Flux Compactification of Type IIB Supergravity

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Based work done with: M. Cvetic and P. Gao

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Introduction

Major problem in string compactifications:

emergence of moduli space of string vacua

moduli appear in two guises:

(i) closed string moduli: deformations of internal cycles
(ii) open string moduli: wrapped branes not rigid

standard model of particle physics & (inflationary) cosmology

$\longrightarrow$ These moduli have to be fixed !

If supersymmetry is broken only at fairly low energies

need fixing mechanism that preserves
some supersymmetry

So far : only fluxes seem to provide mechanism
for lifting *both* moduli while preserving some susy

fluxes = field strength of RR and NS forms
that are non-zero in the vacuum

(i) fluxes: expand (shrink) parallel (perpendicular) cycles

$\rightarrow$ fix closed string (geometrical) moduli

But often: run-away behavior

(ii) they couple to Born-Infeld action $\rightarrow$ generate potential

$\rightarrow$ can also fix open string moduli

they are compact - potential has always extrema

But also : metric fluxes (twisted tori e.g.)

(...Hull, Ferrara, Dall'Agata, D'Auria, Derendinger, Kounas, Zwirner .....)

all fluxes :

related to generalized Scherk-Schwarz reductions

these reductions are especially interesting, because

Scherk-Schwarz reductions yield consistent truncation on massless KK spectrum

In addition: $\longrightarrow$ related to gauged supergravity

$\longrightarrow$ can calculate the potential explicitly

and hence:
- KK spectrum
- number of vacua
- de Sitter extrema

However: gauged supergravity does not yield the deformed (internal) geometry

lift to 10 dimension in most cases not clear !

To get new (internal) geometry

solve directly the 10-d equations

- uncovers embedding of the fluxes
- solution is classified by torsion classes

or the set of globally defined differential forms (G-structures)

<u>remark</u>: branes are charged $\longrightarrow$ fluxes give embedding of branes so that back reaction of the branes on geometry is under control

branes vs. fluxes? - branes = sources in eom or Bianchi identity
- fluxes "near horizon" limit of branes

unforturnately: explicit solutions available only for special cases

→ on IIB side for specific fluxes (3-form fluxes)

→ in massive IIA – nearly Kahler spaces

(→ next section)

BUT on type IIA: fluxes have severe back reaction on geometry !

on type IIB : back reaction often preserve CY-property of internal space

after finding the spaces:

ask (the mathematicians) for moduli spaces related to this geometry

AND : Is there still a landscape of vacua coming from fluxes on non-CY spaces ?

Our results: - we solved the Killing spinor equations explicitly

for internal spinor: SU(3) singlet (ie. SU(3) structure)

(i) type IIA supergravity

- all (geometric) moduli fixed
- fixed dilaton : ratio of fluxes
- chiral matter (gauge group $U(k)^3$)
- internal space cannot be CY !

(ii) type IIB supergravity

- internal space complex
- solution is given by one free holomorphic function
- cosmological constant is zero (flat vacuum)
- axion/dilaton is non-holomorphic (not constant!)

known solutions : only 3-form flux or for flow type solutions

recall : from sugra we can expect to fix all moduli only
if all (even and odd) fluxes are non-zero

Fluxes in (massive) type IIA supergravity

Literature: Polchinski, Strominger, Minasian ,Kaste, Tomasiello, Gauntlett, Martelli
Lüst/Tsimpis , Kachru , Kashani-Poor, Derendinger, Kounas, Petropoulos, Zwirner ....

bosonic fields :

common sector (e, B, ϕ)

RR-sector type IIA (C_1, C_3)

$$F^{(2)} = mB + dC_1 , \quad F^{(4)} = dC_3 + \frac{6}{m} F^{(2)} \wedge F^{(2)}$$

$$dF^{(2)} = m\,H , \quad dF^{(4)} = 12\, H \wedge F^{(2)}$$

$$0 = \delta \psi_M \;=\; [\; D_M + \frac{1}{8} H_M \Gamma_{11} + \frac{1}{8} e^{\phi} (m \Gamma_M + F^{(2)} \Gamma_M \Gamma_{11} + F^{(4)} \Gamma_M) \,] \epsilon$$

$$0 = \delta \lambda \;=\; [\partial \phi + \frac{1}{12} H \Gamma_{11} + \frac{1}{4} e^{\phi} (5m + 3 F^{(2)} \Gamma_{11} + F^{(4)}) \,] \, \epsilon$$

