

# Volume Stabilization and Brane Inflation at One Loop

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Work in collaboration with  
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hep-th/0507nnn

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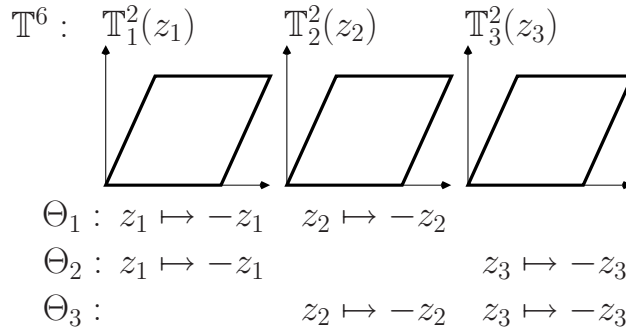
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## Outline

1. One-loop Kähler potential in a  $\mathcal{N} = 1$  orientifold
  - Orientifold  $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$  with D3/D7-branes
  - Classical moduli space and gauge couplings
  - One-loop string threshold corrections
  - Vertex operators for moduli fields
  - An  $\mathcal{N} = 2$  truncation: one-loop prepotential
2. Volume stabilization at one-loop
  - Moduli stabilization with fluxes and gaugino condensation
  - No-scale violation at one-loop
  - One-loop corrections to KKLT minima
3. Brane inflation at one-loop
  - D3/D7-brane inflation with shift symmetry
  - Violation of shift symmetry at one-loop
  - Slow rolling at one-loop

## Orientifold $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D3/D7-branes

- Relevant compactification: “IIB orientifold on  $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ ” [Berkooz, Leigh; and others]



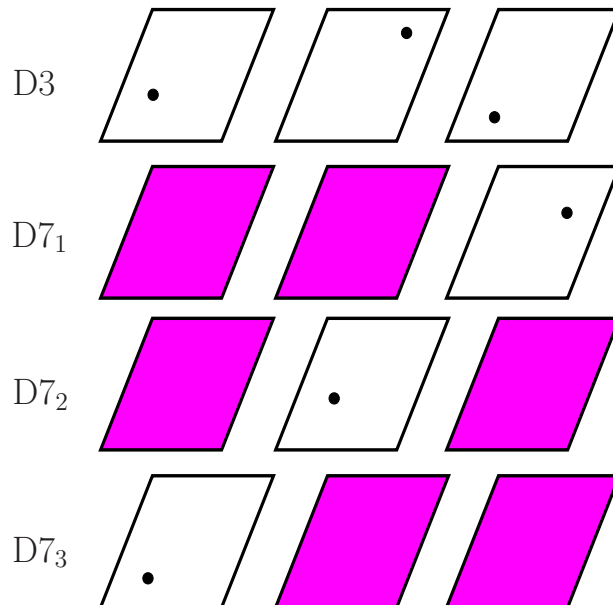
Identification under

orbifold group :  $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \Theta_1, \Theta_2, \Theta_3 = \Theta_1\Theta_2\}$

orientifold :  $\Omega R_6(-1)^{F_R}$ ,  $R_6 : z_I \mapsto -z_I$ ,  $I = 1, 2, 3$

Each element  $\Theta_I$  has a fixed 2-torus  $\mathbb{T}_I^2$ , transverse  $\mathbb{T}_I^4$ .

- Model has D3- and (three types of) D7-branes on  $\mathbb{T}_I^4$



## The classical moduli space

- Untwisted moduli which are considered

$$\text{dilaton} : \quad S \sim e^{-\Phi} + iC$$

$$\mathbb{T}_I^4/\mathbb{T}_I^2 : \quad S_I \sim e^{-\Phi} \text{vol}(\mathbb{T}_I^4) + iC_4|_{\mathbb{T}_I^4} + \dots$$

$$U_I \sim \text{cpl. str. of } \mathbb{T}_I^2$$

$$\text{positions of D3} : \quad A_I = U_I a_{2I} - a_{2I-1}$$

Moduli not considered are: moduli of D7, twisted moduli.

