_1, \quad F^{(4)} = dC_3 + \frac{6}{m} F^{(2)} \wedge F^{(2)}$$
$$dF^{(2)} = mH, \quad dF^{(4)} = 12H \wedge F^{(2)}$$

$$0 = \delta \psi_M = \left[D_M + \frac{1}{8} H_M \Gamma_{11} + \frac{1}{8} e^\phi (m \Gamma_M + F^{(2)} \Gamma_M \Gamma_{11} + F^{(4)} \Gamma_M) \right] \epsilon$$

$$0 = \delta \lambda = \left[\partial \phi + \frac{1}{12} H \Gamma_{11} + \frac{1}{4} e^\phi (5m + 3 F^{(2)} \Gamma_{11} + F^{(4)}) \right] \epsilon$$

Explicitly known solution: with all form fields non-zero

K.B., M. Cvetič / Lüst, Tsimpis

internal space : nearly Kähler e.g. S^6 , CP_3 , $\frac{SU(3)}{U(1) \otimes U(1)}$, $S_3 \otimes S_3$

Einstein spaces with : $dJ \sim \text{Im } \Omega$, $d\Omega \sim J \wedge J$

(strings on flag manifold see also Lüst ; Castellani; Zoupanos)

dilaton and volume fixed :

$$e^\phi = H / F_4 , \quad 1/R \sim m e^\phi$$

external space : anti de Sitter

$$\Lambda \sim -m^2 e^{2\phi}$$

cone over nearly Kähler space = G2 holonomy space (7d)

$$ds^2 = r^2 (-dt^2 + dx_1^2 + dx_2^2) + \frac{1}{(r \Lambda)^2} (dr^2 + r^2 d\Omega_6^{(NK)})$$

$$\varphi = r^2 dr \wedge J - r^3 \text{Im } \Omega \quad \text{with fluxes: } F \sim J \quad \text{and} \quad H \sim \Omega$$

→ No continuous deformation - all moduli fixed !

Moreover : to wrap D6-branes on supersymmetric 3-cycles

consider example

$$S^3 \otimes S^3$$

Acharya, Denef, Lambert

· three supersymmetric 3-cycles, “calibrated” with

$$\operatorname{Re} \Omega \Leftrightarrow \operatorname{Im} \Omega_0 = 0$$

can be decomposed under homology

$$\{(1,0), (0,1), (-1,-1)\} \quad \text{or} \quad \{(-1,0), (0,-1), (1,1)\}$$

Add up to zero in homology

→ wrapping equal # of D6-branes → no orientifolds required!

In local description : branes intersect in SU(3) angle

→ chiral matter at the intersection

Fluxes in type IIB supergravity

Literature: Michelson, Grana, Polchinski, Giddings, Kachru, Schulz, Trivedi, Tripathy, Deneff, Douglas, Louis, Micu, Dall'Agata, Frey, Pilch, Warner,

bosonic fields : RR potentials (C_0, C_2, C_4^+)

with field strengths : $G_3 = \frac{F_3 - T F_3^*}{\sqrt{1 - |T|^2}}$, $F_5 = dC_4 - \frac{1}{8}(C_2 \wedge dB)$

$$dG_3 = -(P - iQ) \wedge G_3, \quad dF_5 = -\frac{1}{8} d C_2 \wedge B$$

$$T = \frac{1 + i\tau}{1 - i\tau}, \quad \tau = C_0 + i e^\phi, \quad P = \frac{dT}{1 - |T|^2}, \quad Q = \frac{\text{Im}(T dT^*)}{1 - |T|^2}$$

$$0 = \delta \psi_M = \left(D_M - \frac{i}{2} Q_M + \frac{i}{480} F \Gamma_M \right) \epsilon - \frac{1}{96} (G \Gamma_M + 6 G_M) \epsilon^*$$

$$0 = \delta \lambda = iP \epsilon^* - \frac{i}{24} G \epsilon$$

notation : $G^{(3)} = G_{MNP}^{(3)} \Gamma^{MNP}$ etc.

ansatz for the bosonic fields (arbitrary function on internal coordinates)

$$\begin{aligned} ds^2 &= e^{-2V(y)} (g_{\mu\nu} dx^\mu dx^\nu + h_{mn} dy^m dy^n) \\ P &= P_m dy^m, \quad G = G_{mnp} dy^m \wedge dy^n \wedge dy^p \\ F &= 5 e^{-4V} \text{vol}_4 \wedge dZ + \text{Hodge-dual} \end{aligned}$$

external space : AdS $\Lambda < 0$ or Minkowski $\Lambda = 0$

ansatz for the fermions : $\epsilon = a \theta \otimes \eta + b^* \theta^* \otimes \eta^*$

With a single internal spinor – can define SU(3) structure

$$\eta \gamma_{mn} \eta^* = i e^{2\alpha} J_{mn}, \quad \eta \gamma_{mnp} \eta = i e^{2(\alpha+i\beta)} \Omega_{mnp}$$

