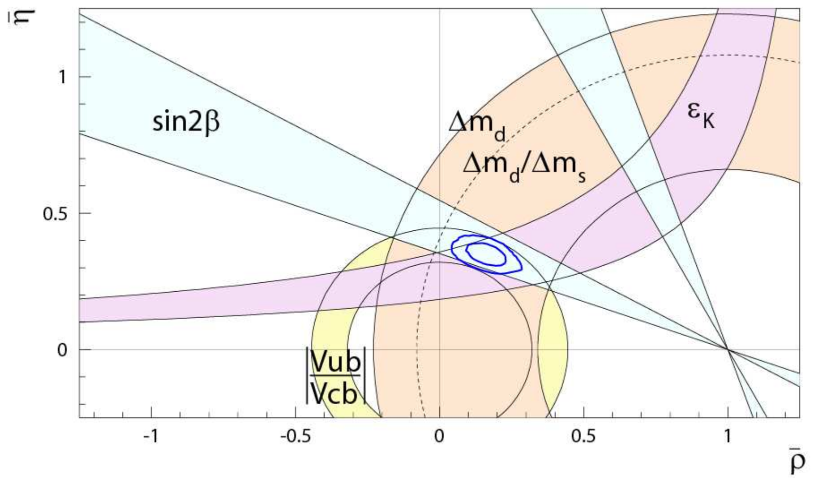
Determination of Unitarity Triangle parameters



Achille Stocchi

LAL-Orsay

Phenomenology Workshop on Heavy Flavours

Ringberg Schloss 28 April – 2 May 2003

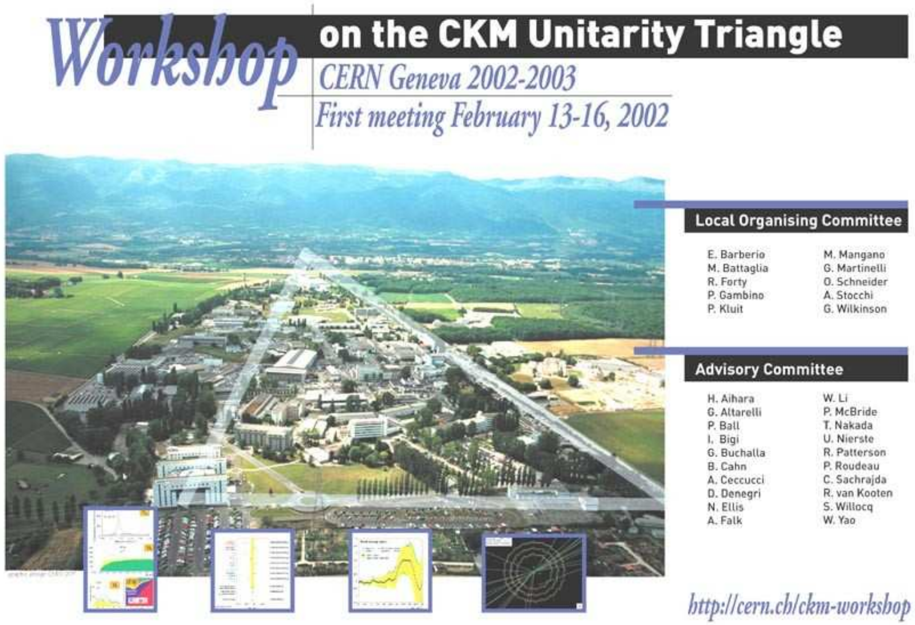
-Introduction (Unitarity Triangle)

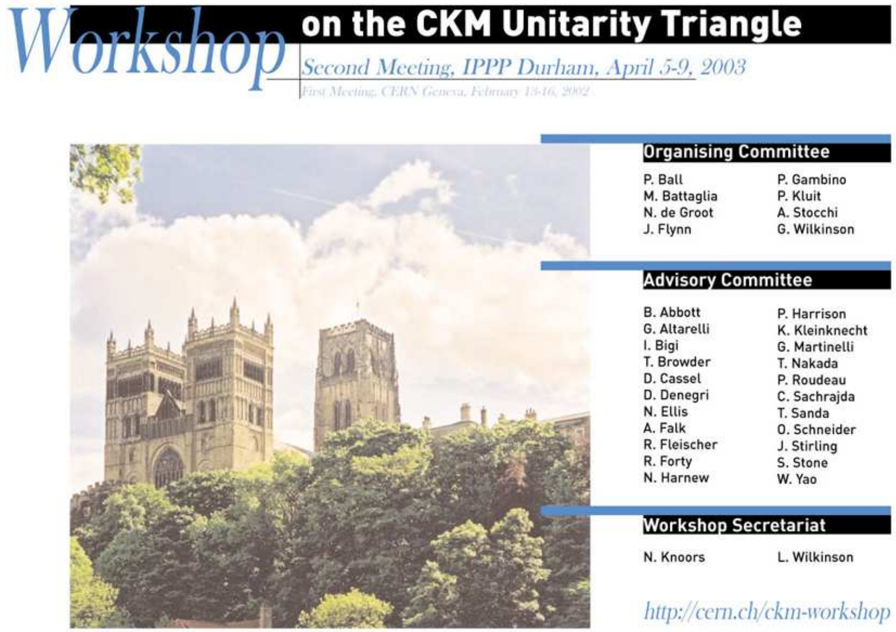
-The measurements/theoretical inputs

- Statistical method – Comparison (very brief)

- Results. Determination of the Unitarity Triangle parameters

All these topics extensively discussed at





hep-ph/0304132

will be submitted as
CERN-Yellow Book

THE CKM MATRIX AND THE UNITARITY TRIANGLE

Based on the workshop held at CERN, 13-16 February 2002

Organised as a coherent document

(work during 12 months between the 2 Workshops)

98 authors

~ 46 theorists
~ 52 experimentalists

330 pages

Editors: M. Battaglia, A.J. Buras, P. Gambino, A. Stocchi

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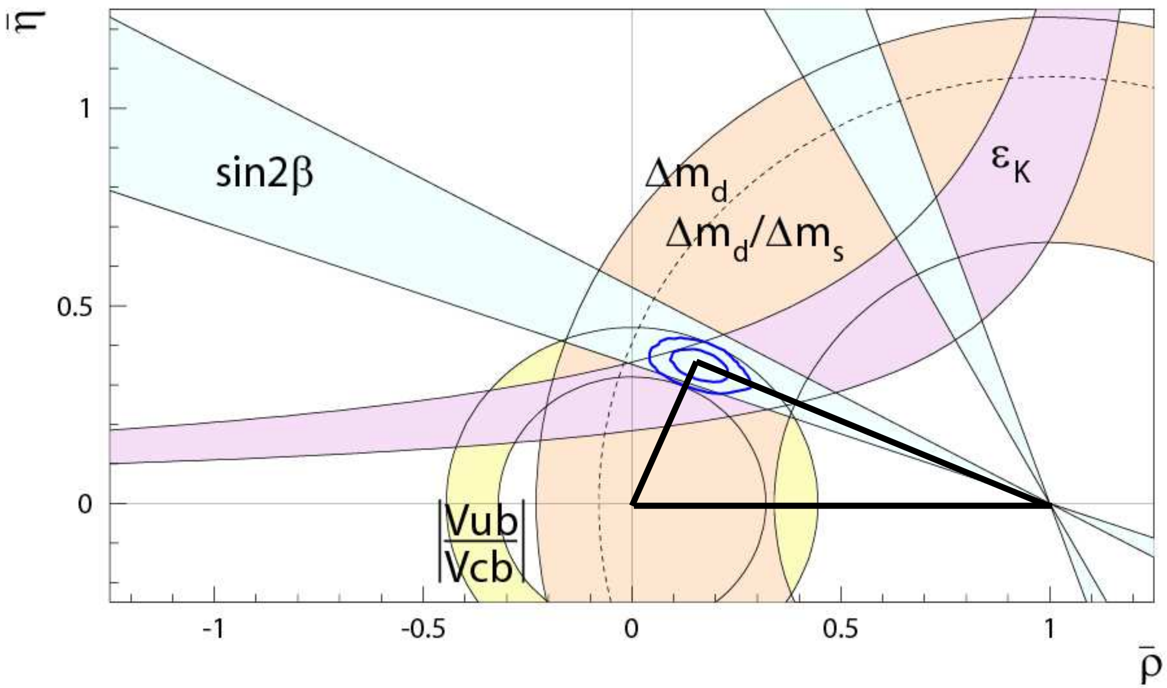


(b→u)/(b→c)	$\bar{\rho}^2 + \bar{\eta}^2$	$\bar{\Lambda}$, λ_1, F(1), …
ε_K	$\bar{\eta}\,[(1-\bar{\rho}) + P]$	B_K
Δm_d	$(1-\bar{\rho})^2 + \bar{\eta}^2$	$f_B^2 B_B$
$\Delta m_d / \Delta m_s$	$(1-\bar{\rho})^2 + \bar{\eta}^2$	ξ
$A_{CP}(J/\psi, K_S)$	$\sin(2\beta)$	-

, m_t

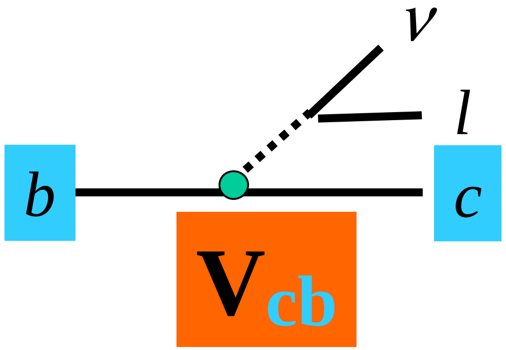
Standard Model + theories which give the link from quarks to hadrons

OPE /HQET/Lattice QCD …. **Need to be tested**

Short review on the inputs

V_{cb} - Inclusive Method

$\Gamma_{sl} = (0.434 \times (1 \pm 0.018))\ 10^{-10}$ *MeV* **2%**



$$\Gamma_{sl}(b \to c l^- \nu) = \frac{Br_{sl}}{\tau_b} = |V_{cb}|^2\ F$$

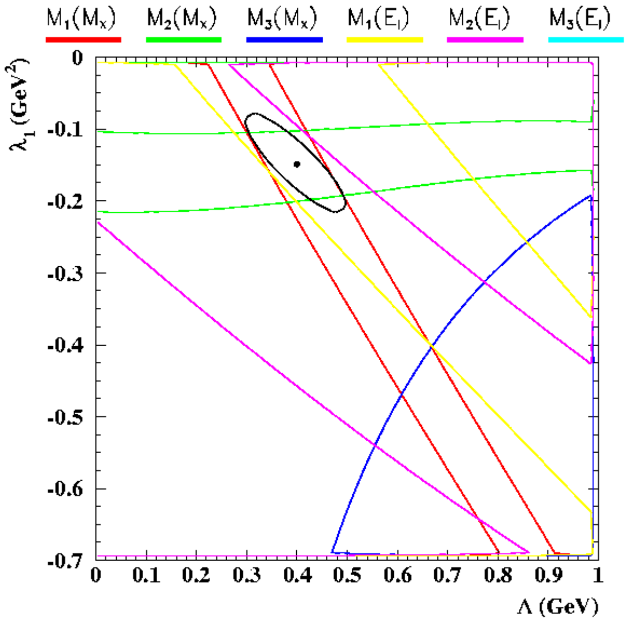
f(μ^2_π , $m_{b,}$, m_c α_s , ρ_D(or $1/m_b^3$))

Based on ***OPE***

m_b	(also named $\overline{\Lambda}$)	μ^2_π	(λ_1 Fermi movement)

Moments of distributions

HADRONIC mass,
LEPTON Momentum ,
Photon energy bà s γ



$M_{b,kin}$(1GeV) = 4.59 ± 0.08 ± 0.01GeV à 4.23(mb(mb))
$m_{c,kin.}$ (1GeV) = 1.13 ± 0.13 ± 0.03GeV
μ_π^2 = 0.31 ± 0.07 ± 0.02GeV²
ρ_D^2 = 0.05 ± 0.04 ± 0.01GeV²
à terms $1/m_b^3$ (under control?)/small !

V_{cb}(inclusive)= (41.4 ± 0.6 ± 0.7(theo.)) 10^{-3}

Exp + (μ^2_π , $m_{b,}$, ρ_D ….absorbed !)