- Classical Kähler potential

$$\mathcal{K}^{(0)} = -\ln \left[ (S + \bar{S}) \left[ (S_I + \bar{S}_I)(U_I + \bar{U}_I) - \frac{1}{2}(A_I + \bar{A}_I)^2 \right] \right]$$

[Atoniadis,  
Bachas, Fabre,  
Partouche,  
Taylor]

for moduli space

$$\left[ \frac{SU(1,1)}{U(1)} \right]_S \times \prod_{I=1}^3 \left[ \frac{SO(3,2)}{SO(3) \times SO(2)} \right]_{S_I, U_I, A_I}$$

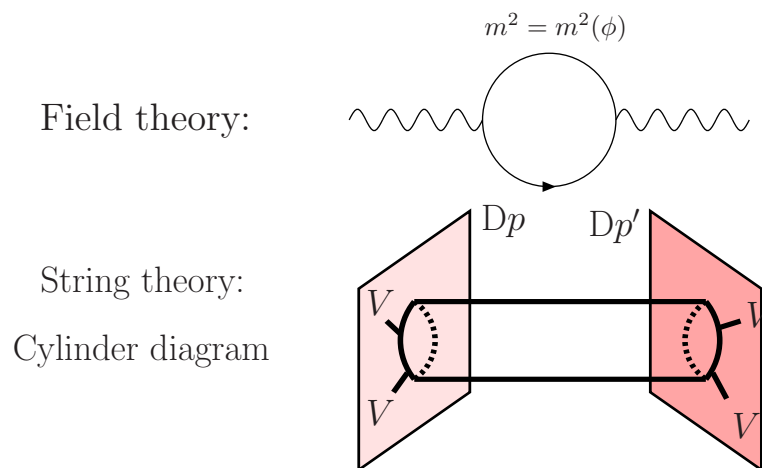
and classical gauge kinetic functions

$$f_{D3}^{(0)} = S, \quad f_{D7_I}^{(0)} = S_I$$

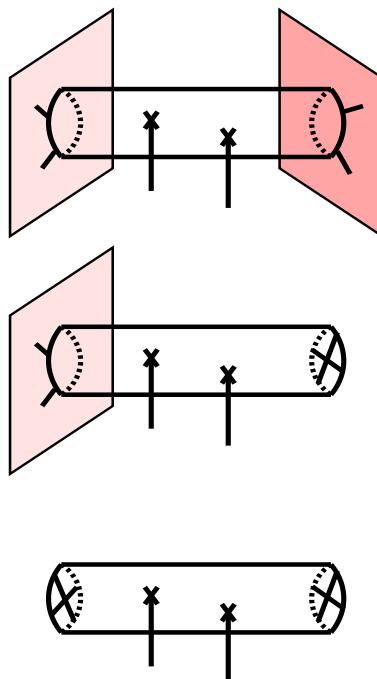
- This defines the  $\mathcal{N} = 1$  effective action (up to superpot.).

## One-loop string threshold corrections

- Threshold correction in string theory:



Also need to add Möbius strip, Klein bottle, torus diagrams. [\[Bachas, Fabre; Antoniadis, Bachas, Dudas; and others\]](#)



And sum over all boundaries: D3- and D7<sub>I</sub>-branes

## Technical remarks

- Important technicality: open/closed string vertex operators

$$\mathcal{S} = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2z \underbrace{G_{ij} \partial X^i \bar{\partial} X^j}_{\text{funct. of } S_I, U_I, A_I} + i \int_{\partial\Sigma} d\tau \underbrace{a_i \partial_{\tau} X^i}_{\text{funct. of } U_I, A_I}$$

Vertex operators of  $U_I, A_I$  have bulk plus boundary pieces!

- With this get “classical” metric from sphere+disc:

$$\begin{aligned} \mathcal{K}_{U_I \bar{U}_I}^{(0)} &= \underbrace{\frac{1}{(U + \bar{U})^2}}_{\mathcal{O}(1)} + \underbrace{\frac{(A + \bar{A})^2}{(U + \bar{U})^3 (S_I + \bar{S}_I + \dots)}}_{\mathcal{O}(e^{\Phi})} \\ &\quad + \frac{1}{4} \underbrace{\frac{(A + \bar{A})^4}{(U + \bar{U})^4 (S_I + \bar{S}_I + \dots)^2}}_{\mathcal{O}(e^{2\Phi})} \end{aligned}$$

“annulus” term from modified vertex operator on disc/sphere.