Explicitly known solution: with all form fields non-zero

K.B., M. Cvetic / Lüst, Tsimpis

internal space : nearly Kähler e.g. $S^6, \; CP_3, \; \frac{SU(3)}{U(1)\otimes U(1)}, \; S_3 \otimes S_3$

Einstein spaces with : $dJ \sim \operatorname{Im}\Omega \quad , \quad d\,\Omega \sim J \wedge J$

(strings on flag manifold see also Lüst ; Castellani; Zoupanos)

dilaton and volume fixed : $e^{\phi} = H / F_4 \; , \quad 1/R \sim m\, e^{\phi}$

external space : anti de Sitter $\Lambda \sim -m^2 e^{2\phi}$

cone over nearly Kähler space = G2 holonomy space (7d)

$$ds^2 = r^2(-dt^2 + dx_1^2 + dx_2^2) + \frac{1}{(r\Lambda)^2}(dr^2 + r^2 d\,\Omega_6^{(NK)})$$

$$\varphi = r^2 dr \wedge J - r^3 \operatorname{Im}\Omega \quad \text{with fluxes:} \quad F \sim J \quad \text{and} \quad H \sim \Omega$$

$\rightarrow$ No continuous deformation - all moduli fixed !

Moreover : to wrap D6-branes on supersymmetric 3-cycles

consider example $S^3 \otimes S^3$ Acharya, Denef, Lambert

. three supersymmetric 3-cycles, "calibrated" with

$$\mathrm{Re}\,\Omega \;\Leftrightarrow\; \mathrm{Im}\,\Omega_0 = 0$$

can be decomposed under homology

$$\{(1,0),(0,1),(-1,-1)\} \quad \text{or} \quad \{(-1,0),(0,-1),(1,1)\}$$

Add up to zero in homology

$\rightarrow$ wrapping equal # of D6-branes $\rightarrow$ no orientifolds required!

In local description : branes intersect in SU(3) angle

$\rightarrow$ chiral matter at the intersection

Fluxes in type IIB supergravity

Literature: Michelson, Grana, Polchinski, Giddings , Kachru, Schulz ,Trivedi, Tripathy, Denef, Douglas, Louis , Micu , Dall'Agata , Frey, Pilch, Warner, ...

bosonic fields : RR potentials (C_0, C_2, C_4^+)

with field strengths : $G_3 = \frac{F_3 - T F_3^*}{\sqrt{1-|T|^2}}$, $F_5 = dC_4 - \frac{1}{8}(C_2 \wedge dB)$

$$dG_3 = -(P - iQ) \wedge G_3, \quad dF_5 = -\frac{1}{8} d\, C_2 \wedge B$$

$$T = \frac{1+i\tau}{1-i\tau}, \quad \tau = C_0 + i e^{\phi}, \quad P = \frac{dT}{1-|T|^2}, \quad Q = \frac{\mathrm{Im}(T\, dT^*)}{1-|T|^2}$$

$$0 = \delta\psi_M = \left(D_M - \frac{i}{2} Q_M + \frac{i}{480} F \Gamma_M \right) \epsilon - \frac{1}{96} (G \Gamma_M + 6 G_M) \epsilon^*$$

$$0 = \delta\lambda = i P \epsilon^* - \frac{i}{24} G \epsilon$$

notation : $G^{(3)} = G^{(3)}_{MNP}\,\Gamma^{MNP}$ *etc.*

ansatz for the bosonic fields (arbitrary function on internal coordinates)

$$
\begin{aligned}
ds^2 &= e^{-2V(y)}\left(g_{\mu\nu} dx^\mu dx^\nu + h_{mn} dy^m dy^n \right) \\
P &= P_m dy^m \quad , \quad G = G_{mnp} dy^m \wedge dy^n \wedge dy^p \\
F &= 5\, e^{-4V} vol_4 \wedge dZ \; + \; Hodge-dual
\end{aligned}
$$

external space : AdS $\Lambda < 0$ or Minkowski $\Lambda = 0$

ansatz for the fermions : $\epsilon = a\ \theta \otimes \eta + b^*\ \theta^* \otimes \eta^*$

With a single internal spinor – can define SU(3) structure

$$\eta \gamma_{mn} \eta^* = i e^{2\alpha} J_{mn} \quad , \quad \eta \gamma_{mnp} \eta = i e^{2(\alpha + i\beta)} \Omega_{mnp}$$

Note: Killing spinor is singlet under structure group

Solving the Killing spinor equations:

external part	$\delta \psi_\mu = 0 \quad , \quad \delta \lambda = 0$	constraints on fluxes
internal part	$\delta \psi_m = 0$	differential eq. for spinor

A-type vacuum

$$\epsilon = a \left(\theta \otimes \eta + \theta^* \otimes \eta^* \right)$$

(Majorana spinor)

results: - only NS flux, all RR fields vanish
- common sector e.g. NS5-brane solution
(Strominger)

B-type vacuum

$$\epsilon = a \; \theta \otimes \eta$$

(Becker,Becker ... Grana, Polchinski, ..)

results: - holomorphic axion/dilaton e.g. D3 and D7-branes

- internal space is still Kaehler or Calabi-Yau.