Pert. QCD. α_s , terms $1/m_b^4$

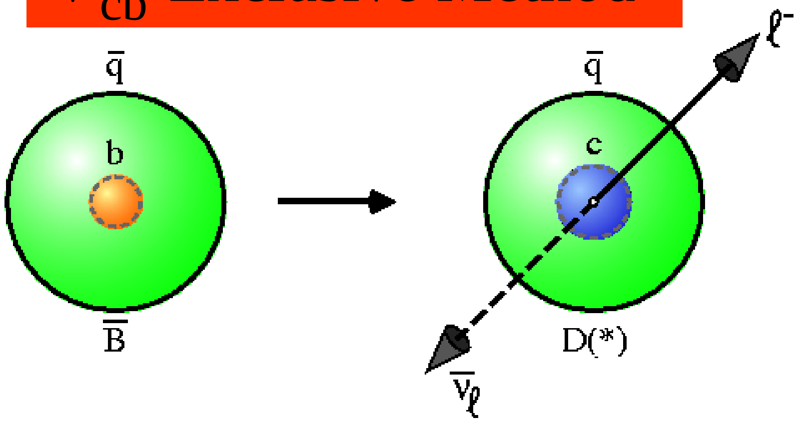
hep-ph/0210027 C.Bauer,Z.Ligeti,M.Luke,A.Manohar

hep-ph/0210319, M.Battaglia et al. (P.Gambino,N.Uraltsev)

hep-ph/0302262 D. Benson,I.Bigi,T.Mannel,N.Uralstev

V_{cb}-Exclusive Method

*Based on **HQET***

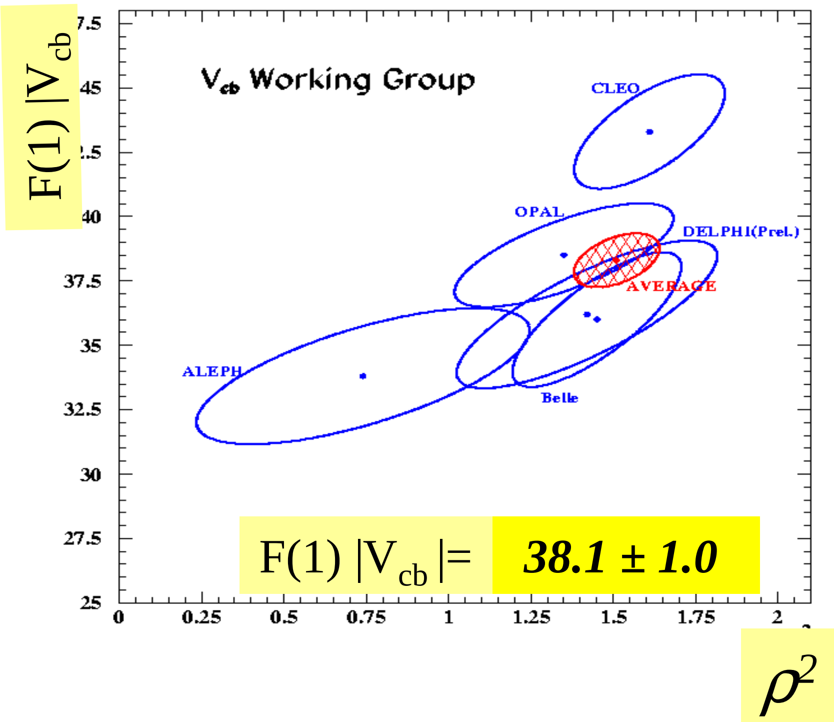


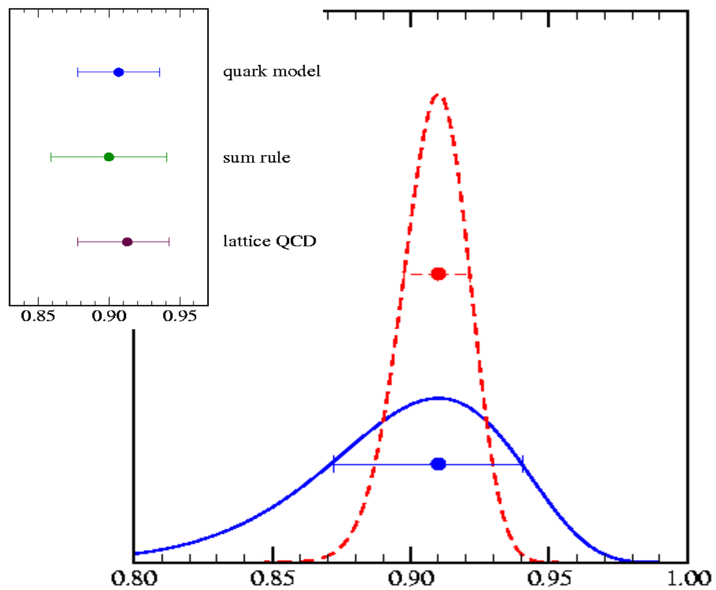
$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^2} \; |V_{cb}|^2 \; |F(w)|^2 \; G(w)$$

$$w = v_B . v_{D^{(*)}}$$

$$w = \frac{m_{D*}^2 + m_B^2 - q^2}{2 m_{D*} m_B}$$

At zero recoil (w=1),
as M_Q à ∞ F(1) à 1



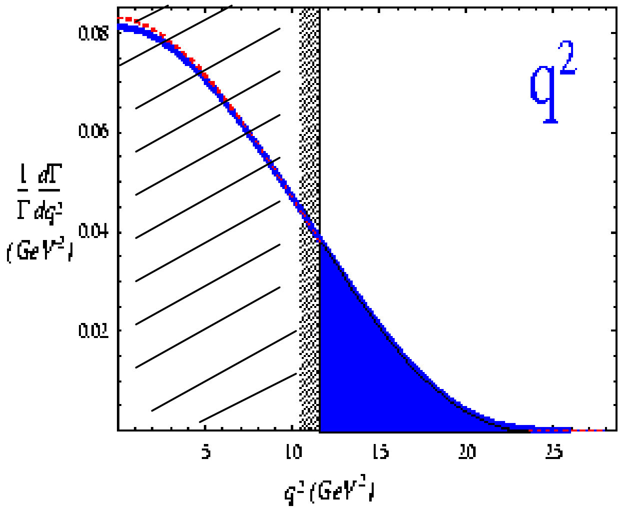


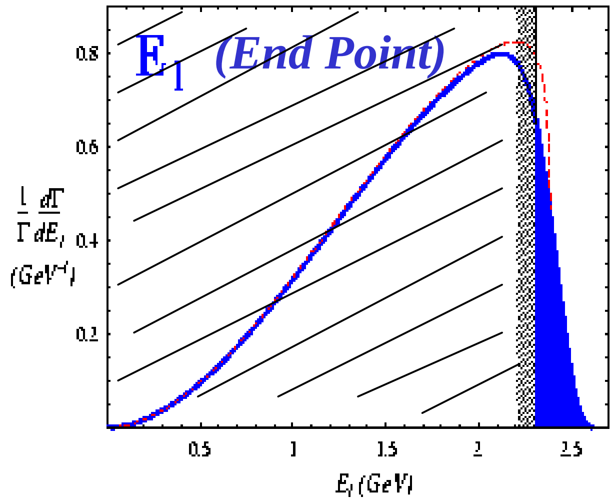
F(1) ~ 0.91 ± **0.04**

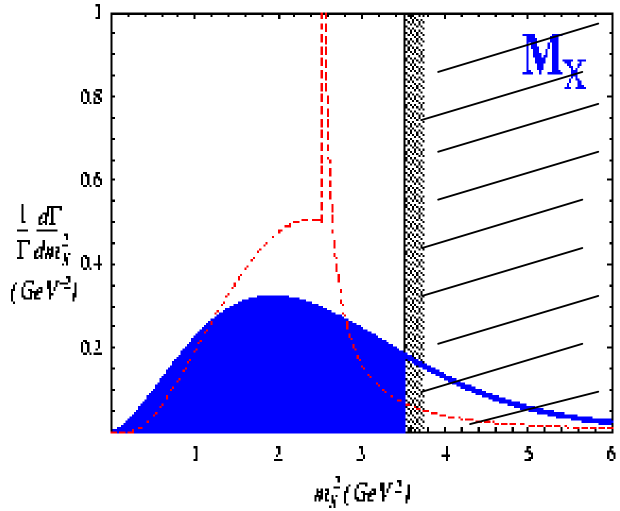
V_{cb}(exclusive)= (42.1± 1.1 ± 1.9) 10^{-3}

