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Lifetime differences of b-flavoured hadrons

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Outline

1. Heavy Quark Expansion and QCD
2. Lifetime differences
3. Summary

1. Heavy quark expansion and QCD

QCD is perturbatively calculable at short distances.
$\Rightarrow$ separate short from long distance physics.

Step 1: $|\Delta B| = 1$ hamiltonian
Exploit the hierarchy $M_W, m_t, M_{\rm new} \gg m_b$:
Interactions mediated by these heavy particles look point-like at the energy scale $m_b \approx 5\,{\rm GeV}$.

Perform an operator product expansion (OPE) resulting in:
Effective hamiltonian for $|\Delta B| = 1$ transitions:

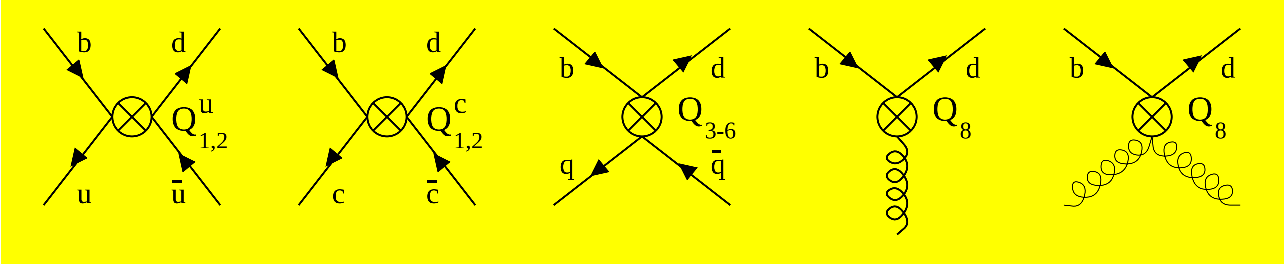
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1}^{2} C_i \left(V_{CKM}\, Q_i^u + V'_{CKM}\, Q_i^c \right) + V''_{CKM} \sum_{i\geq 3} C_i Q_i \right]$$

yields an expansion in $(m_b/M_W)^2$ with

Q_i: effective $|\Delta B| = 1$ operators, e.g. $Q_2^c = \overline{c}\gamma_\mu(1-\gamma_5)b\ \overline{d}\gamma^\mu(1-\gamma_5)c$

C_i: Wilson coefficients = effective couplings, contain short distance structure, perturbative QCD corrections, depend on m_t/M_W.

$V_{CKM}^{('')}$: product of CKM elements



Step 2: Heavy Quark Expansion

developed in

M. A. Shifman and M. B. Voloshin, Sov. Phys. Usp. **26** (1983) 387
M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. **41** (1985) 120
M. A. Shifman and M. B. Voloshin, Sov. Phys. JETP **64** (1986) 698
I. I. Bigi, N. G. Uraltsev and A. I. Vainshtein, Phys. Lett. B **293** (1992) 430

Optical theorem for some inclusive decay rate Γ of a b-flavored hadron H_b:

$$\Gamma \quad \propto \quad \text{Im}\, \langle H_b | \, i \int d^4x \; T \, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | H_b \rangle$$

$\mathcal{H}_{eff}$ is the effective $|\Delta B| = 1$ hamiltonian describing the W-mediated weak decay of the b quark.

HQE $=$ Operator product expansion:

$$\Gamma \quad \propto \quad G_F^2 \sum_j m_b^{8-d_j} \, c_j \, (\mu/m_b) \, \underbrace{\langle H_b | \mathcal{O}_j \, (\mu) \, | H_b \rangle}_{\mathcal{O}\left(\Lambda_{QCD}^{d_j - 3}\right)}$$

c_j: Wilson coefficients containing physics from scales $\geq \mu = \mathcal{O}(m_b)$

$\mathcal{O}_j$: local $\Delta B = 0$ operators with dimension $d_j \geq 3$.

Effect: Expansion of Γ in Λ_{QCD}/m_b and $\alpha_s(m_b)$.
If Γ is the total decay rate of a weakly decaying hadron H_b, this method allows to compute the lifetime $\tau = 1/\Gamma$ of the hadron H_b.