- One-loop moduli dependence from KK sums

$$\pi^2 \sqrt{G_I} \int_0^{\infty} dl l \sum_{\vec{n}} e^{-l \tilde{M}_{\vec{n}}^2} e^{2\pi i \vec{a} \vec{n}} = \frac{1}{\sqrt{G_I}} E_2(A_I, U_I) + \dots$$

with

$$E_s(A, U) = \sum_{n, m \neq (0,0)} \frac{(U + \bar{U})^s}{|n + mU|^{2s}} \exp \left[ 2\pi i \frac{A(n + mU) + \bar{A}(n - m\bar{U})}{U + \bar{U}} \right]$$

When  $A_I = 0$  this becomes Eisenstein series  $E_2(U_I)$ .

## One-loop Kähler potential

- Results for complete one-loop 2-point functions

$$\langle V_{\phi_I} V_{\bar{\phi}_I} \rangle_{\mathcal{K}+\mathcal{A}+\mathcal{M}} = \partial_{\phi_I} \partial_{\bar{\phi}_I} \left[ k \frac{\mathcal{E}_2(A_I, U_I)}{(S + \bar{S})(S_I + \bar{S}_I)} \right] \Big|_{\phi_I=S_I, U_I, A_I}$$

with:  $\mathcal{E}_2(A_I, U_I)$  = sum of  $E_2(A_I, U_I)$  over all diagrams

$$\begin{aligned} \mathcal{E}_2(A, U) = & 16 [E_2(A, U) + E_2(-A, U)] \\ & - [E_2(2A, U) + E_2(-2A, U)] \end{aligned}$$

From here read off one-loop Kähler potential

$$\mathcal{K}^{(1)} = k \sum_{I=1}^3 \frac{\mathcal{E}_2(A_I, U_I)}{(S + \bar{S})(S_I + \bar{S}_I)}$$

- In earlier work have obtained one-loop gauge couplings

$$f_{D7_I}^{(0)} = S_I, \quad f_{D7_I}^{(1)} = k' \ln [\vartheta_1(A_I, U_I)] + \dots$$

[Berg,  
Haack,  
Körs]

- Together this suffices to define the one-loop correction to the potential in models with volume stabilization via gaugino condensation (KKLT). Furthermore, the corrections can be essential for the inflaton dynamics in models of brane-inflation models built on top of KKLT.

## Side-remark: $\mathcal{N} = 2$ special geometry

- One can “truncate”  $\mathbb{T}^6/\mathbb{Z}_2^2$  to three copies of  $\mathbb{T}^2 \times \mathbb{T}^4/\mathbb{Z}_2$

$$\underbrace{\frac{\mathbb{T}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}}_{\mathcal{N}=1} = 3 \text{ copies of } \underbrace{\frac{\mathbb{T}^4}{\mathbb{Z}_2} \times \mathbb{T}^2}_{\mathcal{N}=2} + \text{corrections}$$

But: for contributions to  $\mathcal{K}^{(1)}$  corrections vanish!

- Constraints from  $\mathcal{N} = 2$  special geometry on  $\mathcal{K}$ :

$$\mathcal{K} = -\ln \left[ 2\mathcal{F} + 2\bar{\mathcal{F}} - \sum_N (\phi_N + \bar{\phi}_{\bar{N}})(\mathcal{F}_N + \bar{\mathcal{F}}_{\bar{N}}) \right]$$

with (one-loop exact) prepotential  $\mathcal{F}(\phi)$ ,  $\mathcal{F}_N = \partial_{\phi_N} \mathcal{F}$ .

- In any of the three  $\mathcal{N} = 2$  sectors have

$$\mathcal{K}^{(0)} + \mathcal{K}^{(1)} = -\ln \left[ \underbrace{(S + \bar{S})[(S_I + \bar{S}_I)(U_I + \bar{U}_I) - \frac{1}{2}(A_I + \bar{A}_I)^2]}_{\text{classical}} + \underbrace{k(U_I + \bar{U}_I)\mathcal{E}_2(A_I, U_I)}_{\text{quantum}} \right]$$

and

[Compare to Harvey, Moore; and others]

$$(U + \bar{U})\mathcal{E}_2 = 2f + 2\bar{f} - (U + \bar{U})(\partial_U f + \partial_{\bar{U}} \bar{f}) - (A + \bar{A})(\partial_A f + \partial_{\bar{A}} \bar{f})$$

$$f(A, U) = 16 \left[ h(A, U) + h(-A, U) \right] - \left[ h(2A, U) + h(-2A, U) \right]$$

$$h(A, U) = \frac{2\pi^4}{8i} \left[ \frac{1}{90} U^3 - \frac{1}{3} U A^2 + \frac{2}{3} A^3 \right] + \frac{\pi}{4} Li_3(e^{2\pi i A}) + \frac{\pi}{4} \sum_{m>0} \left[ Li_3(e^{2\pi i(mU-A)}) + Li_3(e^{2\pi i(mU+A)}) \right]$$