V_{ub} Inclusive methods

B à X_u l^+ ν

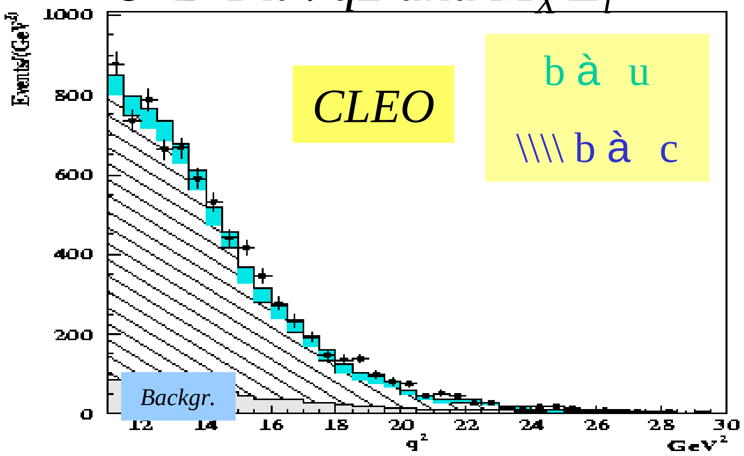


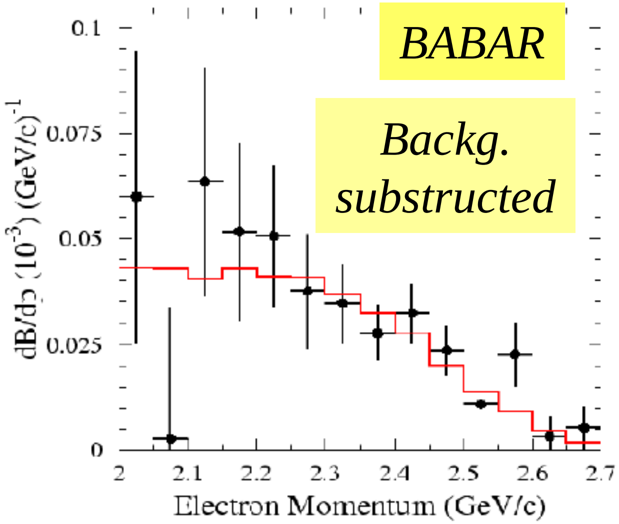


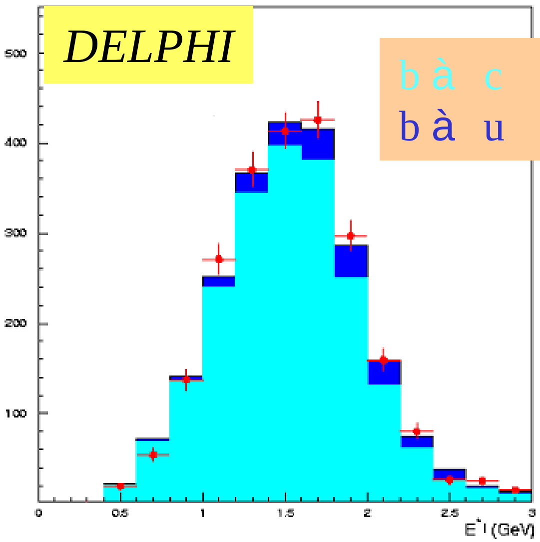


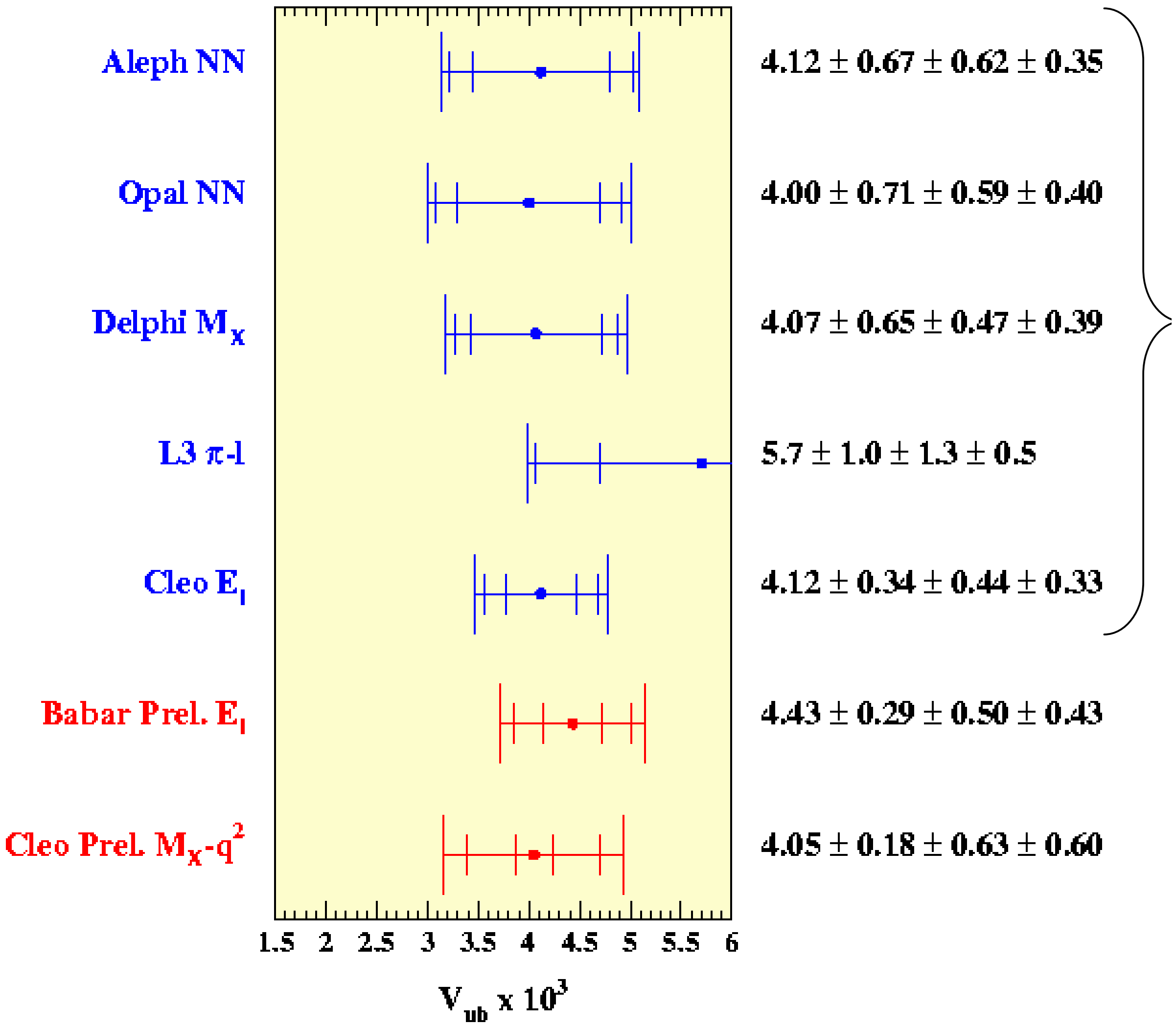
$M^2_{\ell\nu} = q^2 > (M_B - M_D)^2$ $\quad E_\ell > \frac{M_B^2 - M_D^2}{2M_B}$ $\quad M_{uq} < M_{cq}$

3–D Fit : q2 and M_X E_l







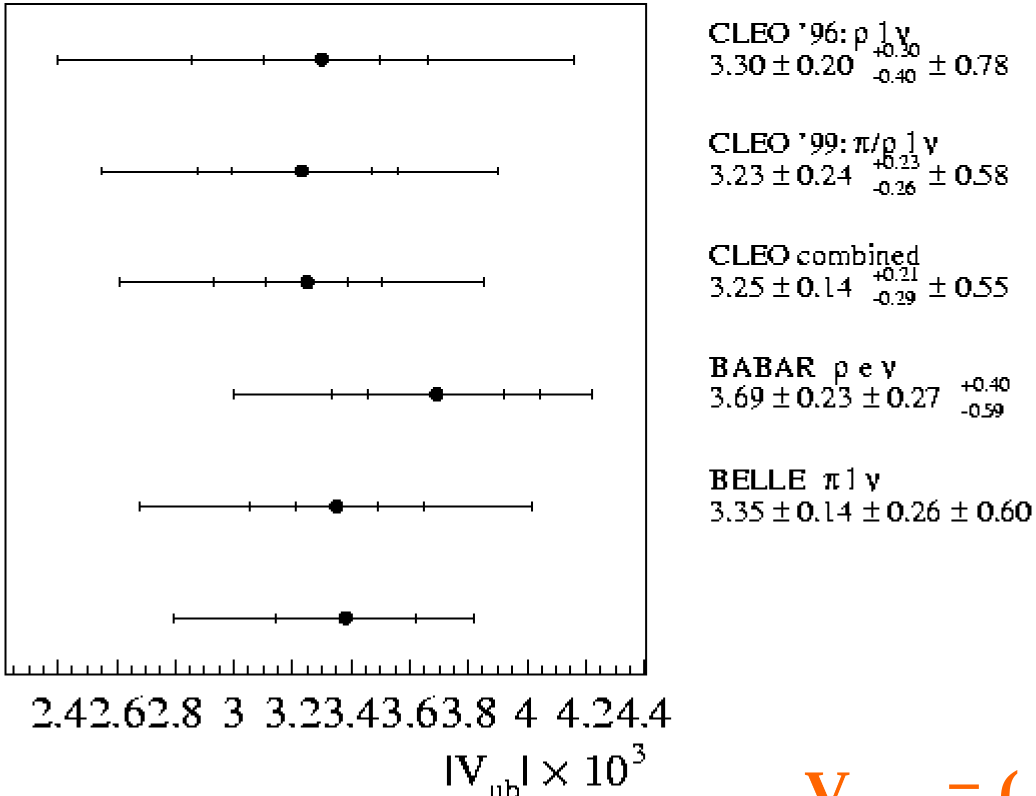


Conservative approach
syst. fully correlated

V_{ub} =
$(4.09 \pm 0.46 \pm 0.36)\ 10^{-3}$

Exclusive methods

B à $(\pi,\rho,\omega)\, l\ \nu$



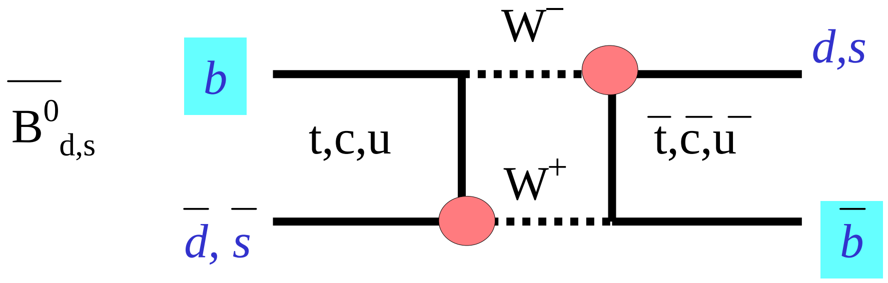
…

Common to all analyses

$$V_{ub} = (\ 3.30 \ \pm 0.24 \pm 0.46)\ 10^{-3}$$

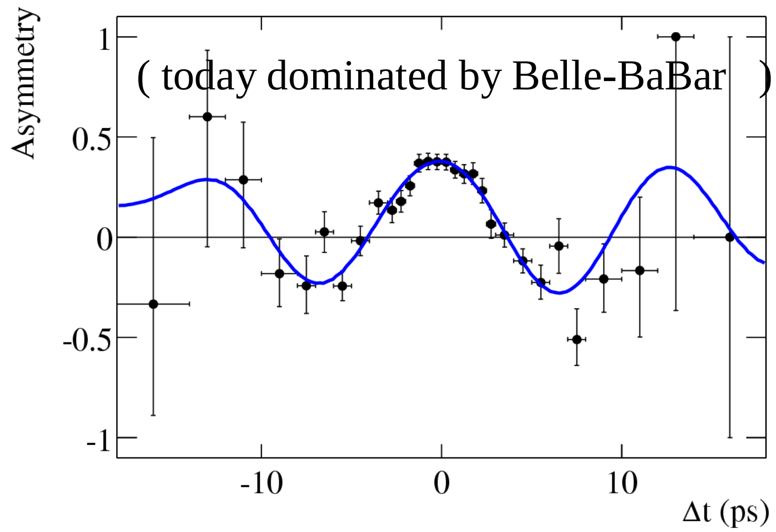
Error : dominated by form factor errors as F(1) in V_{cb}

Oscillations in B^0_d system : Δm_d



→ V_{td} V_{ts}

$$P_{B_q^0 \to B_q^0 (\overline{B_q^0})} = \frac{1}{2} e^{-t/\tau_q} (1 \pm \cos \Delta m_q t)$$



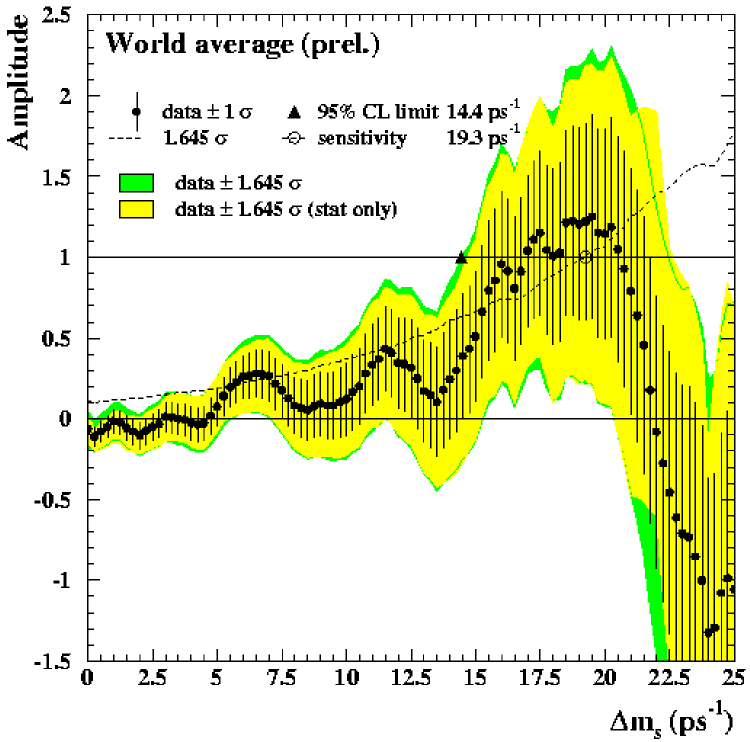
$$\Delta m_d \propto f_{B_d}^2 \, B_{B_d} \left|V_{cb}\right|^2 \lambda^2 \left|V_{td}\right|^2$$

$$\propto f_{B_d}^2 \, B_{B_d} \left|V_{cb}\right|^2 \lambda^2 \left((1-\bar{\rho})^2 + \bar{\eta}^2\right)$$

$\Delta m_d = 0.502 \pm 0.006\ \mathrm{ps}^{-1}$
LEP/SLD/CDF/B-factories

Precise measurement (1.2%)

Oscillations in B^0_s system : Δm_s



$$\Delta m_s \propto f^2_{B_s} B_{B_s} |V_{td}|^2 \propto f^2_{B_s} B_{B_s} |V_{cb}|^2$$

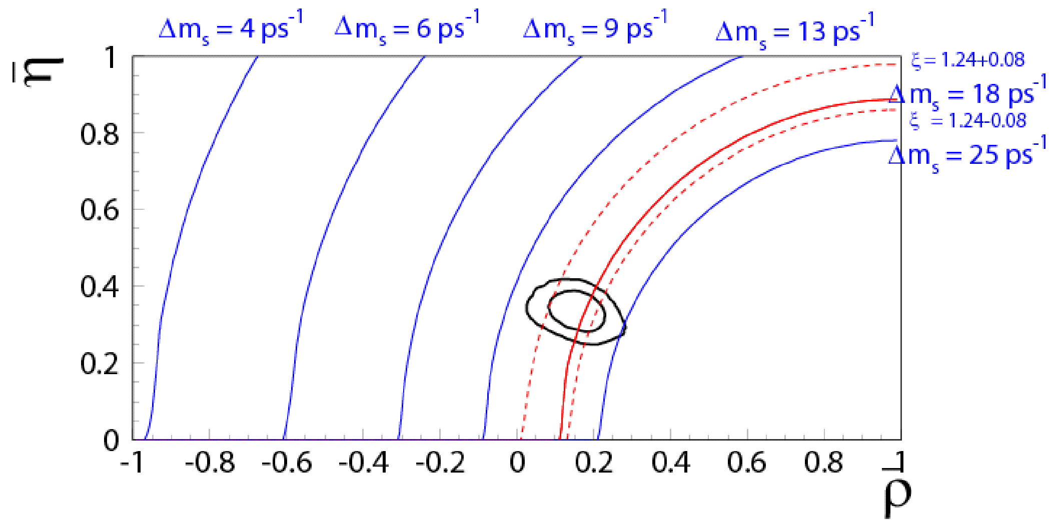
$$\frac{\Delta m_d}{\Delta m_s} \propto \frac{f^2_{B_d} B_{B_d}}{f^2_{B_s} B_{B_s}} \lambda^2 \left((1-\bar{\rho})^2 + \bar{\eta}^2\right)$$

