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The Mysteries of Neutrino Masses: Theory & Phenomenology

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We present the aspects of different models that attempt to successfully explain the patterns in the leptonic sector, as well as their impact on future experiments (and vice versa). Strong constraints often arise from lepton flavour violating processes, but depending on the model, these bounds could be relatively weak. Further information can be gained from neutrino experiments, but their interpretation may not always be straightforward. Dealing carefully with all information is the key requirement to identify a possible successor of the Standard Model of Elementary Particle Physics.

Radiative Neutrino Masses

Observable	Value
m_β	< 2.3 eV
$\lvert m_{ee}\rvert$	< 0.19-0.68 eV
Σ	< 0.61 eV
Δm^2_{21}	$7.59 \cdot 10^{-5}$ eV^2
$\lvert\Delta m^2_{31}\rvert$	$2.40 \cdot 10^{-3}$ eV^2

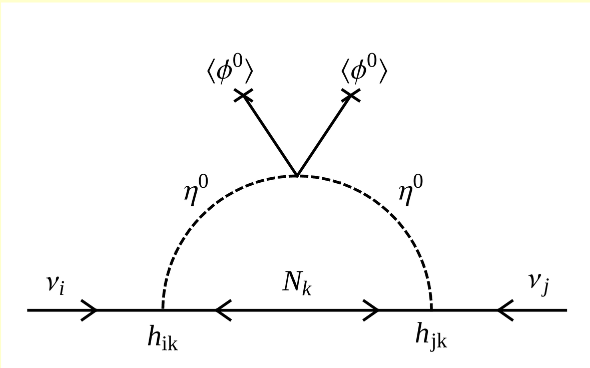
The current neutrino data indicates that the physical neutrino masses must be very small, even though we know from oscillation experiments that they cannot all be zero. Although their mass scale is not known exactly, it is tiny compared to the masses of the other fermions we know. One of the greatest questions on theoretical neutrino physics is to explain this smallness.

A natural explanation for the small masses might be that they are forbidden at tree-level and only arise as (small) 1-loop correction. One of the prime examples for such a situation is Ma's scotogenic model [Phys. Rev. **D73** 077301(2006)]. Apart from explaining the smallness of neutrino masses, this model also yields a Dark Matter candidate and, when combined with left-right symmetry, it even reveals a new mechanism for explaining the neutrino mixing pattern.

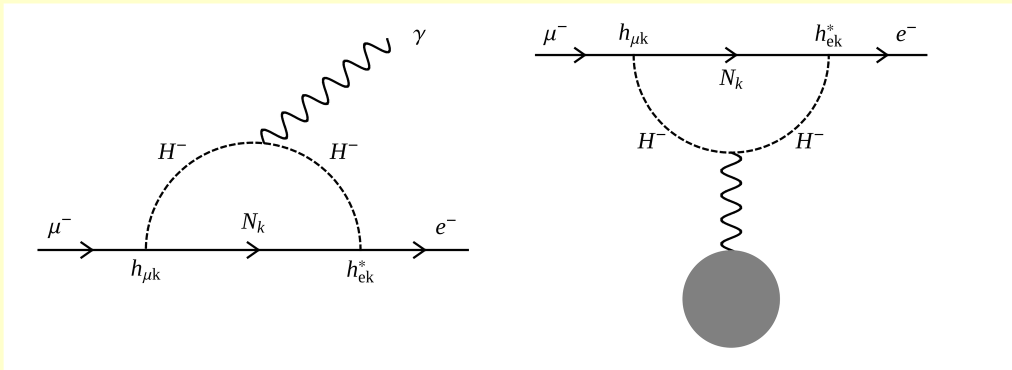
If one, however, tries to explain the leptonic mixing structure by a more conventional way, namely using flavour symmetries, this model (similarly to other models with extended Higgs sectors) nicely reveals the competition between flavour symmetries and extended scalar sectors.

Features of the scotogenic model:

- 3 heavy right-handed (Majorana) neutrinos N_k, which are singlets under $SU(2)$ and have no hypercharge
- a second Higgs doublet η with SM-like quantum numbers that does not obtain a VEV
- an additional Z_2-parity under which all SM-particles are even, while N_k as well as η are odd



Lepton Flavour Violation



The diagrams for $\mu \to e\gamma$ and for μ-e conversion in Ma's scotogenic model. Notice that they look very similar to the 1-loop diagram giving mass to the neutrinos, which is the origin of the deep connection between both sectors.

Lepton flavour violating (LFV) transitions such as $\mu \to e\gamma$ are perfectly allowed by energy, momentum, or angular momentum conservation. However, in the Standard Model, they are forbidden due to an accidental symmetry, and accordingly the bounds on such processes are strong. As there is no known reason for lepton flavour to be conserved, it is generically violated in theories beyond the Standard Model.

