$B \to D^{(*)}$ transitions and $|V_{cb}|$

Theory

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interaction theory probing our control of QCD
dynamics

- Golden way $\Gamma_{\rm sl}(B)$, inclusive decay distributions

- Gold-plated modes: $B \to D^*\,\ell\nu$ and $B \to D\,\ell\nu$
near zero recoil

$1/m_c^k$ corrections are not too small...

Inclusive decays provide a host of dynamic info vital for $B \to D^*$ and $B \to D$ decays

Recent inclusive data fuel advances in the old field

Theory progress:

New HQ sum rules (exact spin sum rules)
exact inequalities

BPS expansion

D' Orsay sum rules HQ relations for higher IW derivatives

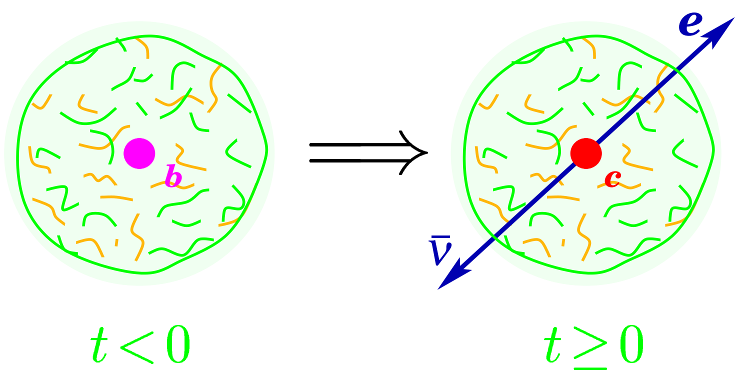
The $1/m_c$ corrections to HQ spin symmetry are too significant
A subgroup, HF symmetry for ground-state pseudoscalar mesons is good

Theoretical fidelity exceeds lattice accuracy, can be used to cross-check lattice simulations

Some problems may be emerging

$B \to D^* + \ell\bar{\nu}$ at zero recoil

$$\mathrm{d}w\,(B \to D^* + \ell\bar{\nu}) \;\sim\; G_F^2 \cdot |V_{cb}|^2 \cdot |\vec{p}\,| \cdot \left|F_{B\to D^*}(\vec{p}\,)\right|^2$$



$F_{B\to D^*}$ is determined by bound state dynamics

If $\vec{p}=0$ $(\,\vec{p}_e=-\vec{p}_{\bar\nu}\,)$

almost nothing has changed!

$F(\vec{p}=0) = 1$ up to 'isotopic effects'

$$F_{n/p}(0) \;=\; 1 + \frac{0}{m_{c,b}} + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m^2_{c,b}}\right) + \mathcal{O}\left(\frac{\Lambda^3_{\rm QCD}}{m^3_{c,b}}\right) + \;...$$

$1/m_{b,c}$ effects are absent

1986 Voloshin, Shifman
1990 Luke

Important to estimate δ_{1/m^2}

Before May 1994 : $\delta_{1/m^2} \simeq -0.02$

OPE $\Longrightarrow$ HQ Sum Rules

SUV, BSUV April 1994
Experiment June 1994

$-\delta_{n/p} > \frac{M^2_{B^*}-M^2_B}{8m_c^2} \simeq -0.04$ rigorous bound on $F(0)$

$F(0) \simeq 0.9$ actual estimate

transition probabilities

A genuinely short-distance quantity

$$F_{D^*}^2 + \sum_{f \neq D^*} |F_{B\to f}|^2 =$$

$$\xi_A^{\text{pert}} - \frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) + \mathcal{O}\left(\frac{1}{m^3}\right)$$

We computed $1/m_Q^3$ terms, $\xi_A^{\text{pert}}(\mu)$ to α_s^2 and to all orders in BLM, $\mathcal{O}(\alpha_s)$ to Wilson coefficients of $1/m^2$ nonperturbative terms 1997, 1999

$\sum_{f \neq D^*} |F_{B\to f}|^2$ is positive $\Longrightarrow$ upper bound on F_{D^*}

Similarly, $\mu_\pi^2 - \mu_G^2$ is a sum of certain transition probabilities

QM meaning of sum rules

Weak decay is an instantaneous replacement $b \to c$

Total probability to hadronize into some final state is 1

Why nonperturbative corrections?