$1/\xi^2$

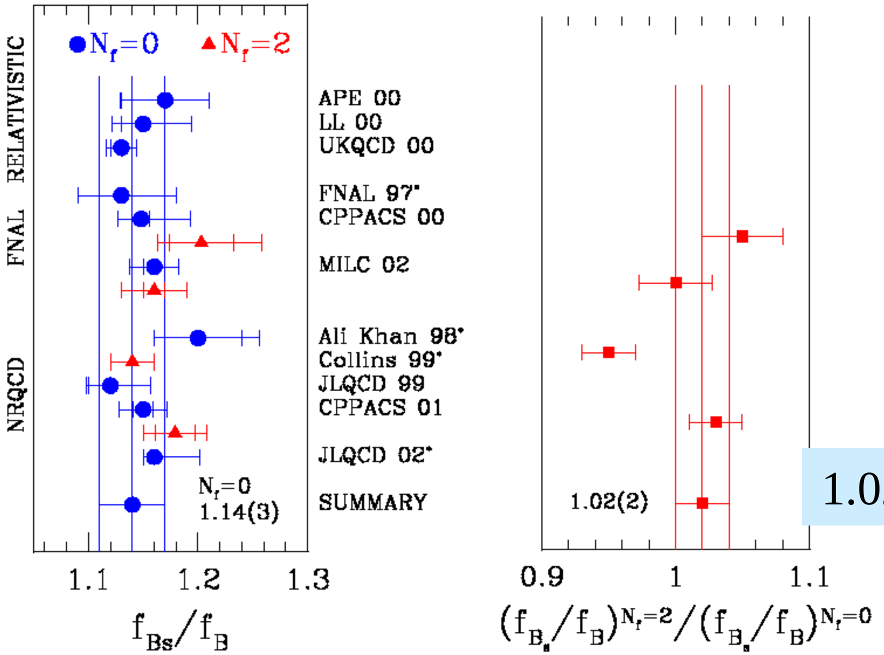
Δm_s > 14.4 ps^{-1} at 95% CL

Sensitivity at 19.3 ps^{-1}

LEP/SLD/CDF-I



ξ



1.02±0.02

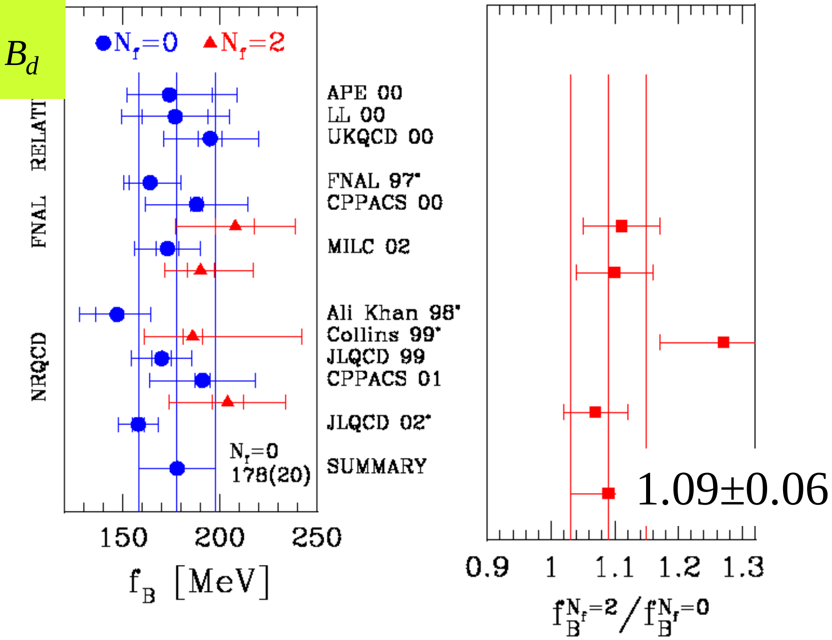
Calculation partially unquenched (N_f=2 or 2+1) in agreement

$$\frac{f_{B_s}}{f_{B_d}} = 1.18 \pm 4^{+12}_{-0}, \quad \frac{B_{B_s}}{B_{B_d}} = 1.00 \pm 0.03$$

$$\xi = \frac{f_{B_s}}{f_{B_d}} \frac{\sqrt{\hat{B}_{B_s}}}{\sqrt{\hat{B}_{B_d}}} = 1.24 \pm 0.04 \pm 0.06$$

Chiral extrapolation : light quarks simulated typically in a range [$m_s/2$ - m_s]

$f_{B_d}\sqrt{\hat{B}_{B_d}}$



Calculation partially unquenched
(N_f=2 or 2+1) in agreement

$$f_{B_d} = 203 \pm 27^{+0}_{-20}\, MeV,\; B_{B_d} = 1.34 \pm 0.12$$

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 223 \pm 33 \pm 12\, MeV$$

$$(Sum-Rules \quad f_{B_d} = 208 \pm 27\, MeV,\; B_{B_d} = 1.67 \pm 0.23)$$

(syst not correlated ~m_b)

$\hat{B}_K$

$$B_K(2GeV) = 0.63 \pm 0.04 \quad \rightarrow \hat{B}_K = 0.86 \pm 0.06$$

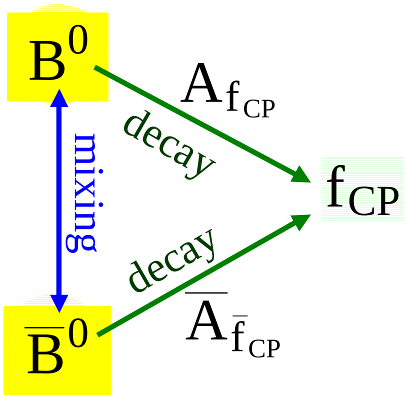
1.05±0.15 unquenching factor

1.05±0.05 SU(3) effects factor

$$\hat{B}_K = 0.86 \pm 0.06 \pm 0.14$$

$\sin 2\beta$

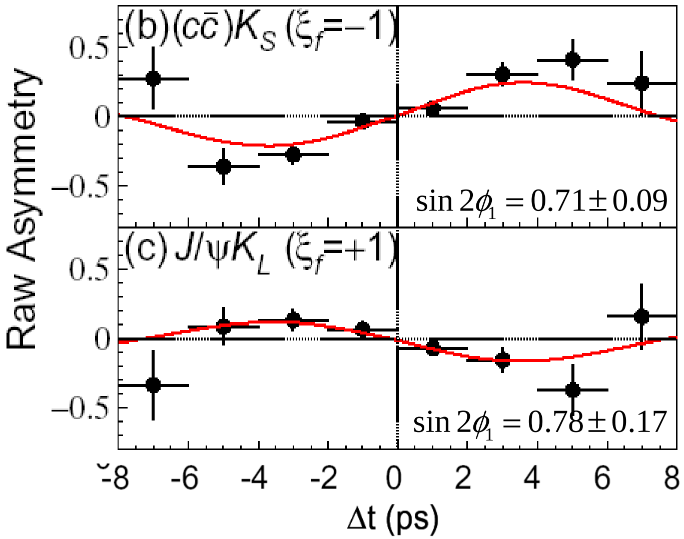
CP violation comes from interference between decays with and without mixing

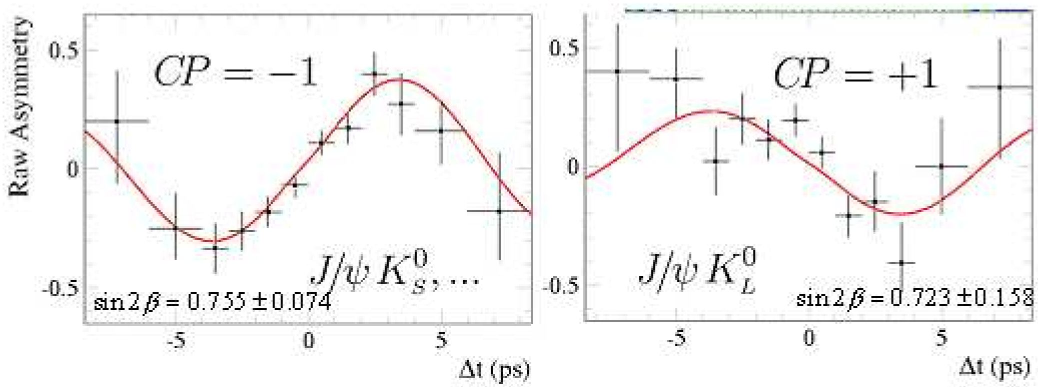


$$a_{f_{CP}}(t) = \frac{\Gamma(\bar{B}^0_{phys}(t)\to f_{CP}) - \Gamma(B^0_{phys}(t)\to f_{CP})}{\Gamma(\bar{B}^0_{phys}(t)\to f_{CP}) + \Gamma(B^0_{phys}(t)\to f_{CP})}$$

$$= C_{f_{CP}} \cos(\Delta m_d t) + S_{f_{CP}} \sin(\Delta m_d t)$$

$$\sim -\ \eta_{J/\psi K^0_{S,L}}\ \sin 2\beta \sin(\Delta m_d t)$$





$$\sin 2\beta = 0.734 \quad \pm 0.054$$

Determination of V_{ud}, V_{us}

K_{l3} decays : K à π l ν

Neutron β decay

β transition of $J^P=0^+$ nuclei

V_{ud} =0.9736 ± 0.0005

Using unitarity $V^2_{ud} + V^2_{us} + V^2_{ub} = 1$, $\sigma(V_{us}) = 1/\lambda\, \sigma(V_{ud})$

V_{us} = 0.2196 ± 0.0026

V_{us} = 0.2269 ± 0.0021

1%

2.2σ discrepancy

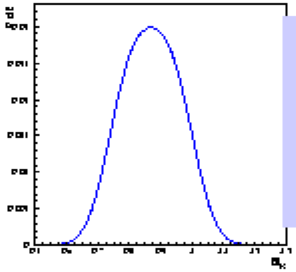
Attributing it to an underestimate of syst. error (theo/exp) or (an unlikely stat. fluctuaction) à **inflate the error**

V_{us} = 0.2240 ± 0.0036

Treatment of the inputs

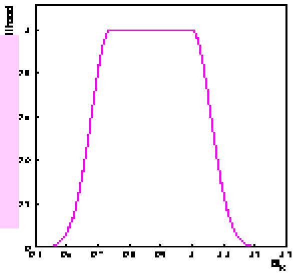
Ex : $B_K = 0.86 \pm 0.06$ (Gaus.) ± 0.14 (theo.)