General case: $\epsilon = a\ \theta \otimes \eta + b^*\ \theta^* \otimes \eta^*$

results: $W_1 = W_2 = 0\ ,\ \Lambda = 0$ $\quad b + i\,a = e^{-V/2 + i\alpha}$

- internal space: complex (can use compl. coordinates z^i)
- external space: flat Minkowski

3-form:

$$G = \frac{1}{4} e^{-2V} J \wedge \left(\cot\alpha\ P_i dz^i + \tan\alpha\, P_{\bar{i}} dz^{\bar{i}} \right) + G^{prim}$$

with $\quad J \wedge G^{prim} = \Omega \wedge G^{prim} = \bar{\Omega} \wedge G^{prim} = 0$

remaining fields can be written as :

$$Q = d\theta - \frac{d\tau}{2\tau_2}\ ,\quad P = i\,e^{2i\theta} \frac{d\tau}{2\tau_2}\ ,\quad G_3 = i\,\frac{e^{i\theta}}{\sqrt{\tau_2}} (d A_2 - \tau\, dB_2)$$

$$F_5 \sim dZ \wedge vol_4 + Hodge\,dual$$

with $(\tau\ ,\ \theta\ ,\ Z)$ given by :

solution :

$$\tau = c_0 + i e^{\phi_0} \frac{|f|^2 \cos 2\alpha}{f \sin^2\alpha + \bar{f}\cos^2\alpha}$$

$$e^{-4V} = \frac{\mathrm{Re}\, f}{4|f|^2} \frac{\sin^2 2\alpha}{\cos 2\alpha}$$

$$Z = \frac{|f^2|}{\mathrm{Re}\, f} \frac{\cos^2 2\alpha}{\sin 2\alpha}$$

$$\tan\theta = -\frac{\mathrm{Im}\, f}{\mathrm{Re}\, f} \cos 2\alpha$$

where: $f = \dfrac{\tilde{Z}}{\cos 2\alpha + i \tan\theta} = f(z^i)$ $\tilde{Z} = Z \dfrac{\sin^2 2\alpha}{\cos 2\alpha}$

Remark: Supersymmetry leaves one function un-fixed !

$\rightarrow$ fixed by equations of motion or Bianchi identities

here α ; other possibility: Z can be fixed by $dF_5 = \Delta Z = \dfrac{5i}{12} G \wedge \bar{G}$

Discussion

$f = constant$

Apart from trivial solution, includes the flow type solution

$\leftrightarrow$ fields depend only on one real coordinate

for $G \wedge \bar{G} = 0$

Z harmonic $\leftrightarrow$ is monoton (and singular!)

UV-regime $\leftrightarrow$ poles of Z (or warp factor)

$\sin 2\alpha = 0 \rightarrow$ internal space becomes CY

geometry can be regular $\rightarrow AdS_5 \otimes S^5$

IR regime $Z \simeq 0$ or $\cos 2\alpha \simeq 0$

singular, NOT CY ! However, regular iff

$G \wedge \bar{G} \neq 0$

$\rightarrow$ lower cut-off for warp factor

$\rightarrow$ confinement

$$f \neq constant$$

Non-constant holomorphic function on compact space has zeros and poles

zeros : $\sin 2\alpha \simeq 0$

→ B-type vacuum:
- Kaehler geometry (e.g. Calabi Yau)
- axion/dilaton is holomorphic
- prototype sugra solution: D3/D7-branes

poles : $\cos 2\alpha \simeq 0$

→ A-type vacuum:
- all RR fields are trivial
- prototype sugra solution: NS5-branes

Conclusion

to fix the moduli = one of the important issues in string theory

to get contact to particle physics and cosmology

fluxes ~ only possibility that preserves some supersymmetry (?)

results for SU(3) structures:

(i) type IIA supergravity

- explicit solution with all (geometric) moduli fixed
- chiral matter (gauge group $U(k)^3$)
- internal space cannot be CY !

(ii) type IIB supergravity

- internal space complex
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open questions: all moduli fixed, chiral matter ?