Normalization of the weak current $\bar{c}...b$ is not exactly 1, depends on the external field

$$\bar{c}\gamma_k\gamma_5 b \xrightarrow{\text{QM}} \varphi_c^+\sigma_k\varphi_b - \varphi_c^+\left\{\frac{(\vec{\sigma}\vec{\pi})^2}{8m_c^2}\sigma_k + \sigma_k\frac{(\vec{\sigma}\vec{\pi})^2}{8m_b^2} - \frac{(\vec{\sigma}\vec{\pi})\sigma_k(\vec{\sigma}\vec{\pi})}{4m_c m_b}\right\}\varphi_b + \ldots$$

$$\vec{\pi} = -i\,\vec{D} = -i\vec{\partial} - g_s\vec{A}$$

Come from Foldy-Wouthuysen transformation — missed in HQET

plagued Falk & Neubert, Neubert 1994+

$$\Psi_B \xrightarrow{\bar{c}\gamma_k\gamma_5 b} \Psi_{\text{charm}} = \sigma_k\Psi_B - \left\{\frac{(\vec{\sigma}\vec{\pi})^2}{8m_c^2}\sigma_k + \sigma_k\frac{(\vec{\sigma}\vec{\pi})^2}{8m_b^2} - \frac{(\vec{\sigma}\vec{\pi})\sigma_k(\vec{\sigma}\vec{\pi})}{4m_c m_b}\right\}\Psi_B$$

$$||\Psi_{\text{charm}}||^2 = ||\Psi_B||^2 - \frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) + \ldots$$

$$||$$

$$1$$

Numerical estimates of F_{D^*}

$$F_{D^*} = \left[\xi_A(\mu) - \frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) - \sum_{f \neq D^*}^{\epsilon<\mu} |F_{B\to f}|^2 + \mathcal{O}\left(\frac{1}{m^3}\right)\right]^{\frac{1}{2}}$$

$$2\delta_{1/m^2}(\mu)$$

$\xi_A^{\frac{1}{2}}(\mu)$ is the short-distance renormalization factor 0.97 ± 0.01

$$\sum_{f \neq D^*}^{\epsilon<\mu} |F_{B\to f}|^2 = \chi\left[\frac{\mu_G^2}{3m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) + \mathcal{O}\left(\frac{1}{m^3}\right)\right]$$

χ describes the w f overlap deficit guess: $0 < \chi \leq 1$ SUV 1994

$$F_{D^*} \simeq \xi_A^{\frac{1}{2}} - (1+\chi)\left[\frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) + \Delta_{\frac{1}{m^3}}\right]$$

if $\chi = 0.5 \pm 0.5$ $\mu \approx 0.8\,\text{GeV}$

$$F_{D^*} \simeq 0.89 - 0.015\,\frac{\mu_\pi^2 - 0.4\,\text{GeV}^2}{0.1\,\text{GeV}^2} \pm 0.03_{\text{exc}} \pm 0.01_{\text{pert}}$$

$1/m_c^3$ correction is significant!

$F_{D^*} \lesssim 0.92$ $\chi \simeq 0$

$\chi^{\text{pert}} = 1$ @ $\mathcal{O}(\alpha_s^1)$

't Hooft model: $\chi = \frac{13}{21} + \frac{5}{21}\frac{m^2 - \beta^2}{\overline{\Lambda}^2 - m^2 + \beta^2} - \frac{4}{21}\left(\varrho^2 - \frac{3}{4}\right) \simeq 0.55$

Burkardt, N.U. 2000

Other estimates:

Falk & Neubert, 1993: $\delta_{1/m^2} \simeq -(2 \div 3)\%$ analysis wrong

Mannel, 03/1994: $\delta_{1/m^2} \simeq -(1 \div 5)\%$

pre-QCD summary: Neubert, 04/1994 $\delta_{1/m^2} = -(2 \pm 1)\%$

HQ sum rules, 05/1994: $F_{D^*} \simeq 0.9$

Kronfeld *et al.* cite $F_{D^*} \simeq 0.91$ from quark models – **not true!**

Nonrelativistic quark models yielded small corrections, but there have been `no correct calculations` reason is understood

Neubert 1994, 1995: $\delta_{1/m^2} = -(5.5 \pm 2.5)\%$

Express δ_{1/m^2} via δ^V_{BD}, $\delta^V_{B^*D^*}$, μ_π^2, μ_G^2 and a correlator λ_G^2

Expressing a formfactor of interest in terms of five other obscure hadronic quantities of which only one expectation value is known, cannot yield anything!