Experiment	Status	Process	BR-Limit/Sensitivity
MEGA	Past	$\mu \to e\gamma$	$1.2 \cdot 10^{-11}$
MEG	On-going	$\mu \to e\gamma$	$1.0 \cdot 10^{-13}$
BELLE	Past	$\tau \to \mu\gamma$	$4.5 \cdot 10^{-8}$
Babar	Past	$\tau \to e\gamma$	$1.1 \cdot 10^{-7}$
MECO	Cancelled	μAl $\to e$Al	$2.0 \cdot 10^{-17}$
SINDRUM II	Past	μTi $\to e$Ti	$6.1 \cdot 10^{-13}$
PRISM/PRIME	Future	μTi $\to e$Ti	$5.0 \cdot 10^{-19}$
SINDRUM II	Past	μAu $\to e$Au	$7.0 \cdot 10^{-13}$
SINDRUM II	Past	μPb $\to e$Pb	$4.6 \cdot 10^{-11}$

The limits and sensitivities of several past, on-going, or future experiments. The number for the MEG-experiment is the final sensitivity goal rather than the present limit.

The Radiative Transmission of Hierarchies

The 1-loop neutrino mass matrix in the scotogenic model is given by $(\mathcal{M}_\nu)_{ij} = h_{ik} h_{jk} \Lambda_k$, where

$$\Lambda_k = \frac{M_k}{16\pi^2}\left[\frac{m^2(\eta^0)}{m^2(\eta^0) - M_k^2} \ln\left(\frac{m^2(\eta^0)}{M_k^2}\right) - (\eta^0 \to A^0)\right]$$

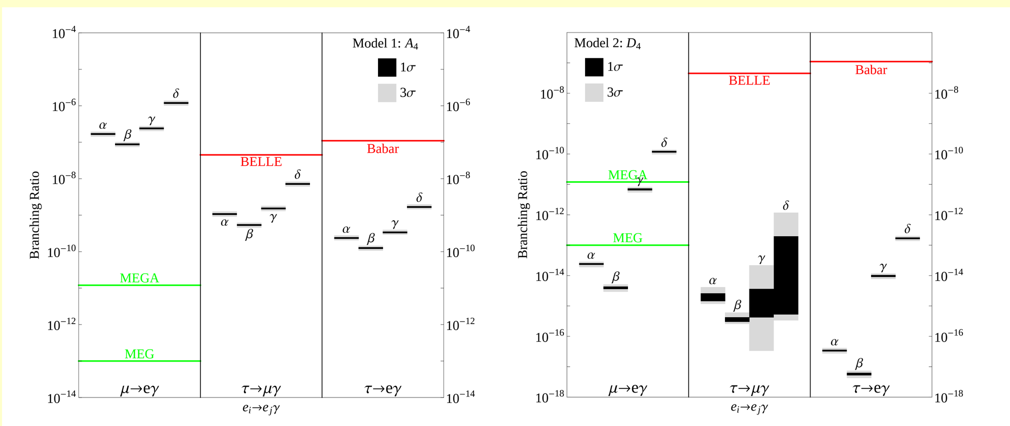
depends on the Yukawa couplings h of the neutrinos to η and on the heavy neutrino masses M_k.

If an additional left-right symmetry is imposed, the Yukawa couplings will be proportional to the charged lepton masses. If furthermore the new Higgs is significantly lighter than the heavy neutrinos (which is natural in this framework), the form of the mass matrix is altered to $(\mathcal{M}_\nu)_{ij} = h_{l,i} \Lambda_{ij} h_{l,j}$, where Λ_{ij} is roughly given by $\frac{2}{16\pi^2}\frac{\lambda_5 v^2}{M_{ij}} \ln\left(M^2_{N,ij}/m^2(\eta^0)\right)$. Absorbing the logarithm into the modified mass matrix $\tilde{M}_N$ yields seesaw-like mass formula:

$$(\mathcal{M}_\nu)_{ij} = \frac{2\lambda_5}{16\pi^2} m_{l,i} \left(\tilde{M}_N^{-1}\right)_{ij} m_{l,j}$$

If the heavy neutrinos obey a (Froggat-Nielsen) hierarchy, this can be compensated by the known hierarchy in the charged leptons, resulting in an anarchic neutrino mass matrix. Such a mechanism is a natural explanation for large but non-maximal mixing angles.