Lifetimes

There is a lot of theory literature on heavy hadron lifetimes, which, however, have a bad press:

"...the lifetime...of the Λ_b baryon...still needs understanding."

"...such corrections cannot explain the observed difference between $\tau_{\Lambda_b^0}$ and $\tau_{B_d^0}$..."

"...the still present problem of the $\tau_{\Lambda_b^0}/\tau_{B_d^0}$ ratio..."

"...the escalating problem of the $\tau_{\Xi_c^+}/\tau_{\Lambda_c^+}$ ratio..."

"It is...conceivable that the OPE based description fails for charm decays..."

Lifetime differences in the bottom system

Weakly decaying b-flavored hadrons:

$B^+ \sim \bar{b}u$	$B_d^0 \sim \bar{b}d$	$B_s^0 \sim \bar{b}s$
$\Lambda_b^0 \sim bud$	$\Xi_b^- \sim bds$	$\Xi_b^0 \sim bus$
	$\Omega_b \sim bss$	

Contribution of the terms in the HQE to lifetime differences:

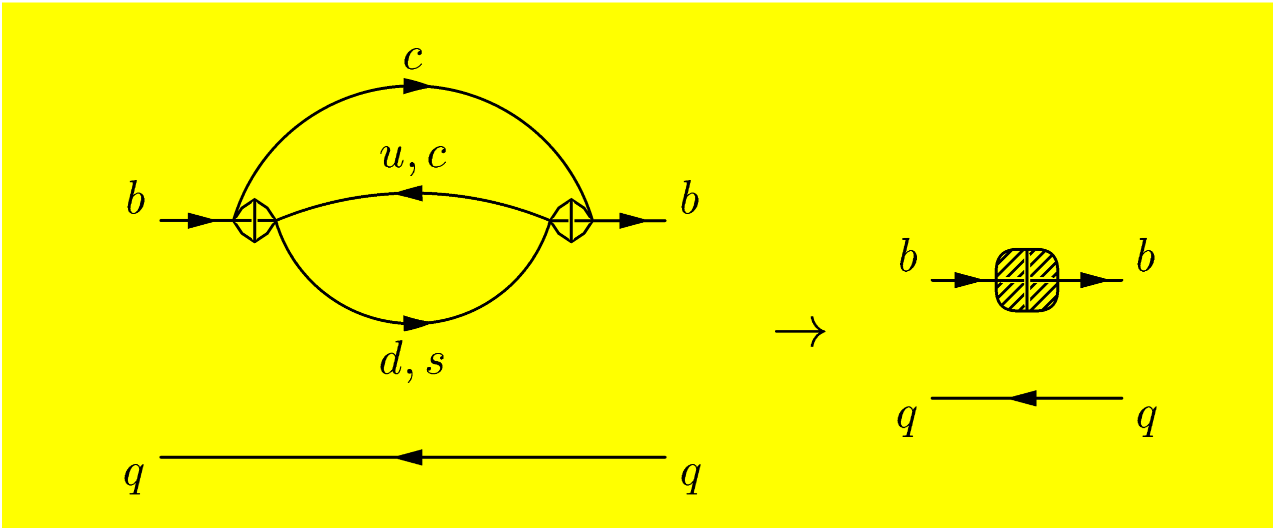
order:	1	Λ_{QCD}^2/m_b^2	$16\pi^2\Lambda_{QCD}^3/m_b^3$
contribution:	0	$\mathcal{O}(1\%)$	$\mathcal{O}(3-20\%)$

The leading contribution is the QCD corrected free quark decay rate (spectator decay), the order Λ_{QCD}/m_b is absent. The dominant source of lifetime differences is the participation of the valence quark in the weak decay.

Exception: Lifetime differences within the U-spin doublets (B_s^0,B_d^0) and (Λ_b^0,Ξ_b^0), which mainly stem from $SU(3)_F$ breaking in the $\mathcal{O}\left(\Lambda_{QCD}^2/m_b^2\right)$ matrix elements.