In fact, the same HQ sum rules were used `with the` **`same assumption`** $\chi = 0.5 \pm 0.5$ put in different variables

μ_π^2 was `generously` varied from $0.36\,\mathrm{GeV}^2$ to $0.4\,\mathrm{GeV}^2$
Consequently resulted in literally the same our prediction with $\mu_\pi^2 = 0.4\,\mathrm{GeV}^2$

Neubert's comparison with the quark models has the same mistake which missed the leading $1/m_c^2$ terms

Naive quark models are doomed here since the major corrections come from chromomagnetic spin-related effects

B mesons are highly relativistic bound states

A fresh perspective can be gained using the BPS expansion

address later

FNAL, lattice:

$F(0) \simeq 0.88$ order $1/m_Q^2$

$F(0) \simeq 0.91$ order $1/m_Q^3$

higher orders in $1/m_c$?

$$F(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003\,+0.000\,+0.006}_{-0.014\,-0.016\,-0.014}$$

Significant part of the correction is added theoretically rather than directly emerged from the lattice simulation

$B \to D\ell\nu$ near zero recoil

$$\langle D(p_2)|\bar{c}\gamma_\nu b|B(p_1)\rangle = f_+(p_1+p_2)_\nu + f_-(p_1-p_2)_\nu$$

$f_\pm \equiv f_\pm(\vec{q}^{\,2})$

One amplitude $J_0 = (M_B+M_D)f_+(0) + (M_B-M_D)f_-(0)$ at $\vec{q}=0$

HQ limit: $f_+ = \frac{M_B+M_D}{2\sqrt{M_B M_D}}, \qquad f_- = -\frac{M_B-M_D}{2\sqrt{M_B M_D}}$

$$\frac{J_0}{2\sqrt{M_B M_D}} = 1 - a_2\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2 - a_3\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2\left(\frac{1}{m_c}+\frac{1}{m_b}\right) + \ldots$$

Corrections are well under control and small quantify later

Any amplitude with massless leptons depends, however solely on f_+, while only the combination of f_+ and f_- has no $1/m$ corrections

$F_+ \equiv \frac{2\sqrt{M_B M_D}}{M_B+M_D} f_+$ has $1/m_Q$ corrections since $\vec{J}$ has such a term...

Good news: we know it!

$$F_+ = 1 + \left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{M_B - M_D}{M_B + M_D} - \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

Homework: derive (on $\le$ two lines)

hint: take $m_c \to \infty$ and a textbook with Dirac equation (B & D, v.I)

Thanks to inclusive decays and exact sum rules we know it (very small)

Moreover, we know `all` power corrections are small, the concern is rather exponential terms $\sim e^{-2m_c/\mu_{\rm hadr}}$

$$\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) = 1.035 \pm 0.025$$

This formfactor is known better than for 'gold-plated' $B \to D^*$

differs from existing estimates

If you can measure this, you don't need anything else

`exclusive`

Sum Rules in the HQ Limit

$\varrho^2 - \frac{1}{4} = 2\sum_m |\tau_{3/2}^{(m)}|^2 + \sum_n |\tau_{1/2}^{(n)}|^2$	Bj	1990
$\frac{1}{2} = 2\sum_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n |\tau_{1/2}^{(n)}|^2$	N.U.	2000
$\frac{\overline{\Lambda}}{2} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2$	Voloshin	1992
$\overline{\Sigma} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2$	N.U.	2000
related to ORSAY SUM RULE	Le Yaouanc et al.	2000
$\frac{\mu_\pi^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2$	BSUV	1994
$\frac{\mu_G^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2$	BSU	1997
$\frac{\rho_D^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2$	Chow, Pirjol	1994
$-\frac{\rho_{LS}^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2$	BSU	1997