## Supergravity potentials: no-scale

- General form of any  $4d \mathcal{N} = 1$  potential

$$\mathcal{V}(\phi, \bar{\phi}) = e^{\mathcal{G}} [\mathcal{G}^{N\bar{M}} \mathcal{G}_N \bar{\mathcal{G}}_{\bar{M}} - 3] + \Re(f_{ab}(\phi)) D_a D_b$$

with definitions

$$\mathcal{G} = \mathcal{K}(\phi, \bar{\phi}) + \ln |W(\phi)|^2 ,$$

$$\mathcal{G}_N = \partial_{\phi^N} \mathcal{G} , \quad \mathcal{G}_{N\bar{M}} = \partial_{\phi^N} \partial_{\bar{\phi}^{\bar{M}}} \mathcal{G} = \partial_{\phi^N} \partial_{\bar{\phi}^{\bar{M}}} \mathcal{K}$$

- Completely determined by three (sets of) functions

$$\mathcal{K}(\phi, \bar{\phi}) = \mathcal{K}^{(0)} + \mathcal{K}^{(1)} + \sum_{n=2}^{\infty} \mathcal{K}^{(n)} + \mathcal{K}_{\text{nper}}$$

$$W(\phi) = W_{\text{cl}} + W_{\text{nper}}$$

$$f_a(\phi) = f_a^{(0)} + f_a^{(1)} + (f_a)_{\text{nper}}$$

Blue = what we know (at least in part)

- Classical no-scale property of the potential

$$\mathcal{K}(\phi, \bar{\phi}) = \mathcal{K}^{(0)} , \quad W(\phi) = W_{\text{cl}}(U_I)$$

leads to

$$\mathcal{G}^{\Sigma\bar{\Gamma}} \mathcal{G}_{\Sigma} \bar{\mathcal{G}}_{\bar{\Gamma}} = 3 , \quad \phi_{\Sigma} = S_I, A_I \text{ not } U_I$$

and

$$\mathcal{V}(\phi, \bar{\phi}) = \mathcal{V}(U_I, \bar{U}_I) \geq 0$$

a positive definite potential, and flat in  $S_I$  (the volume).

## Breaking the no-scale property

- Setting  $U_I, A_I$  to constants, the one-loop Kähler potential

$$\mathcal{K}^{(0)} + \mathcal{K}^{(1)} = -\ln \left[ \prod_{I=1}^3 (S_I + \bar{S}_I) \right] + \sum_{I=1}^3 \frac{k_I}{S_I + \bar{S}_I}$$

$$k_I = k \left\langle \frac{\mathcal{E}_2(A_I, U_I)}{S + \bar{S}} \right\rangle$$

- There is another known (tree-level)  $\alpha'$  corr. to Kähler pot.

$$\mathcal{K}_{\alpha'}^{(0)} = -\ln \left[ \prod_{I=1}^3 (S_I + \bar{S}_I) \right] + \frac{\xi}{\left[ \prod_{I=1}^3 (S_I + \bar{S}_I) \right]^{1/2}}$$

[Becker, Becker, Haack, Louis; Balasubramanian, Berglund, Conlon, Quevedo]

At large volume,  $S_I + \bar{S}_I \gg 1$ ,  $\mathcal{K}^{(1)}$  is dominant correction!

- Both corrections break no-scale!
- Set all  $S_I = v_6^{2/3}$ ,  $k_I = k$ , and use  $\mathcal{K} = \mathcal{K}_{\alpha'}^{(0)} + \mathcal{K}^{(1)}$  to get

$$\mathcal{V}(v_6^{2/3}) = \left[ 3 \frac{(v_6 + k v_6^{1/3} - \xi)^2}{v_6^2 + 2k v_6^{4/3} - \frac{5}{2} \xi v_6} - 3 \right] |W_{\text{cl}}|^2 \xrightarrow{v_6=0} 0$$

which (apparently) does not lead to minima.

## Moduli stabilization with fluxes and KKLT

- Superpotential from background fluxes (in orientifolds)

$$W_{\text{cl}}(S, U_I) = \int \Omega_3 \wedge G_3$$

[Gukov, Vafa, Witten; Taylor, Vafa; many others]

Sufficiently generic fluxes can fix all  $U_I$ . (Decoupling?)