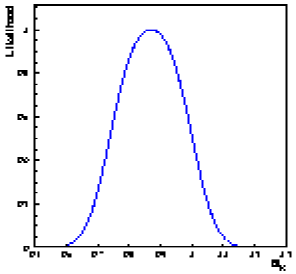
Scan	Rfit	Bayesian



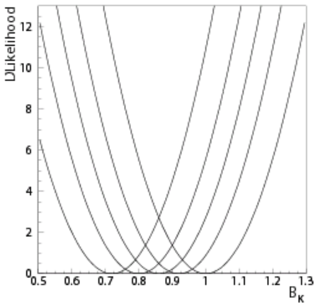
p.d.f.
from convolution
(**sum in quadrature**)

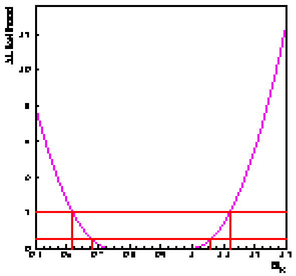
Likelihood
su**mming linearly**
the two errors

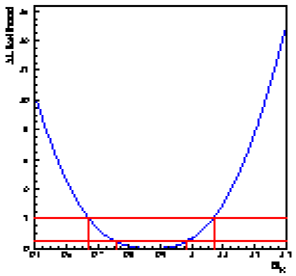




Likelihood





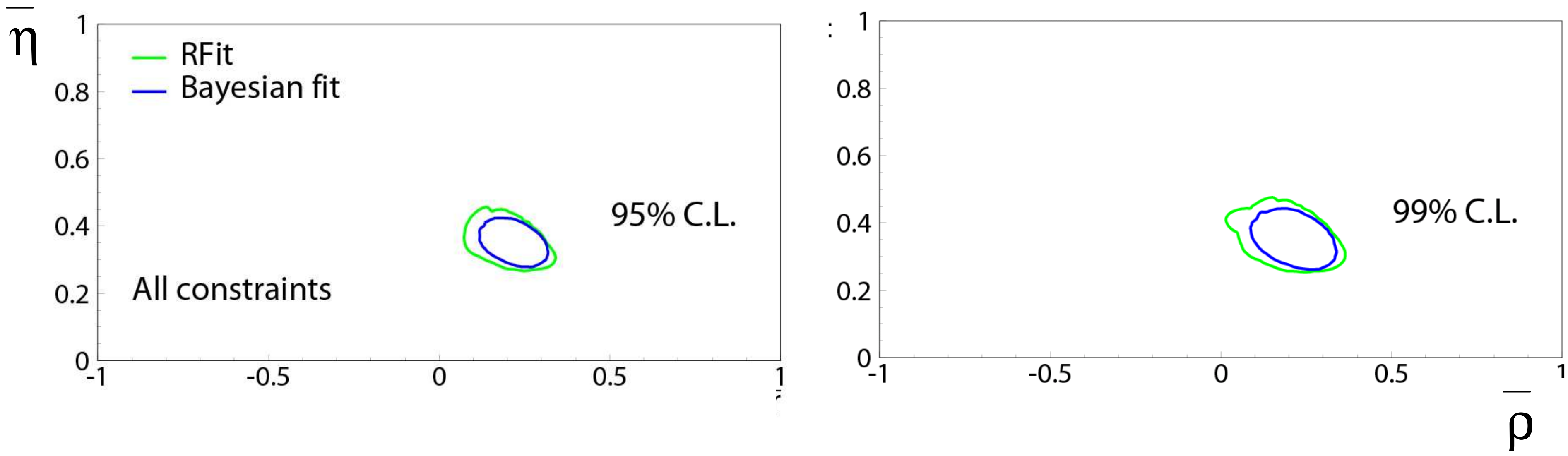


Delta Likelihood

Delta Likelihood

[0.68-1.06] [0.76-0.98] **At 68% CL**

FIT COMPARISON-same inputs



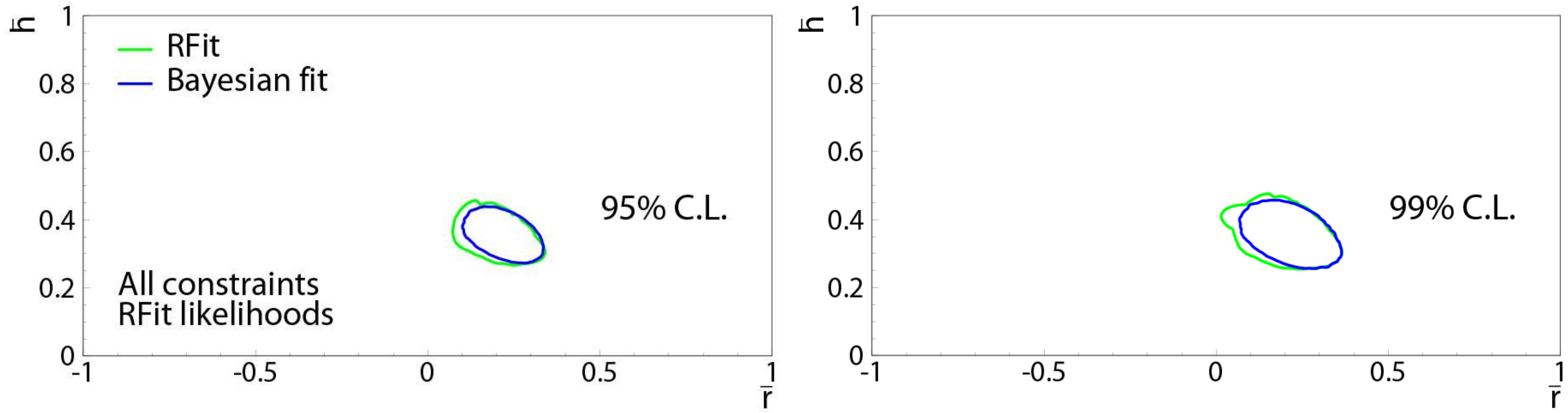
Ratio RFit/Bayesian Method			
Parameter	5% CL	1% CL	0.1% CL
$\bar{\rho}$	1.42	1.34	1.12
$\bar{\eta}$	1.18	1.12	1.05
$\sin 2\beta$	1.16	1.16	1.17
γ°	1.51	1.31	1.09

Ratio between sizes of intervals corresponding to a given CL

Quantitative differences in the selected (ρ,η) regions between Bayesian and frequentist are small

“The main origin of the difference on the output quantities between the Bayesian and the Rfit method comes from the likelihood associated to the input quantities”

Both methods use the same likelihood



Conclusion of the CERN Workshop:

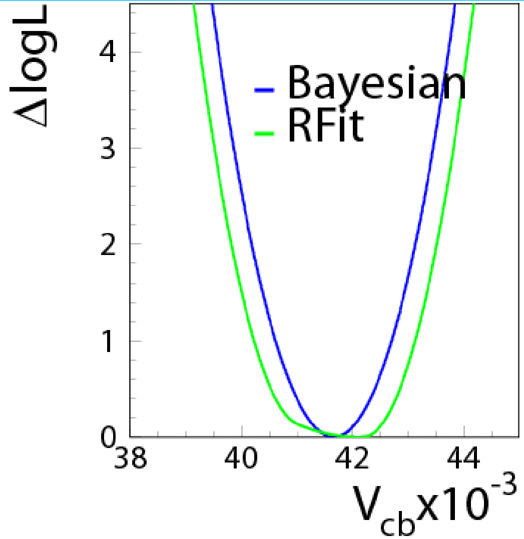
“If same (and any) likelihood are used the output results are very similar”

Parameter	Value	Error(Gaussian)	Error(Flat)
λ	0.2240	0.036	
V_{cb} ($\times 10^{-3}$) (excl.)	42.1	2.1	
V_{cb} ($\times 10^{-3}$) (incl.)	41.6	0.7	0.6
V_{ub} ($\times 10^{-4}$) (excl.)	33.0	2.4	4.6
V_{ub} ($\times 10^{-4}$) (incl.)	40.9	4.6	3.6
Δm_d (ps^{-1})	0.503	0.006	
Δm_s (ps^{-1})	> 14.4 ps^{-1} at 95% CL		
m_t (GeV)	167	5	
m_c (GeV)	1.3		0.1
$f_{B_d}\sqrt{\hat{B}_{B_d}}$ (MeV)	223	33	12
ξ	1.24	0.04	0.06
B_K	0.86	0.06	0.14
$\sin 2\beta$	0.734	0.054	

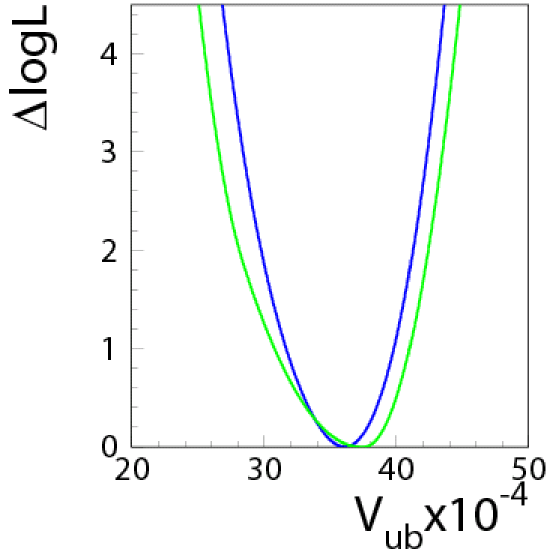
Combination of V_{cb} and V_{ub} incl/excl

No correlation between the incl/excl measurements

V_{cb} = (41.5 ± 0.8) 10^{-3}



V_{ub} = (35.7 ± 3.1) 10^{-4}



Differences if :

If the theoretical/statistical errors are
- Convoluted (Bayesian)
- Linearly (frequentist-Rfit)

(difference of ~20% @ 95% C.L.)