Second and Fourth sum rules are superconvergent

$$\epsilon_k = M_k - M_B$$

$$\langle B(v)|\,\bar{b}\,\gamma_0\,b\,|B(0)\rangle = 1 - \varrho^2\frac{\vec{v}^{\,2}}{2} + \mathcal{O}(\vec{v}^{\,4})$$

$$\langle P^{(1/2)}(v_2)|\bar{b}\gamma_\mu\gamma_5 b|B(v_1)\rangle = -\tau_{1/2}\,(v_1 - v_2)_\mu$$

$$\langle P^{(3/2)}(v_2)|\bar{b}\gamma_\mu\gamma_5 b|B(v_1)\rangle = -\frac{1}{\sqrt{2}}\,i\,\tau_{3/2}\,\epsilon_{\mu\alpha\beta\gamma}\,\varepsilon^{*\alpha}\,v_2^\beta\,v_1^\gamma$$

spin of light cloud is $\begin{cases} \frac{1}{2} & \text{in } P^{(1/2)} \\ \frac{3}{2} & \text{in } P^{(3/2)} \end{cases}$

D′ Orsay Sum Rules

Le Yaouanc, Oliver, Raynal 10/2002

OPE for nonforward scattering amplitude

$$\varrho_L^2 = (2L+1)\sum_n \left|\tau^{(n)}_{L+\frac{1}{2}}\right|^2 \qquad \varrho_L^2 \equiv \frac{(-1)^L}{L!}\frac{\mathrm{d}^L\xi(w)}{(\mathrm{d}w)^L}\Big|_{\mathrm{w=1}}$$

$$L\sum_n \left|\tau^{(n)}_{L+\frac{1}{2}}\right|^2 - \sum_k \left|\tau^{(k)}_{L-\frac{1}{2}}\right|^2 = \frac{2L-1}{4}\sum_n \left|\tilde{\tau}^{(n)}_{(L-1)+\frac{1}{2}}\right|^2$$

Divergent – undergo renormalization...

Remarkable: only L-th orbital waves enter for L-th derivative !

Interpretation – N.U. 2002

For instance

$$\varrho_2^2 \geq \frac{5}{4}\varrho^2 \geq \frac{15}{16}$$

↑ IW curvature ↑ IW slope

$$\varrho_L^2 \geq \frac{(2L+1)!!}{2^{2L}L!}\varrho^2$$

Extended BPS limit: All $\tau^2_{L-\frac{1}{2}}$ suppressed ?!

all ‘spin’ inequalities are approximately saturated

$$\xi_{\mathrm{BPS}}(w) = \left(\frac{2}{w+1}\right)^{\frac{3}{2}}$$

Can be directly measured in $B \to D\,\ell\nu$

$\overline{\Lambda}(1\,\mathrm{GeV}),\ \ \mu_\pi^2(1\,\mathrm{GeV}),\ \ \mu_G^2(1\,\mathrm{GeV}),\ \ \ldots$

Using the same accurate regularized definition for kinetic $(\{j,k\})$ and chromomagnetic $([j,k])$ operators allows precision numerical evaluation

Product of covariant derivatives $\bar{Q}(x)iD_j\, P\exp\, iD_k Q(0)$ offset along t direction $\;it\sim 1/\mu$

$M_{B^*}-M_B \simeq \frac{2}{3}\frac{\mu_G^2}{m_b}$

$$\mu_G^2(1\,\mathrm{GeV}) = 0.35^{+.03}_{-.02}\,\mathrm{GeV}^2$$ N.U. 2001

$\mu_\pi^2(\mu) > \mu_G^2(\mu)$ **at any** μ rigorous inequality

BSUV, Voloshin 1993–1994

Physical observables, renormalon-free

"Such a $\mu_\pi^2(1\,\mathrm{GeV})$ can be absolutely arbitrary, even $100\,\mathrm{GeV}^2$"

M.Neubert, SLAC December 5, 2001

μ_π^2 is an observable $\Rightarrow$ has a definite value in Nature

Actual uncertainty is about 1000 times smaller

Experiment: typically $\mu_\pi^2(1\,\mathrm{GeV}) \simeq (0.37\pm 0.1)\,\mathrm{GeV}^2$

$\mu_\pi^2(1\,\mathrm{GeV}) > 0.45\,\mathrm{GeV}^2$ excluded ?