Note: Killing spinor is singlet under structure group

Solving the Killing spinor equations:

external part
internal part

$$\delta \psi_\mu = 0 \quad , \quad \delta \lambda = 0$$
$$\delta \psi_m = 0$$

constraints on fluxes
differential eq. for spinor

A-type vacuum

$$\epsilon = a \left(\theta \otimes \eta + \theta^* \otimes \eta^* \right)$$

(Majorana spinor)

results: - only NS flux, all RR fields vanish
- common sector e.g. NS5-brane solution
(Strominger)

B-type vacuum

$$\epsilon = a \theta \otimes \eta$$

(Becker, Becker ... Grana, Polchinski, ..)

results: - holomorphic axion/dilaton e.g. D3 and D7-branes
- internal space is still Kaehler or Calabi-Yau.

General case: $\epsilon = a \theta \otimes \eta + b^* \theta^* \otimes \eta^*$

results:

$$W_1 = W_2 = 0 \quad , \quad \Lambda = 0$$

$$b + i a = e^{-V/2 + i\alpha}$$

- internal space: complex (can use compl. coordinates z^i)
- external space: flat Minkowski

3-form:

$$G = \frac{1}{4} e^{-2V} J \wedge (\cot \alpha P_i dz^i + \tan \alpha P_{\bar{i}} dz^{\bar{i}}) + G^{prim}$$

$$\text{with} \quad J \wedge G^{prim} = \Omega \wedge G^{prim} = \bar{\Omega} \wedge G^{prim} = 0$$

remaining fields can be written as :

$$Q = d\theta - \frac{d\tau}{2\tau_2} \quad , \quad P = i e^{2i\theta} \frac{d\tau}{2\tau_2} \quad , \quad G_3 = i \frac{e^{i\theta}}{\sqrt{\tau_2}} (dA_2 - \tau dB_2)$$

$$F_5 \sim dZ \wedge vol_4 + \text{Hodge dual}$$

with (τ, θ, Z) given by :

solution :

$$\begin{aligned}\tau &= c_0 + i e^{\phi_0} \frac{|f|^2 \cos 2\alpha}{f \sin^2 \alpha + \bar{f} \cos^2 \alpha} \\ e^{-4V} &= \frac{\operatorname{Re} f \sin^2 2\alpha}{4|f|^2 \cos 2\alpha} \\ Z &= \frac{|f|^2 \cos^2 2\alpha}{\operatorname{Re} f \sin 2\alpha} \\ \tan \theta &= -\frac{\operatorname{Im} f}{\operatorname{Re} f} \cos 2\alpha\end{aligned}$$

where: $f = \frac{\tilde{Z}}{\cos 2\alpha + i \tan \theta} = f(z^i) \qquad \tilde{Z} = Z \frac{\sin^2 2\alpha}{\cos 2\alpha}$

Remark: Supersymmetry leaves one function un-fixed !

→ fixed by equations of motion or Bianchi identities

here α ; other possibility: Z can be fixed by $dF_5 = \Delta Z = \frac{5i}{12} G \wedge \bar{G}$

Discussion

$$f = \text{constant}$$

Apart from trivial solution, includes the flow type solution

\leftrightarrow fields depend only on one real coordinate

for $G \wedge \bar{G} = 0$

Z harmonic \leftrightarrow is monoton (and singular!)

UV-regime \leftrightarrow poles of Z (or warp factor)

$\sin 2\alpha = 0 \rightarrow$ internal space becomes CY

geometry can be regular $\rightarrow AdS_5 \otimes S^5$

IR regime $Z \simeq 0$ or $\cos 2\alpha \simeq 0$

singular, NOT CY ! However, regular iff

$$G \wedge \bar{G} \neq 0$$

\rightarrow lower cut-off for warp factor

\rightarrow confinement

$$f \neq \text{constant}$$

Non-constant holomorphic function on compact space
has zeros and poles

zeros : $\sin 2\alpha \simeq 0$

- **B-type vacuum**: - Kaehler geometry (e.g. Calabi Yau)
- axion/dilaton is holomorphic
- prototype sugra solution: D3/D7-branes

poles : $\cos 2\alpha \simeq 0$

- **A-type vacuum**: - all RR fields are trivial
- prototype sugra solution: NS5-branes

Conclusion

to fix the moduli = one of the important issues in string theory
to get contact to particle physics and cosmology

fluxes ~ only possibility that preserves some supersymmetry (?)

results for SU(3) structures:

(i) type IIA supergravity

- explicit solution with all (geometric) moduli fixed
- chiral matter (gauge group $U(k)^3$)
- internal space cannot be CY !

(ii) type IIB supergravity

- internal space complex
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open questions: all moduli fixed, chiral matter ?