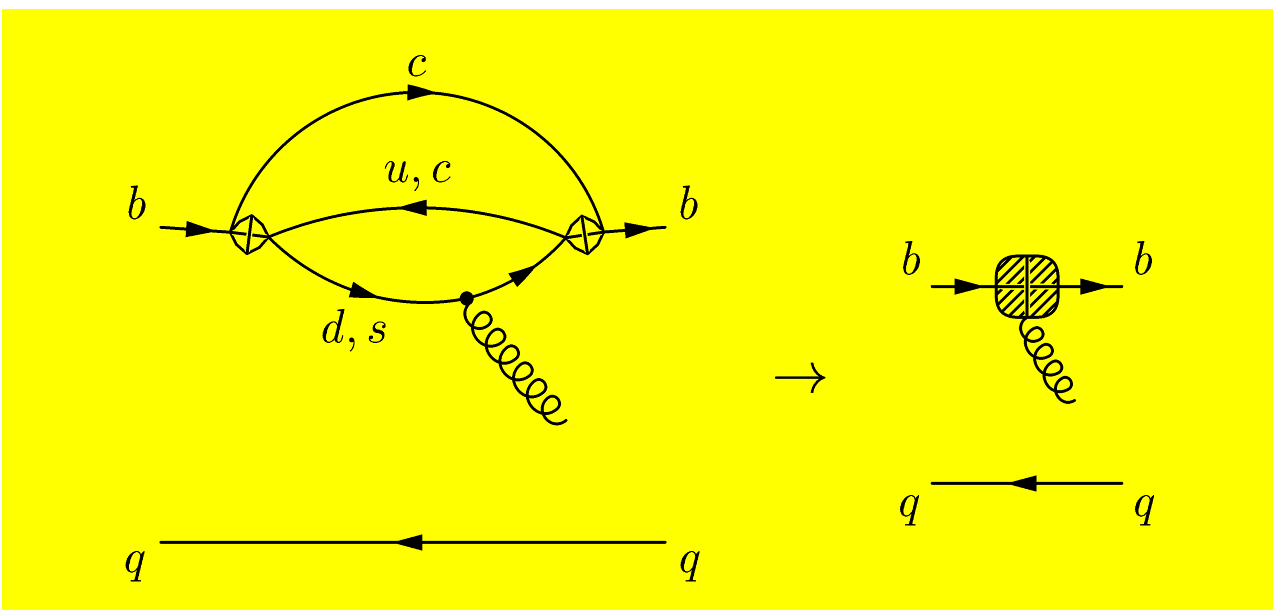
$\Rightarrow$ Lifetime differences probe the HQE at the third order in Λ_{QCD}/m_b.

Leading power:



$$\langle\, H_b\, |\bar{b}b|\, H_b\,\rangle = 1 + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{m_b^2}\right)$$

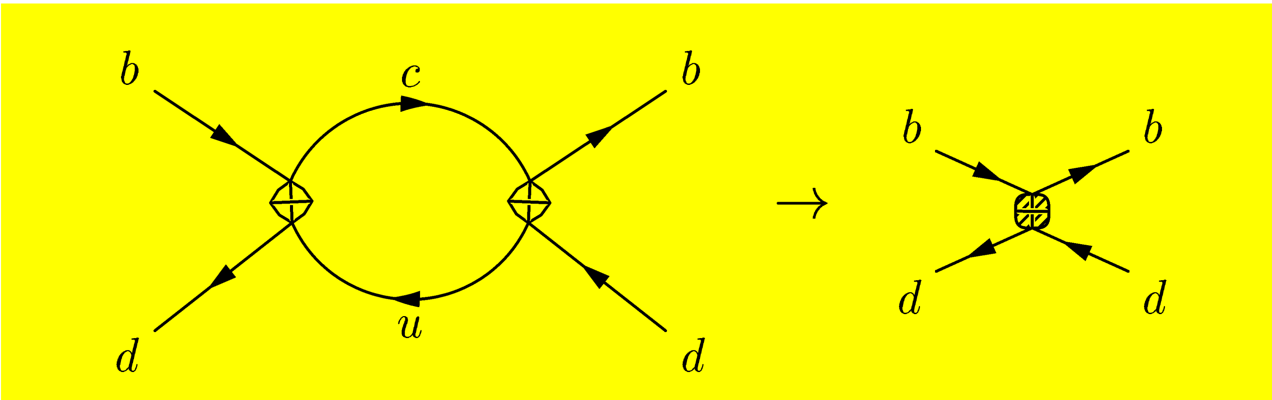
$\mathcal{O}(\Lambda^2_{QCD}/m_b^2)$:



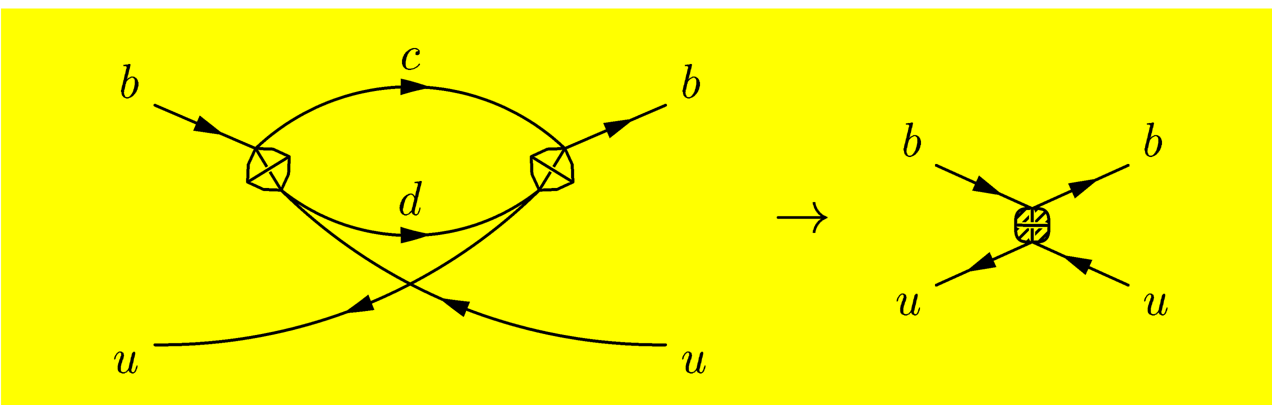
$$\langle\, H_b\, |\bar{b}\sigma_{\mu\nu}T^a b\, G^{\mu\nu,a}|\, H_b\,\rangle = \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{m_b^2}\right)$$

$\mathcal{O}(16\pi^2\,\Lambda^3_{QCD}/m_b^3)$:

Weak annihilation contribution to the B_d^0 lifetime:



Pauli interference contribution to the B^+ lifetime:



All diagrams shown receive short distance QCD corrections from gluon exchange.

What could go wrong?

Terms like

$$\frac{\sin(-c\, m_b/\Lambda_{QCD})}{m_b^n}$$

are not reproduced by the HQE. They might come from instanton effects and possibly induce the

"violation of quark-hadron duality"

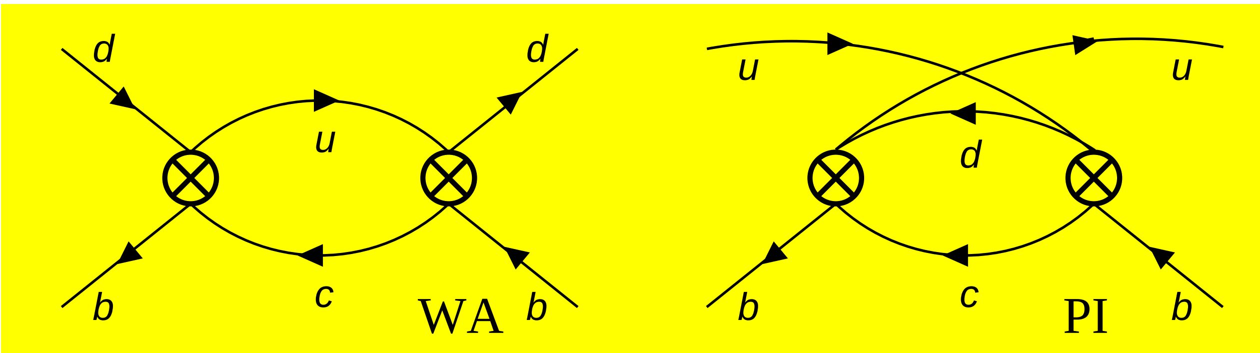
At present this issue can only be addressed experimentally, by confronting data with sufficiently precise theory predictions. Most lifetime ratios are insensitive to new physics and therefore unambiguously probe the HQE.