Sum rules yield strict inequalities

$$\varrho^2 > \tfrac{3}{4}\,, \qquad \overline{\Lambda} > 2\bar{\Sigma}\,, \qquad \mu_\pi^2 > \mu_G^2\,, \qquad \rho_D^3 > -\rho_{LS}^3$$

$$\rho_D^3 > |\rho_{LS}^3|/2$$

Likewise

$$\mu_\pi^2 \ge \frac{3\overline{\Lambda}^2}{4\varrho^2 - 1}\,, \quad \rho_D^3 \ge \frac{3}{8}\,\frac{\overline{\Lambda}^3}{(\varrho^2 - \frac{1}{4})^2}\,, \quad \rho_D^3 \ge \frac{(\mu_\pi^2)^{3/2}}{\sqrt{3(\varrho^2 - \frac{1}{4})}}$$

Similarly for W_- moments

Positivity for many non-local correlators

Hold in our renormalization schemes

Regardless of theory: what practically can be gained ?

A good behavior of the (standard) perturbative expansion

N.U. 1995
explicit in DELPHI analysis, 2002

Maximal physical information – the case of 'kinetic' mass and other definitions based on the SV sum rules

Preserves almost everything $\Rightarrow$ respects inequalities, allows QM probabilistic interpretation, direct relation to observables

the reason for a number of nontrivial constraints

There is no explicit independence on the heavy quark velocity – the $\vec{v} = 0$ frame is special

Inconvenience? This is a physical preference!

Good example: $\varrho^2 > \frac{3}{4}$ N.U., 2000

Dynamic, much stronger than the Bjorken's $\varrho^2 > \frac{1}{4}$

Moreover

$$\mu_\pi^2(\mu) - \mu_G^2(\mu) = 3\tilde{\varepsilon}^2 \cdot \left(\varrho^2(\mu) - \tfrac{1}{4} - S(\mu)\right) \qquad 0.5\,\mathrm{GeV} < \tilde{\varepsilon} < \mu$$

$$S(\mu) = 2\sum_{\varepsilon<\mu} |\tau_{3/2}^{(m)}|^2 - |\tau_{1/2}^{(n)}|^2 \underset{\mu\to\infty}{\longrightarrow} \tfrac{1}{2} + 0$$

If the first spin sum rule is saturated at $\mu = 1\,\mathrm{GeV}$ then

$$\mu_\pi^2 - \mu_G^2 = 3\tilde{\varepsilon}^2 \cdot (\varrho^2 - \tfrac{3}{4})$$

Quite a constraint: $\left(\varrho^2 - \frac{3}{4}\right) = \frac{\mu_\pi^2 - \mu_G^2}{3\tilde{\varepsilon}^2} \lesssim 0.2 \;\; (0.3)$

at $\mu_\pi^2 = 0.43\;(0.5)\,\mathrm{GeV}^2$ since $\tilde{\varepsilon} > 0.4\,\mathrm{GeV}$