Confronting Flavour Symmetries with LFV



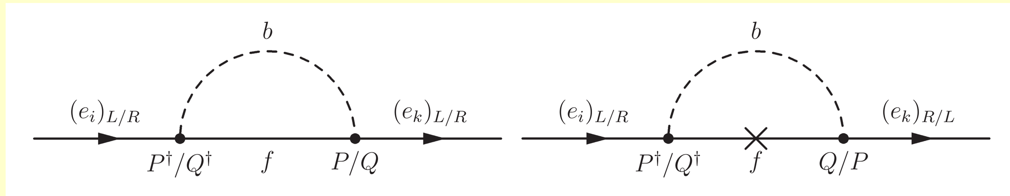
Practically any model with an extended scalar sector will lead to LFV-processes like $\mu \to e\gamma$. The corresponding amplitudes are typically proportional to combinations of Yukawa couplings such as $|h_{11}^* h_{21} + h_{12}^* h_{22} + h_{13}^* h_{23}|$, which can avoid all bounds if cancellations appear.

However, using a flavour symmetry to explain mixing patterns imposes certain relations on the Yukawa couplings, which can easily destroy the possibilities for a cancellation, therefore putting the bite on the respective model.

We have exemplified this using the scotogenic model. Imposing an A_4-symmetry (3 free parameters) rules out the model completely. Even a more moderate D_4-symmetry (7 free parameters - more than twice as many) is ruled out or disfavored for certain benchmark points in the scalar potential parameter space.

General Conditions for LFV

As LFV processes occur very generically in theories beyond the Standard Model, it is very useful to systemize the conditions for LFV to occur. At tree-level, this is simple, but at 1-loop level it is very helpful to take a step back and have a look at the general features of a 1-loop LFV diagrams (neglecting everything that has nothing to do with the flavour violation itself, such as external photons):

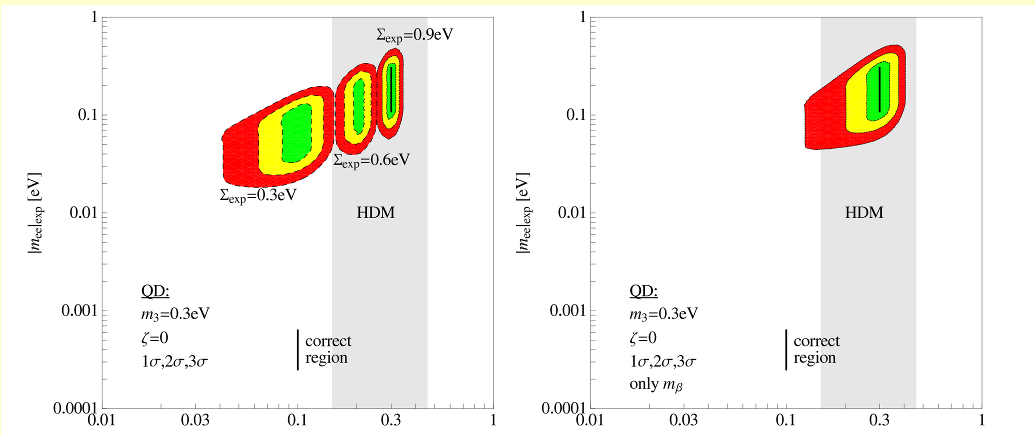


The conditions a model has to fulfill to ensure absence of LFV are very simple, and it is sufficient that one of the three is fulfilled:

- not exactly one fermion and one boson in the loop to ensure Lorentz invariance
- $b \otimes f \not\supseteq \mathbf{1}$ (under one gauge group except $SU(2)_L \times U(1)_Y$)
- $\forall b, f:\ b \otimes f \not\supseteq (\mathbf{2}_L, Y = -1)$ & $b \otimes f \not\supseteq (\mathbf{1}_L, Y = -2)$

Also the GIM-mechanism can be generalized: As soon as the loop-function of the diagram can be expanded in some small mass ratio [e.g. *fermionic GIM*: small m_{f_j} in $F(m_i, m_k, m_{f_j}, m_b)$], GIM will occur if $PP^\dagger =$ diagonal, $QQ^\dagger =$ diagonal, and $PQ^\dagger =$ diagonal.

Statistical Analysis of future Experiments

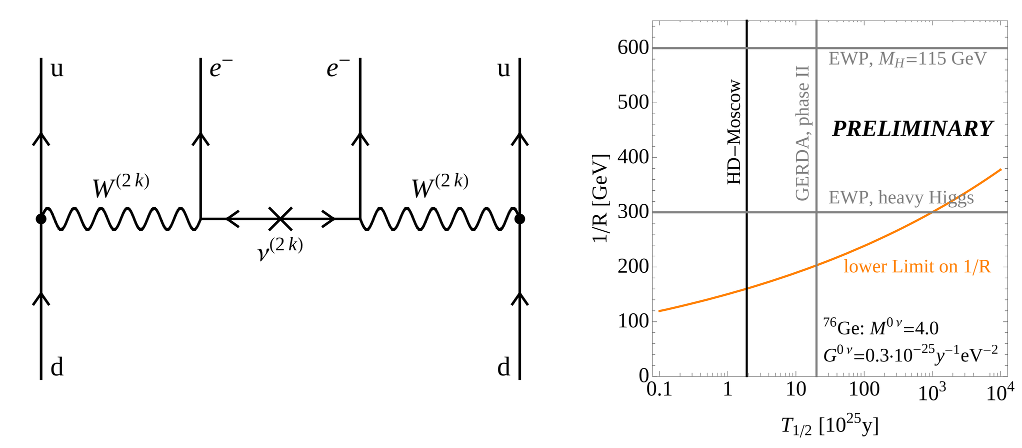


The interplay between different observables that depend on the neutrino mass relies on how much we can trust the data. We have analyzed the impact of a wrong cosmological observation of the sum Σ of all neutrino masses using three benchmark scenarios:

Scenario	m_3 [eV]	$\lvert m_{ee}\rvert$ [eV]	m_β [eV]	Σ [eV]
$\mathcal{QD}$	0.3	$0.11-0.30$	0.30	0.91
$\mathcal{INT}$	0.1	$0.04-0.11$	(0.11)	0.32
$\mathcal{IH}$	0.003	$0.02-0.05$	(0.05)	(0.10)

If all observations are consistent,a perfect reconstruction of the scenario $\mathcal{QD}$ is possible. If, however, the observed value of Σ is incorrect, one might reconstruct a *wrong* region for the neutrino mass. In such a case (if the inconsistency is known), one way out is to dismiss the cosmological data for the cost of accuracy, but with the value of reconstructing the correct mass (see right plot).

$0\nu\beta\beta$ and Extra Dimensions

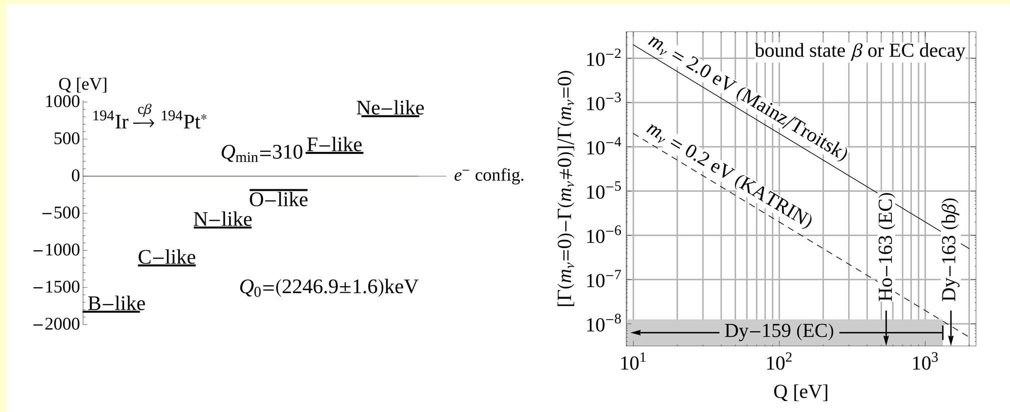


An interesting aspect of neutrino-less double beta processes is the sensitivity to contributions from new physics, which arises since the leading light neutrino contribution is very small.

In scenarios with extra spatial dimensions, the so-called *Kaluza-Klein* towers of particles arise in the 4D effective theory, similar to the excited states in an infinite quantum well. If a whole tower of such states contributes to the process, the corresponding rate will be enhanced. On the contrary, if $0\nu\beta\beta$ is not seen, a limit can be set on the compactification scale $1/R$.

Although this limit may be complicated by issues like the bad knowledge on nuclear matrix elements, it can well be competitive to bounds from electroweak precision experiments and it strongly exceeds the direct bound from gravity. Future $0\nu\beta\beta$ experiments can yield complementary information on certain UED models.

Decays with ultra-low Q Values



The energy release (Q value) in a nuclear decay will always depend on the binding energies of the electrons in the atomic shell, unless the atom is completely ionized. Hence, by adjusting the atomic shell to a state of excitation energy E^*, it is possible to modify the Q_0 value (the mass differences of parent and daughter atom):

$$\beta^-:\ Q_0 - (B_{Z+1,Z} - B_{Z,Z}) \lesssim E^* \lesssim Q_0 + B_{Z,1} - B_{Z+1,2}$$
$$\beta^+:\ Q_0 - 2m_e \lesssim E^* \lesssim Q_0 - 2m_e + B_{Z,Z} - B_{Z-1,Z}$$
$$\text{EC}:\ Q_0 - B_{Z,1} \lesssim E^* \lesssim Q_0$$

The smaller Q, the more sensitive the transition will be to the neutrino mass. However, the decay rate decreases with a power of Q except for a small region close to the spectral endpoint, where it is independent of the exact value of Q. If a suitable (allowed) decay mode with high enough rate could be identified, one might be able to come close to (or to even go beyond) the KATRIN limit.

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