The lifetime splitting most sensitive to new physics is the one between the two mass eigenstates of the B_s meson.

2. Lifetime differences

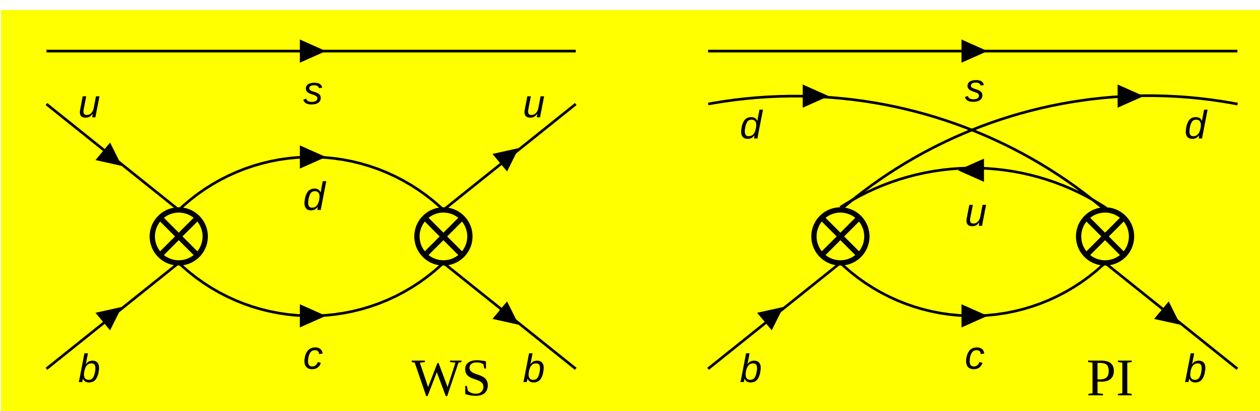
So expect all b-hadron lifetimes to be equal up to $\mathcal{O}(3-20\%)$ corrections.

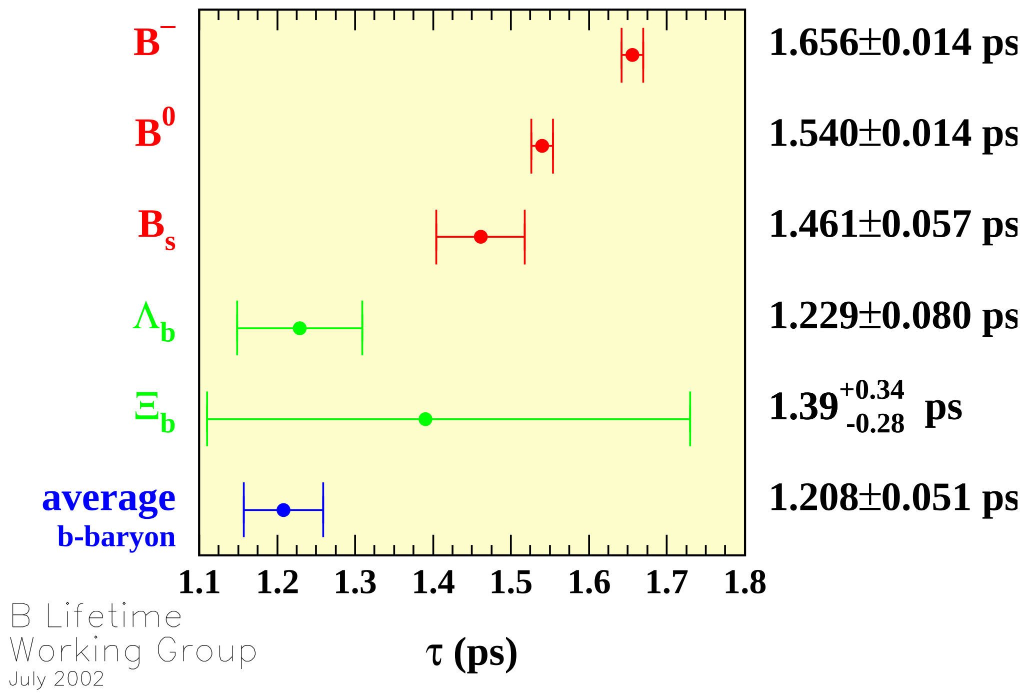
To $\mathcal{O}(\alpha_s^0)$ the lifetime differences stem from:



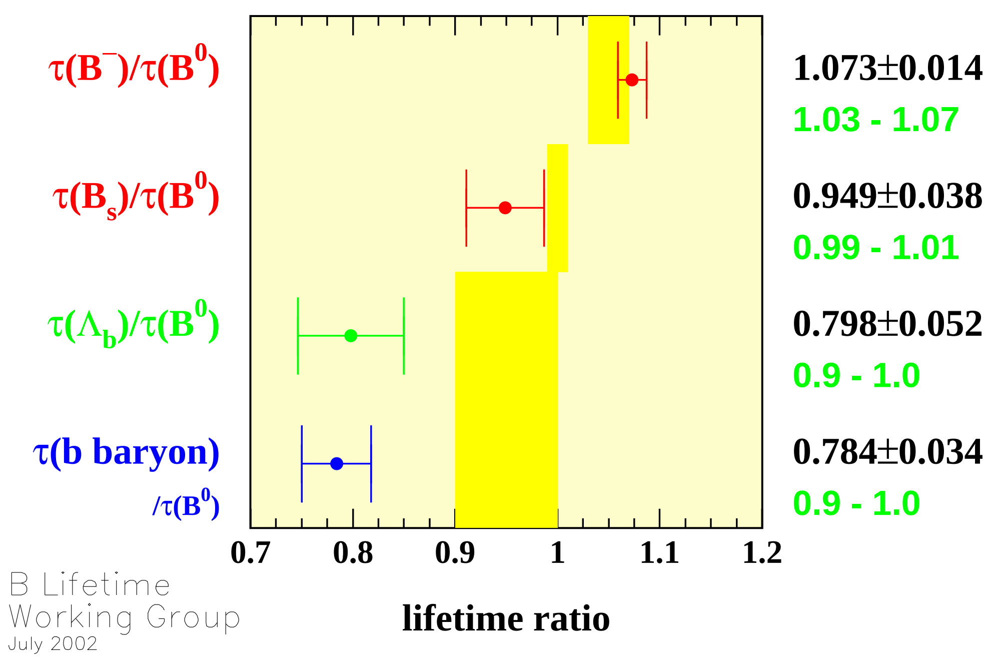
Bigi, Shifman, Uraltsev, Vainshtein, Neubert, Sachrajda

mesons:	weak annihilation	Pauli interference
baryons:	Pauli interference	weak scattering