ϱ^2 is measured in experiment

important for V_{cb}

radically improves $B \to D^*$ extrapolation to zero recoil

Neubert, 1993: $\hat{\varrho}^2 \simeq \varrho^2 - 0.09$

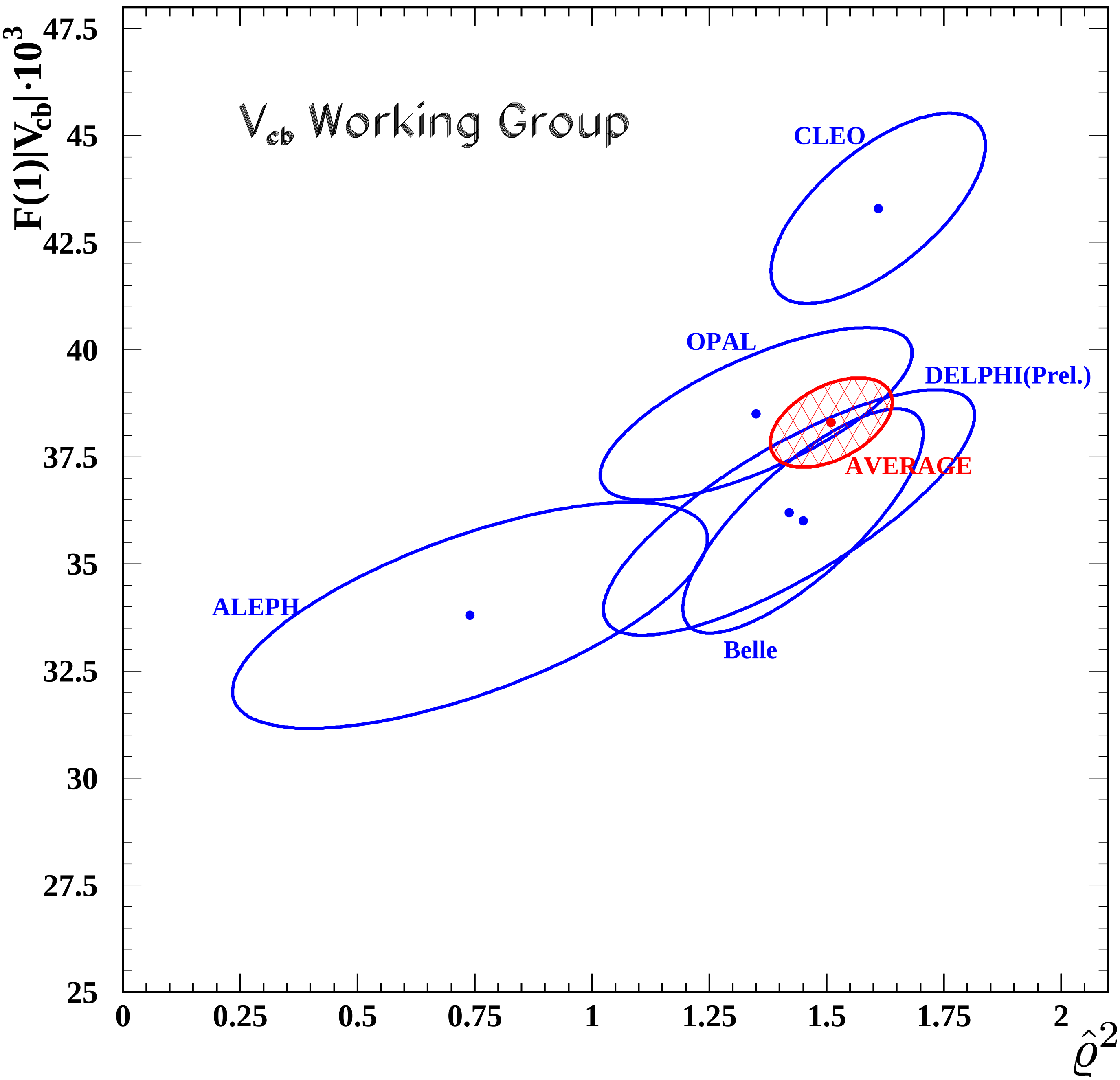
hardly correct

Excluded by experiment

Recent UKQCD lattice is quite compatible with the prediction:

$$\varrho^2 = 0.83^{+.15\,+\,.24}_{-.11\,-\,.01}$$

hep-lat/0202029



Question to experiment and fits:

What is your value for $F(1)\cdot|V_{cb}|$ with the constraint $\hat{\varrho}^2 < 1.2$?

The whole set of the sum rule constraints is even more interesting

If indeed $\mu_\pi^2 \gtrsim 0.45\,\mathrm{GeV}^2$, i.e. $\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2, \mu_G^2$

BPS expansion

N.U. 11/2001

Expand around $\mu_\pi^2 - \mu_G^2 = 0$

- $\varrho^2 - \frac{3}{4}$, $\overline{\Lambda} - 2\overline{\Sigma}$, $\mu_\pi^2 - \mu_G^2$, $\rho_D^3 + \rho_{LS}^3$, ... are all moments of one and the same HQ positive structure function which then must be suppressed

- At $\mu_\pi^2 = \mu_G^2$ there is a `functional` relation $\vec{\sigma}\vec{\pi}|B\rangle = 0$