V_{cb} know at ~2%

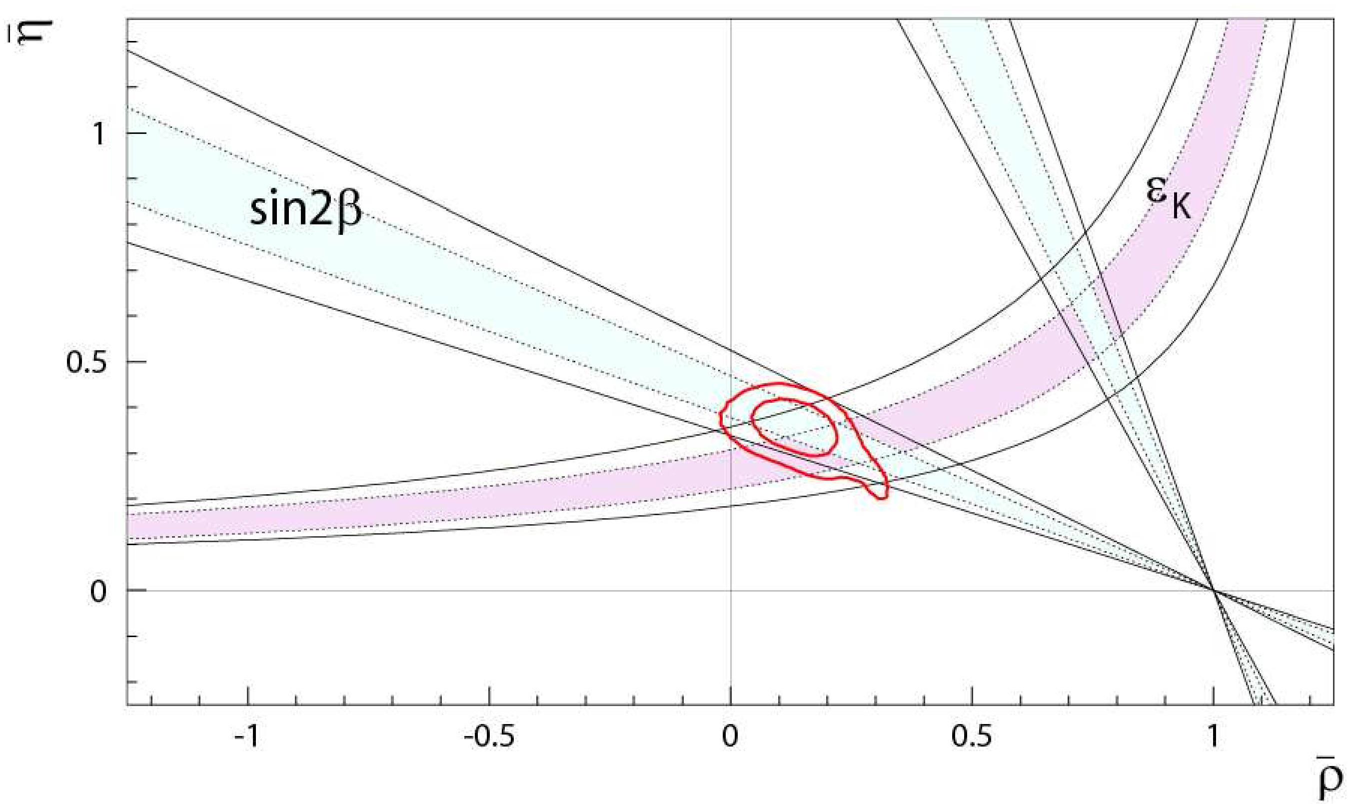
Precision driven by incl. method

V_{ub} know at ~10%

Results on
Unitarity Triangle parameters

Buras,Ciuchini,Franco,Lubicz,Martinelli,Parodi,Roudeau,Silvestrini,Stocchi

Crucial Test of the SM in the fermion sector

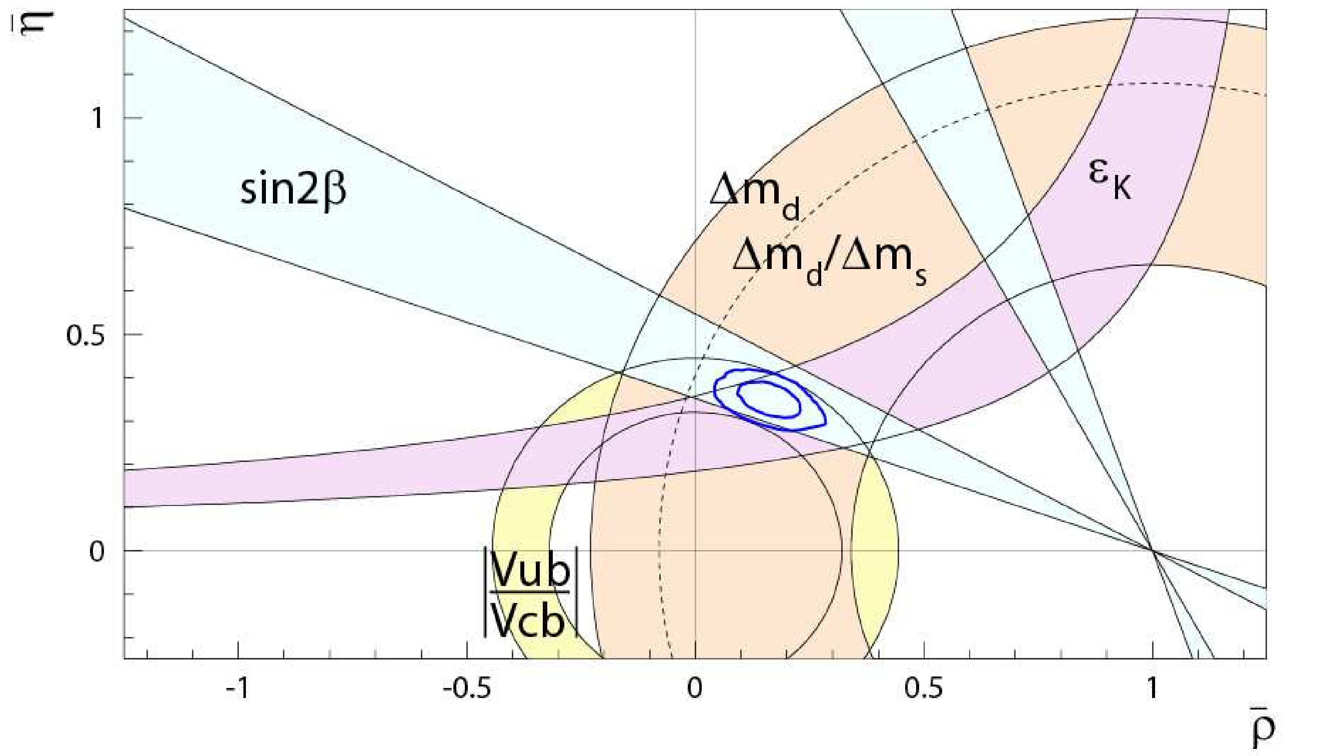


$\sin 2\beta = 0.734 \pm 0.054\ (0.628 - 0.840)$ B à J/ψ K^0_s

$\sin 2\beta = 0.695^{+0.054}_{-0.045} \quad (0.562 - 0.789)$ *from sides-only*

Coherent picture of CP Violation in SM

Fit of the Unitarity Triangle in SM



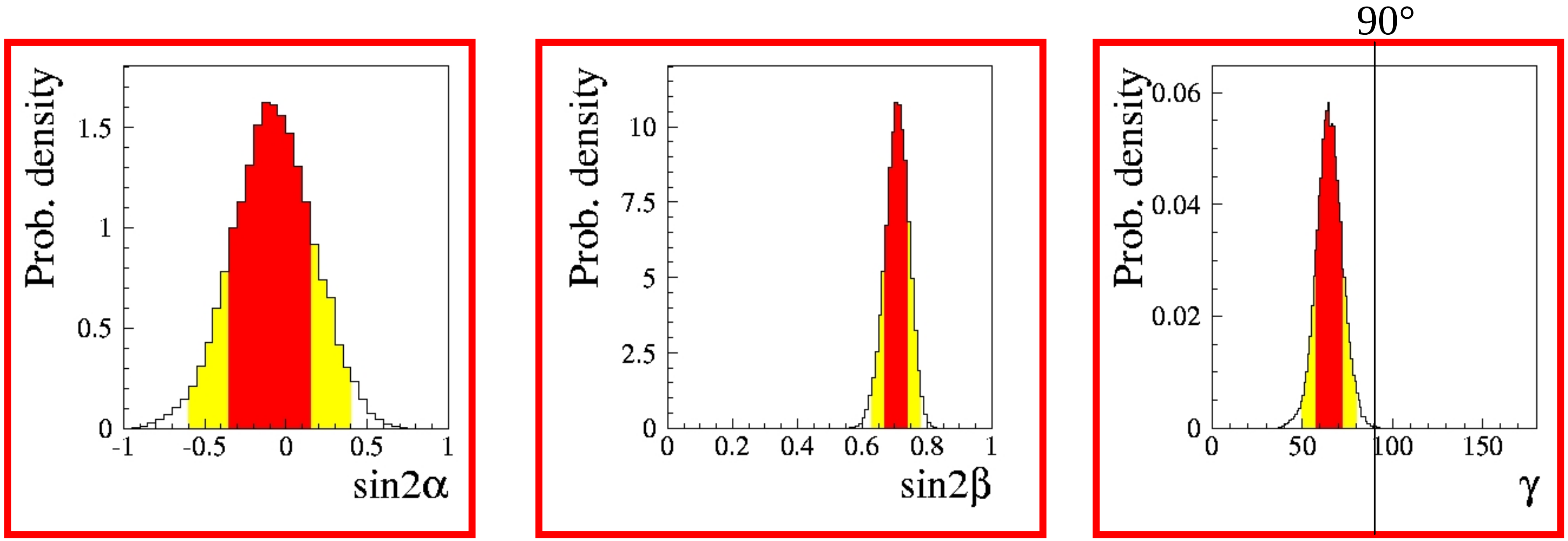
$$\bar{\rho} = 0.162 \pm 0.046$$

$$[0.067 - 0.255] \quad @95\% \quad C.L.$$

$$\bar{\eta} = 0.347 \begin{array}{l} + 0.029 \\ - 0.026 \end{array}$$

$$[0.291 - 0.403] \quad @95\% \quad C.L.$$

Indirect determination of the UT angles : sin2α, sin2β and γ



$$\sin 2\alpha = -0.13 ^{+0.28}_{-0.23}$$

$[-0.58 - 0.41]$ @95% *C.L.*

$$\sin 2\beta = 0.705 \pm 0.035$$

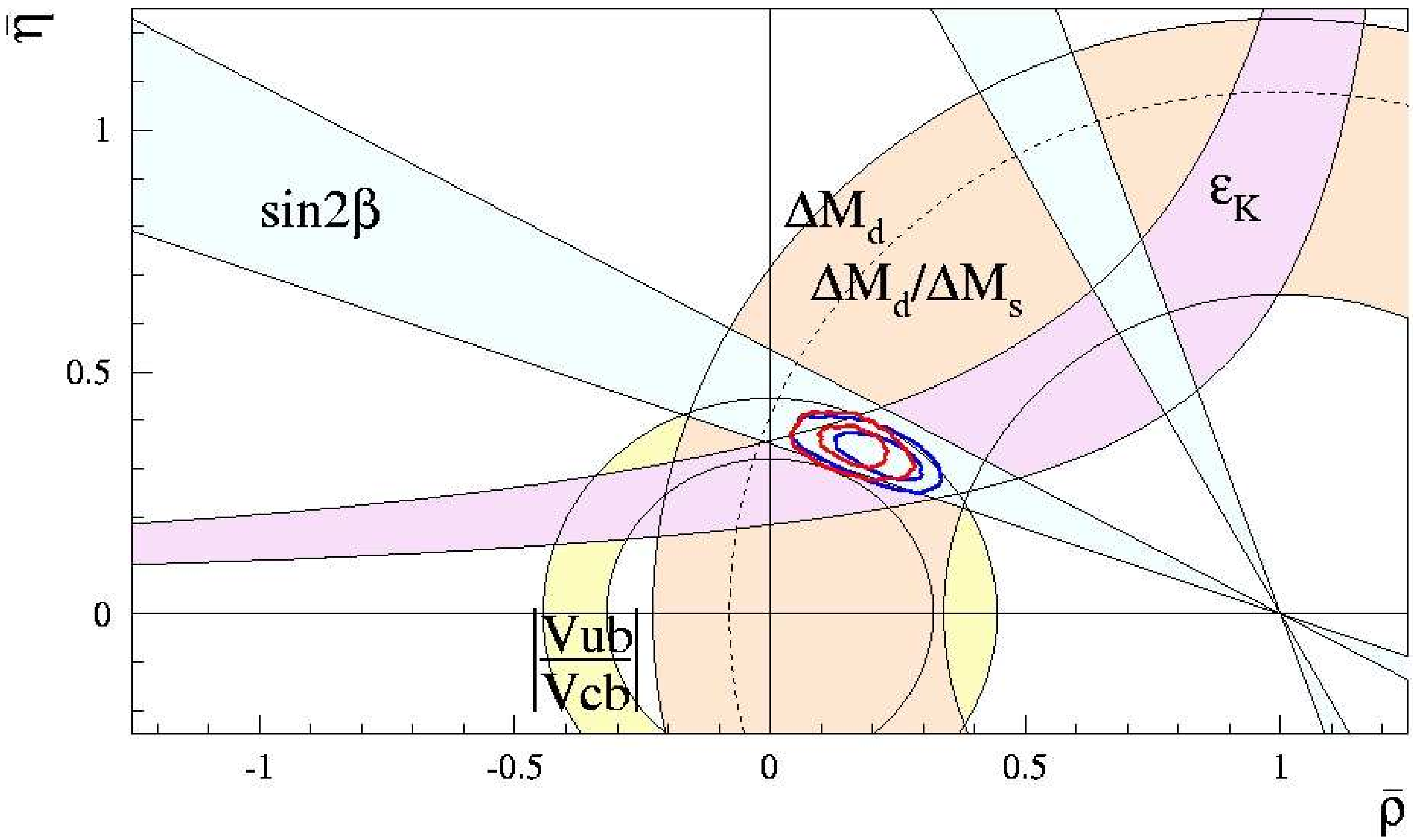
$[0.631 - 0.777]$ @95% *C.L.*

$$\gamma = (65 \pm 7)\ \text{degree}$$

$[50.5 - 79.8]$ @95% *C.L.*

$\gamma > 90°$ Prob~0.001

Without Δm_s $\gamma > 90°$ Prob~0.005



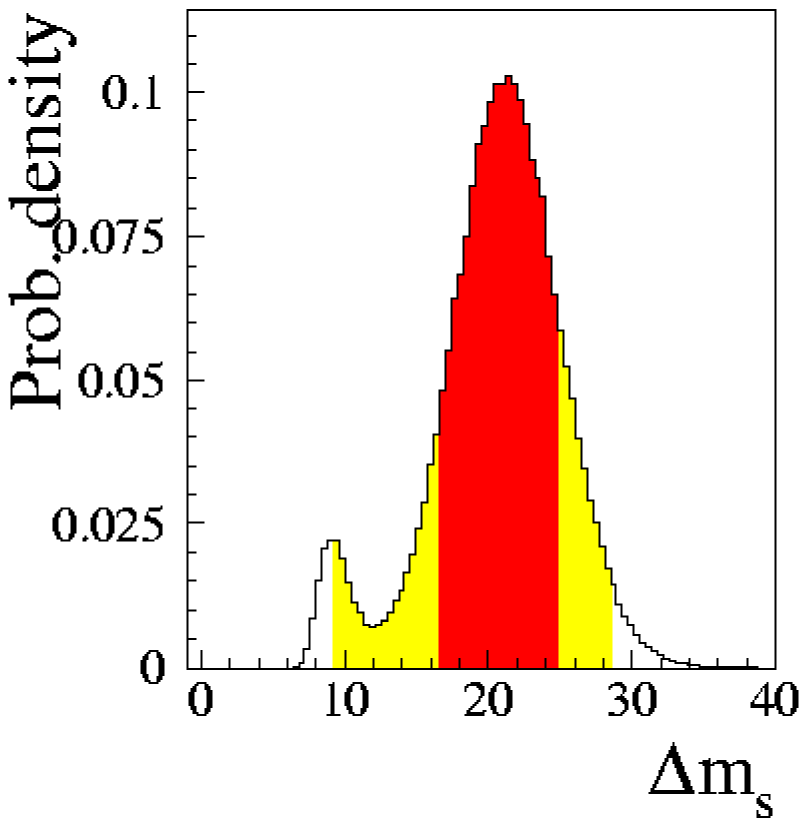
RED: WITH ALL CONSTRAINTS / BLUE: WITHOUT Δm_s

By removing the constraint from Δm_s: $\gamma = (65 \pm 7)°$ → $\gamma = (60 \pm 9)°$