- Superpotential from gaugino condensation

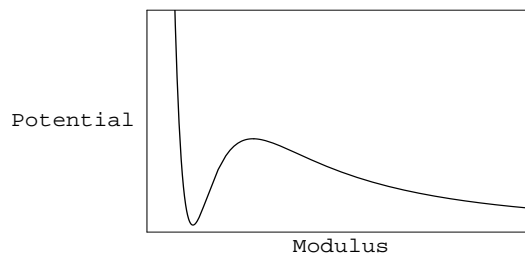
$$W_{\text{nper}}(S_I, U_I, A_I) = \sum_{I=1}^3 \Lambda_{\text{D7}_I}^3 e^{-bf_{\text{D7}_I}(S_I, U_I, A_I)}$$

- $D$ -term from D3-brane tension (“anti-branes”)

$$\mathcal{V}_D(S_I, \bar{S}_I, \dots) \sim \left[ \prod_{I=1}^3 (S_I + \bar{S}_I + \dots) \right]^{-3}$$

- With these ingredients can get meta-stable minima: KKLT

[Kachru, Kallosh, Linde, Trivedi; many others]



Positive vacuum energy through combining  $D$ -terms and  $W_{\text{nper}}$ .

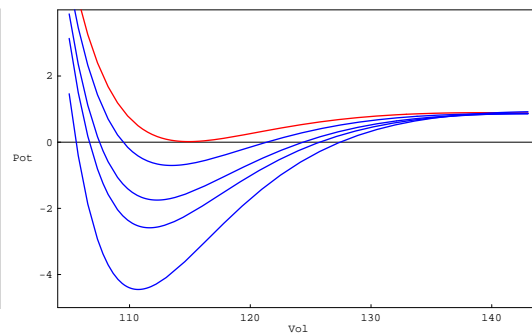
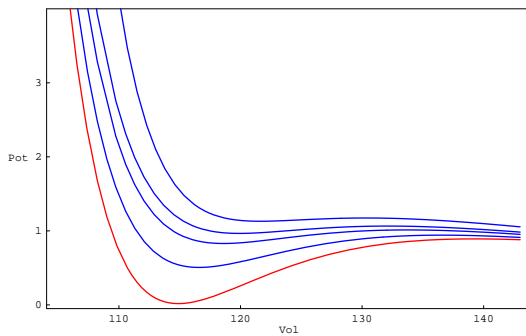
- Only survivors:  $A_I \longrightarrow$  inflaton candidates of brane inflation

## Impact of one-loop corrections on KKLT minima

- Use same parameter as KKLT
- Set all  $S_I + \bar{S}_I \sim \text{vol}(\mathbb{T}_I^4) = v_6^{2/3}$  equal.
- Add one-loop correction to Kähler potential:

$$f_{D7_I} = f_{D7_I}^{(0)}(S_I) = v_6^{2/3}, \quad \mathcal{K} = \mathcal{K}^{(0)} + \mathcal{K}^{(1)} = \mathcal{K}^{(0)} + k v_6^{-2/3}$$

Potential  $\mathcal{V}(v_6^{2/3}) \times 10^{15}$  looks like (note  $\mathcal{K}^{(0)} \sim 10$ )



$$k \in \{0, 50, 100, 130, 180\}$$

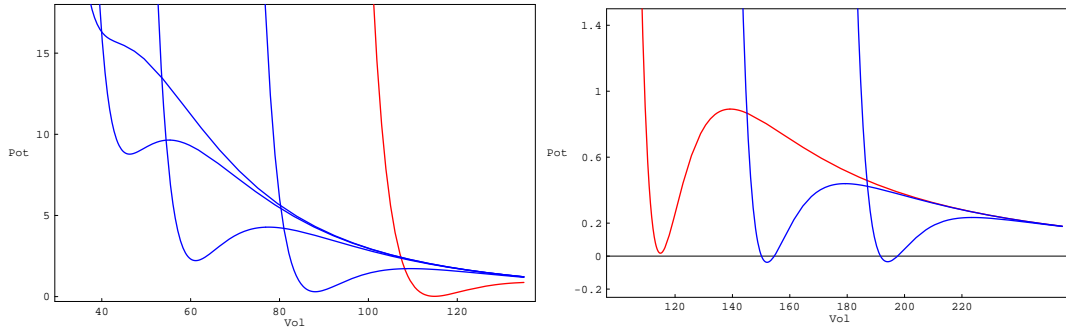
$$k \in \{0, -50, -100, -130, -180\}$$

## Impact of one-loop correction on KKLT minima

- Same for one-loop gauge couplings:

$$\mathcal{K} = \mathcal{K}^{(0)}, \quad f_{D7_I} = f_{D7_I}^{(0)}(S_I) + f_{D7_I}^{(1)}(U_I, A_I) = v_6 + f_{D7_I}^{(1)}$$