Reminiscent to a "BPS"-saturated state   not literally

$$\mathcal{H}_Q = A_0 + \frac{(\vec{\sigma}\vec{\pi})^2}{2m_Q}$$

Yet

$$\mathcal{P}_z|B_{\frac{1}{2}}\rangle = 0\,, \qquad \mathcal{P}_x - i\mathcal{P}_y\,|B_{\frac{1}{2}}\rangle = 0$$

Remarkable limit in many respects

Infracted by hard gluons   rather a property of soft dynamics

Often extends Heavy Flavor (but not Spin) symmetry to all orders in $1/m_Q$;   Example: in $B \to D$ zero recoil $\delta_{1/m^k} = 0$

No formal power corrections to $m_b - m_c = M_B - M_D$

$\varrho^2 = \frac{3}{4}, \qquad \overline{\Lambda} = 2\overline{\Sigma}, \qquad \rho_D^3 + \rho_{LS}^3 = 0, \; ...$

$\rho_{\pi G}^3 = -2\rho_{\pi\pi}^3, \qquad \rho_A^3 + \rho_{\pi G}^3 = -(\rho_{\pi\pi}^3 + \rho_S^3), \; ...$

In practice:

$$m_b - m_c = M_B - M_D + \frac{\lambda^2}{2}\left(\frac{1}{m_c} - \frac{1}{m_b}\right) + \frac{\lambda^3}{4}\left(\frac{1}{m_c^2} - \frac{1}{m_b^2}\right) + ...$$

$$\lambda \ll \sqrt{\mu_G^2} \simeq 0.6\,\mathrm{GeV}$$

Should be used instead of traditional $M_{\bar{B}} - M_{\bar{D}}$ to constrain $m_b - m_c$, if necessary. Can be checked from the upcoming data

A chain of higher-order corrections is suppressed

A class of higher-order nonperturbative effects becomes tractable

Zero recoil $B \to D$: $\quad \delta_{1/m^k} = 0 \quad$ regardless of mass ratio

Moreover, at arbitrary velocity power corrections in $B \to D$ vanish (or only kinematic)

Decay rate directly gives the IW function

Experiment: $B \to D$ slope much closer to $\varrho^2 \simeq 0.9$

$$B \to D^*: \qquad \delta_{1/m^2} = -(1+\chi)\frac{\mu_G^2}{6m_c^2}$$

usually assume $0 < \chi < 1$

If $1 - \frac{\mu_G^2}{6m_c^2}$ itself is just normalization, what about the overlap, maybe χ is small as well?

corrections, χ is significant in $B \to D^*$ and δ_{1/m^2}, δ_{1/m^3}, ... are not suppressed at all

$$\mathrm{BPS:} \qquad F_{D^*} \lesssim 0.9$$

Likewise corrections to the `shape` of the $B \to D^*$ formfactor are way too significant

Irreducible uncertainties:

$$e^{-\frac{2m_c}{\mu_{\mathrm{hadr}}}} \sim \text{ a few } \%$$

Conclusions:

Inclusive studies yield crucial info for HQ physics, exclusive amplitudes *ibidem*   Formerly viewed as antipodes

Power corrections to HQ symmetry are very significant in charm.   There is a subset of relations which are stable, they are limited to the ground-state pseudoscalar B and D mesons, but exclude `spin` symmetry for charm

$B \to D$ decays can be reliable theory-wise

Inclusive decays: we do not rely on charm expansion, only on $1/m_b$. Study decays of B, not of B^* – additional safety

FNAL lattices do not seem to reproduce smallness of $\mu_\pi^2 - \mu_G^2$. Their other numerical results seem to be in a qualitative agreement with a large μ_π^2 – may be taken with a grain of salt

Experiment must verify the actual value of the kinetic expectation value, with higher accuracy and fidelity in inclusive decays

If $\mu_\pi^2 \lesssim 0.43\,\mathrm{GeV}^2$ is confirmed then

$\mathcal{F}_+(0) \simeq 1.035$ is a reliable prediction for $B \to D$

Slope ϱ^2 is close to 1 -

Fits of $B \to D^*$ should incorporate constrains on $\hat{\varrho}^2$