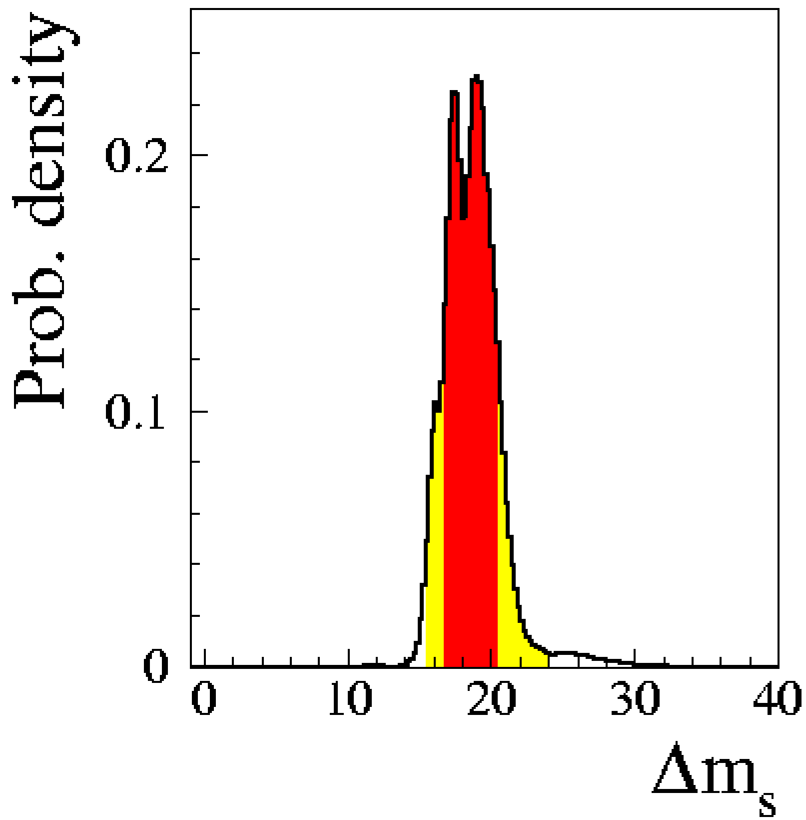
$\gamma > 90°$ Prob~0.005

Prediction for Δm_s

Without limit on Δm_s



With limit on Δm_s



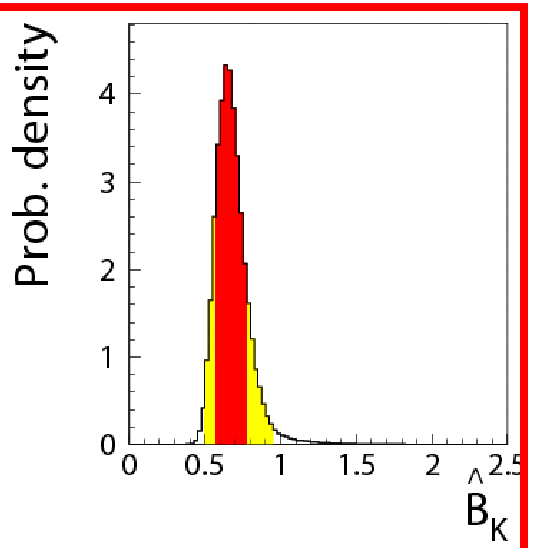
$$\Delta m_s = 20.9 \pm 4.0\, ps^{-1}$$

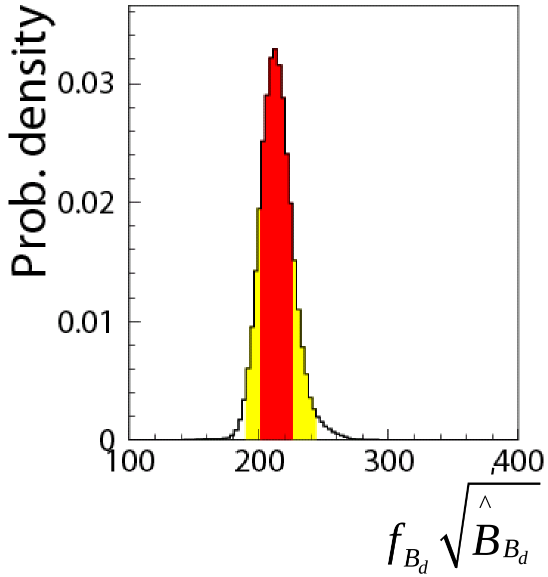
$[9.1 - 28.7]$ $@95\%$ $C.L.$

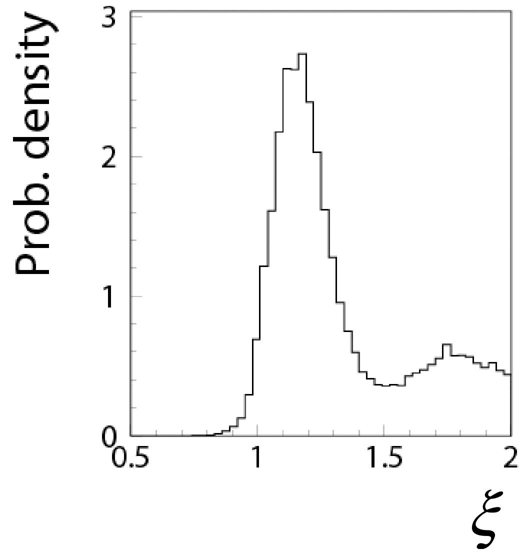
$$\Delta m_s = 18.6 \pm 1.7\, ps^{-1}$$

$[15.6 - 23.7]$ $@95\%$ $C.L.$

Indirect determination of the non-perturbative QCD parameters







$$\hat{B}_K = 0.66 \, {}^{+\,0.11}_{-\,0.09}$$

$[0.51 - 0.95] \quad @95\% \quad C.L.$

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 212.5 \pm 12.0 \quad \text{MeV}$$

$[190.5 - 243.5] \quad \text{MeV} \quad @95\% \quad C.L.$

$\hat{B}_K = 0.86 \pm 0.06 \pm 0.14$

$f_{B_d}\sqrt{\hat{B}_{B_d}} = 223 \pm 33 \pm 12 MeV$

$\xi = 1.24 \pm 0.04 \pm 0.06$

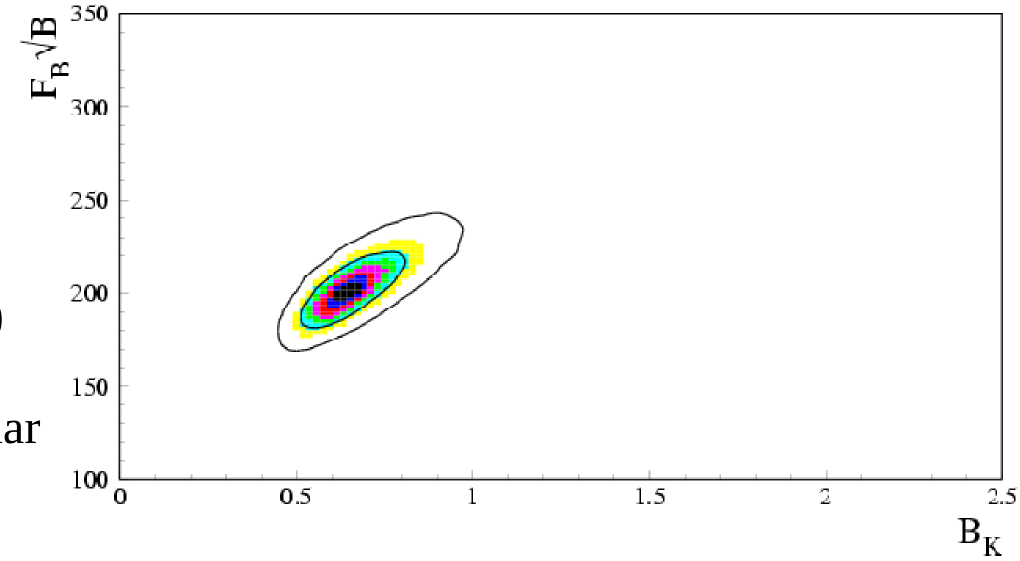
Using :

V_{ub}/ V_{cb}

Δm_d/ Δm_s

$A_{CP}(J/\psi ,K_S)$

So in particular knowing ξ



$$\hat{B}_K = 0.65 \begin{matrix} + \ 0.12 \\ - \ 0.08 \end{matrix}$$

$$[0.51 - 0.96] \quad @95\% \quad C.L.$$

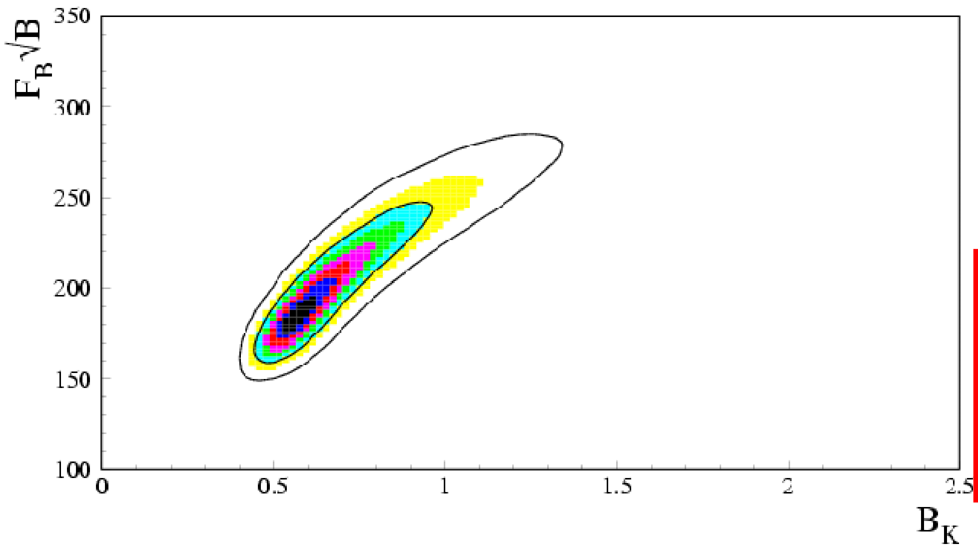
$$f_{B_d} \sqrt{\hat{B}_{B_d}} = 203 \begin{matrix} +17 \\ -13 \end{matrix} \quad \text{MeV}$$

$$[180 - 242] \quad \text{MeV} \quad @95\% \quad C.L.$$

Using :

V_{ub}/ V_{cb}

$A_{CP}(J/\psi ,K_S)$



$$\hat{B}_K = 0.67 \begin{matrix} + \ 0.26 \\ - \ 0.13 \end{matrix}$$

$$[0.47 - 1.27] \quad @95\% \quad C.L.$$

$$f_{B_d} \sqrt{\hat{B}_{B_d}} = 206 \begin{matrix} +38 \\ -28 \end{matrix} \quad \text{MeV}$$

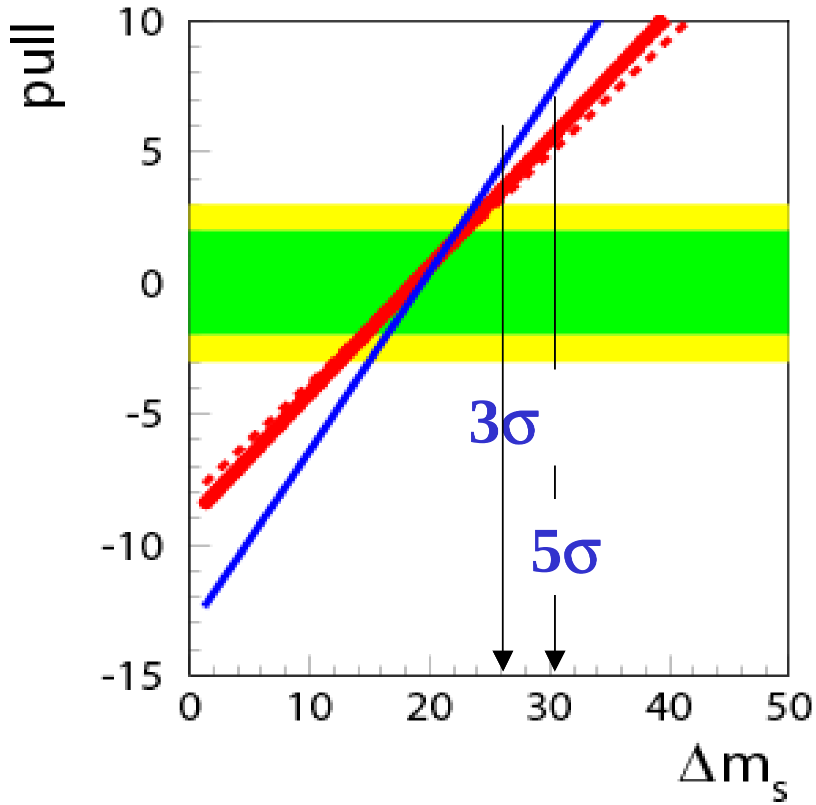
$$[162 - 278] \quad \text{MeV} \quad @95\% \quad C.L.$$