Potential  $\mathcal{V}(v_6^{2/3}) \times 10^{15}$  looks like



$$f_{D7_I}^{(1)} \in \{0, 25, 50, 65, 71\}$$

$$f_{D7_I}^{(1)} \in \{0, -35, -75\}$$

## The inflaton mass of D3/D7-inflation

- Model has **classical** shift symmetry for  $\mathfrak{S}(A_I)$

$$\delta A_I = ic \Rightarrow \delta \mathcal{V} = 0$$

since

$$\mathcal{K}^{(0)} = - \sum_{I=1}^3 \ln [(S_I + \bar{S}_I)(U_I + \bar{U}_I) - \frac{1}{2}(A_I + \bar{A}_I)^2]$$

$$W = W_{\text{cl}}(U_I) + \sum_{I=1}^3 \Lambda_I^3 e^{-b[f_{\text{D}7_I}^{(0)} + \dots]}, \quad f_{\text{D}7_I}^{(0)} = S_I$$

Can  $\mathfrak{S}(A_I)$  satisfy slow rolling without fine-tuning?

- Corrections violate shift symmetry: non-trivial dynamics

$$\begin{aligned} \mathcal{K} &= \mathcal{K}^{(0)} + \mathcal{K}^{(1)} + \dots \\ f_{\text{D}7_I} &= f_{\text{D}7_I}^{(0)} + f_{\text{D}7_I}^{(1)} = S_I + f_{\text{D}7_I}^{(1)}(A_I, U_I) \end{aligned}$$

In particular:  $f_{\text{D}7_I}^{(1)}$  holomorphic in  $A_I$ .

- Compute slow-roll parameter

$$\eta = \frac{\mathcal{V}''}{\mathcal{V}} \stackrel{!}{\lesssim} 0.01$$

for inflaton candidate  $\mathfrak{S}(A_I)$  as function of moduli/parameters.

[Hsu, Kallosh, Prokushkin; Kallosh, Linde; Firouzjahi, Tye; and others]

[Berg, Haack, Körs]

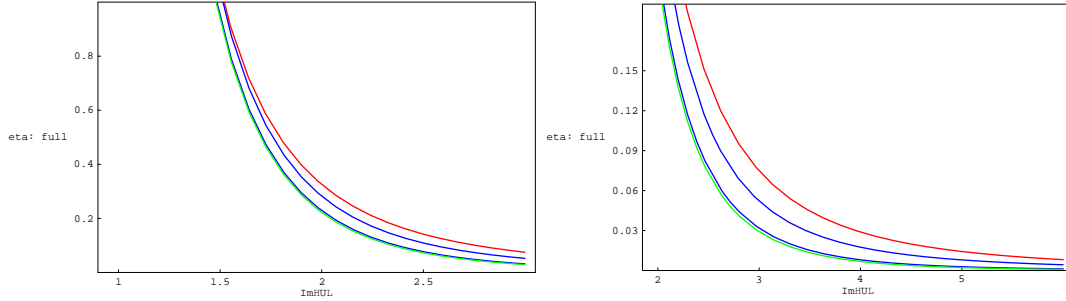
## The inflaton mass of D3/D7-inflation

- The  $\eta$  parameter of  $\mathfrak{S}(A)$  for

$$b = 0.1, \quad \Lambda_I = \Lambda, \quad W_0 = -10^{-4}$$

$$S = 10, \quad S_I = 100, \quad U_I = iu_2, \quad A_I = i\phi$$

as a function of  $u_2$ :  $\eta = \eta(u_2)$ ; and for various  $\Lambda$



$$\Lambda \in \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{10} \right\}$$

- Slow-roll is possible ( $u_2 \gtrsim 5$ ), but very parameter-dependent. Varying the expectation value of  $S + \bar{S}$  (string coupling) or the value for  $\Lambda_I$  has significant impact on the range of values for  $\eta$ .