You can take this examples to show how the system is starting to be overconsrained

Looking for "New physics" by measuring Δm_s

Red lines: $\sigma(\Delta m_s)$ = 1.0, 0.5, 0.2, 0.1 ps^{-1}

Blue line: all errors divided by 2

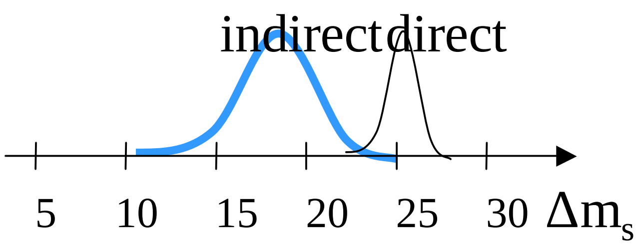


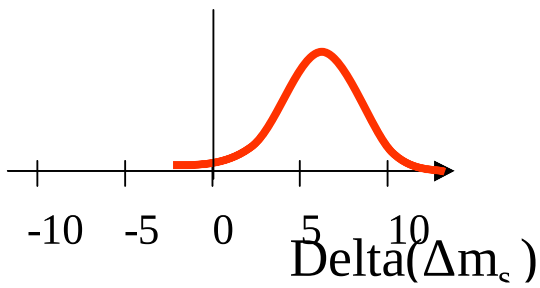
$\Delta m_s > 26 ps^{-1}$ New Physics at 3σ

$> 30.5 ps^{-1}$ New Physics at 5σ

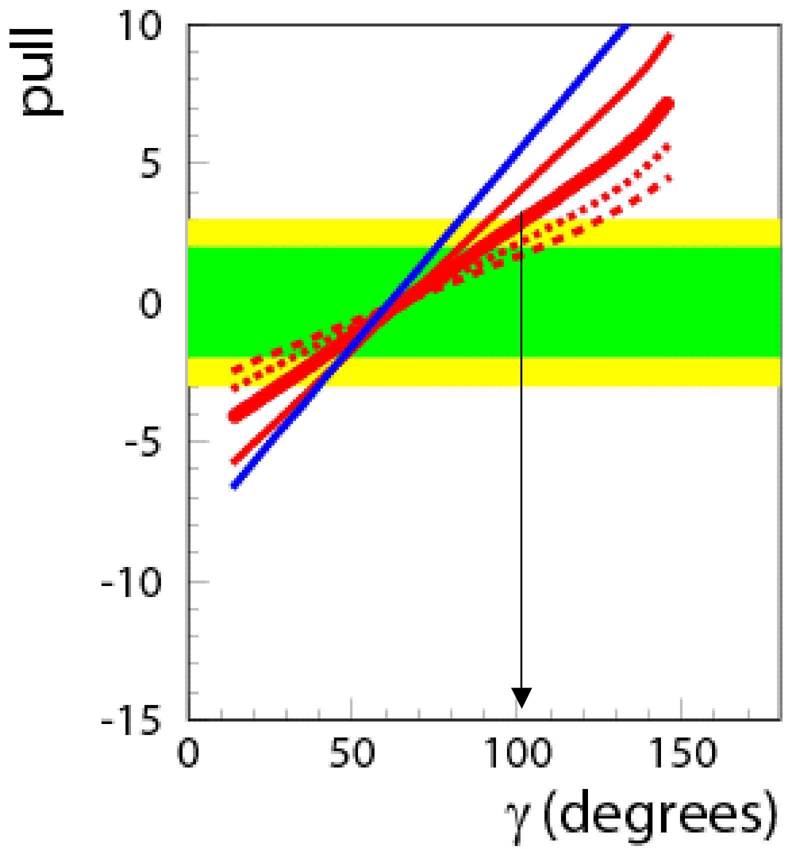
Almost independently of the precision on the measurement of **Δm_s**

If the value measured for **Δm_s** will fall in the SM region [(12-26)**ps^{-1}**] important theoretical improvements have to be forseen to test the SM





Looking for "New physics" by measuring γ



Red lines: σ = 20, 15, 10, 5 degrees

Blue line: all errors divided by 2

Suppose γ can be measured with an error of 10^{o}

à $\gamma > 100^{o}$ **New Physics at 3σ**

with an error of 5^{o}

à $\gamma > 90^{o}$ **New Physics at 3σ**

Importance of reducing some theo. errors (ξ,B_K…) to perform a more powerful test of the SM

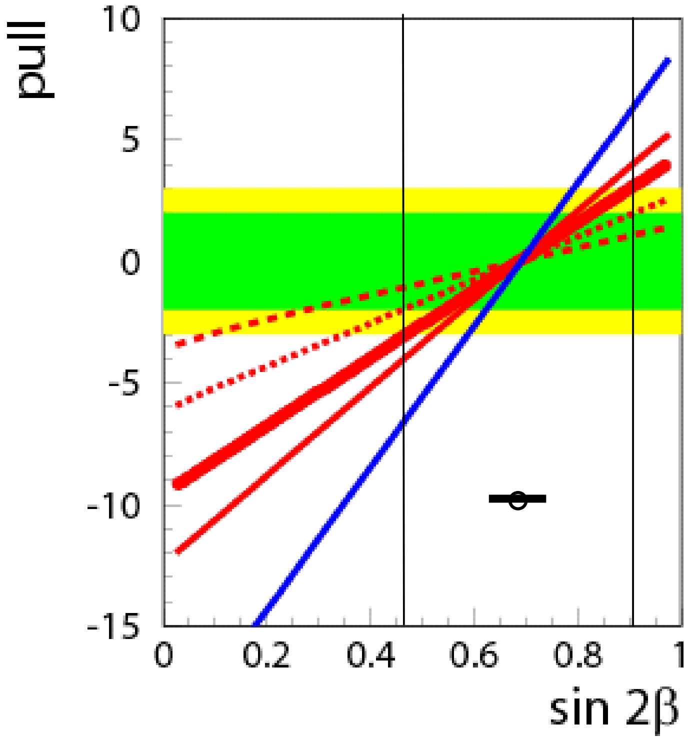
for ex: if all theo. errors/2

à $\gamma > 80^{o}$ **New Physics at 3σ**

Red lines: σ(sin2β) = 0.2, 0.1, 0.05, 0.02   **Blue line: all errors divided by 2**

SM prediction without A(J/ψK_s)

sin2β = 0.685 ± 0.055



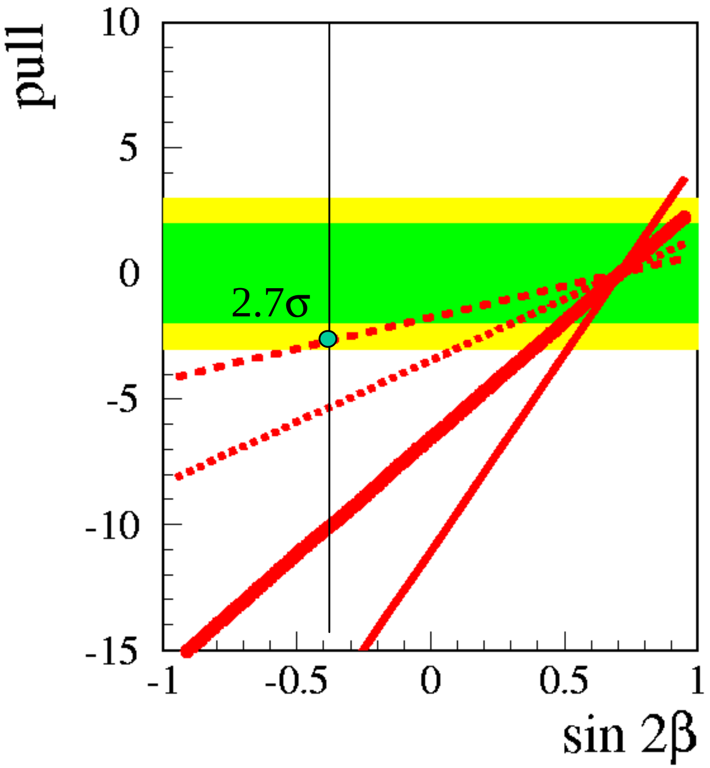
[0.52-0.85] 3σ SM region if σ (sin2β)=0.02
[0.46-0.91] 3σ SM region if σ (sin2β)=0.05 (today)

Improving the precision on sin2β using A(J/ψK_s) ?

Obviously difficult to find any discrepancy with SM

SM prediction with $A(J/\psi K_s)$

$\sin 2\beta = 0.705 \pm 0.035$



Measuring sin2β using
a different channel **$B \rightarrow \varphi K_s$?**

$A(\varphi K_s) = -0.39 \pm 0.41$

2.7σ

Red lines: $\sigma(\sin 2\beta)$ = 0.4, 0.2, 0.1, 0.05

If the experimental error
goes down by a factor 2
(à 0.2)

$A(\varphi K_s) \sim -0.4$ 5.7σ

$A(\varphi K_s) < 0$ 4σ

Important progress in the last years

V_{cb} ~2%

V_{ub} ~10%

Δm_d ~ 1.2%

Δm_s > 14.4 ps^{-1} at 95% CL

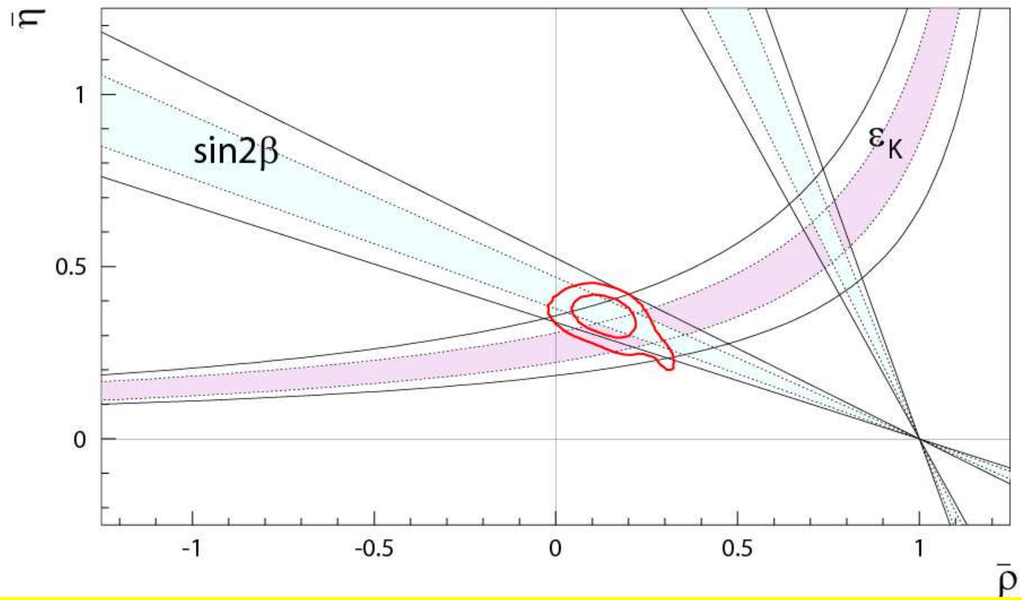
m_t ~ 3%

sin2β ~ 7%

B_K ~ 15%

$f_{B_d}\sqrt{\hat{B}_{B_d}}$ ~ 15%

ξ-1 ~ 20-25%



Success of SM + LQCD/OPE/HQET

Standardisssssimo

Next (in a ~year-time scale) hopes :
Δm_s , A(**φK$_s$**)

ADDITIONAL MATERIAL

The Bayes Theorem:

$$f(\bar{\rho}, \bar{\eta}, \mathbf{x} | c_1,\ldots,c_m) \sim \prod_{j=1,m} f_j(\mathbf{c} | \bar{\rho}, \bar{\eta}, \mathbf{x}) \prod_{i=1,N} f_i(x_i)\; f_o(\bar{\rho}, \bar{\eta})$$

$$\Rightarrow \quad f(\bar{\rho}, \bar{\eta} | \mathbf{c}) \sim \mathcal{L}(\mathbf{c} | \bar{\rho}, \bar{\eta})\; f_o(\bar{\rho}, \bar{\eta})$$

$$\mathbf{x} \equiv x_1,\ldots,x_n = m_t, B_K, F_B \ldots.$$

$$\mathbf{c} \equiv c_1,\ldots,c_m = \boldsymbol{\varepsilon_{K}}, \boldsymbol{\Delta m_d / \Delta m_s, A_{CP}(J/\psi, K_S)}$$