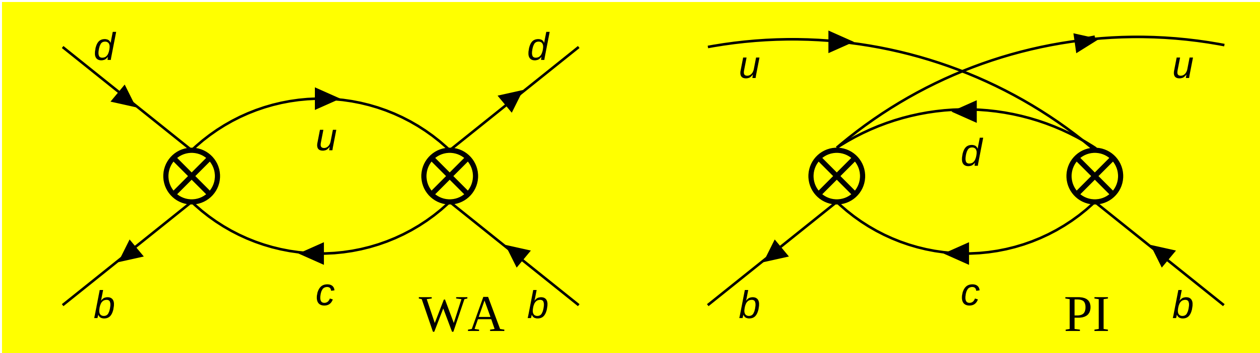


B⁻
B⁰
B_s
Λ_b
Ξ_b
average
b-baryon
1.656±0.014 ps
1.540±0.014 ps
1.461±0.057 ps
1.229±0.080 ps
1.39 +0.34 -0.28 ps
1.208±0.051 ps
1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8
B Lifetime
Working Group
July 2002
τ (ps)



τ(B⁻)/τ(B⁰)
τ(B_s)/τ(B⁰)
τ(Λ_b)/τ(B⁰)
τ(b baryon)
/τ(B⁰)
1.073±0.014
1.03 - 1.07
0.949±0.038
0.99 - 1.01
0.798±0.052
0.9 - 1.0
0.784±0.034
0.9 - 1.0
0.7 0.8 0.9 1 1.1 1.2
B Lifetime
Working Group
July 2002
lifetime ratio

Leading contribution to the B_d^0–$B^{\pm}$ lifetime difference:
From



the coefficient functions of the four $\Delta B = 0$ operators

$$
\begin{aligned}
Q^q &= \overline{b}\gamma_\mu(1-\gamma_5)q\,\overline{q}\gamma^\mu(1-\gamma_5)b, \\
Q_S^q &= \overline{b}(1-\gamma_5)q\,\overline{q}(1+\gamma_5)b, \\
T^q &= \overline{b}\gamma_\mu(1-\gamma_5)T^a q\,\overline{q}\gamma^\mu(1-\gamma_5)T^a b, \\
T_S^q &= \overline{b}(1-\gamma_5)T^a q\,\overline{q}(1+\gamma_5)T^a b,
\end{aligned}
$$

are determined to order α_s^0. The width difference $\Gamma(B_d^0)-\Gamma(B^+)$ then involves the hadronic matrix elements

$$
\begin{aligned}
\langle B^+|(Q^u - Q^d)(\mu_0)|B^+\rangle &= M_B^2 f_B^2 B_1(\mu_0), \\
\langle B^+|(Q_S^u - Q_S^d)(\mu_0)|B^+\rangle &= M_B^2 f_B^2 B_2(\mu_0), \\
\langle B^+|(T^u - T^d)(\mu_0)|B^+\rangle &= M_B^2 f_B^2 \epsilon_1(\mu_0), \\
\langle B^+|(T_S^u - T_S^d)(\mu_0)|B^+\rangle &= M_B^2 f_B^2 \epsilon_2(\mu_0).
\end{aligned}
$$

These matrix elements parametrized by $f_B^2 B_1, \ldots f_B^2 \epsilon_2$ must be calculated non-perturbatively.
μ_0: renormalization scale.

Why calculate lifetime differences to $\mathcal{O}(\alpha_s)$?

- to control the dependence on renormalization scales
- to define the renormalization scheme of the quark masses entering the result
- consistent use of $\Lambda_{\overline{MS}}$
- meaningful use of lattice results for the hadronic parameters $f_B^2 B_1, \ldots f_B^2 \epsilon_2$
- QCD corrections are of order 30%.
- Test of quark-hadron duality ($=$ validity of the HQE): Need to go beyond leading logarithmic approximation.

Status of complete NLO calculations:

1. Width difference $\Delta\Gamma_{B_s}$ between the mass eigenstates of the B_s–$\overline{B}_s$ system:

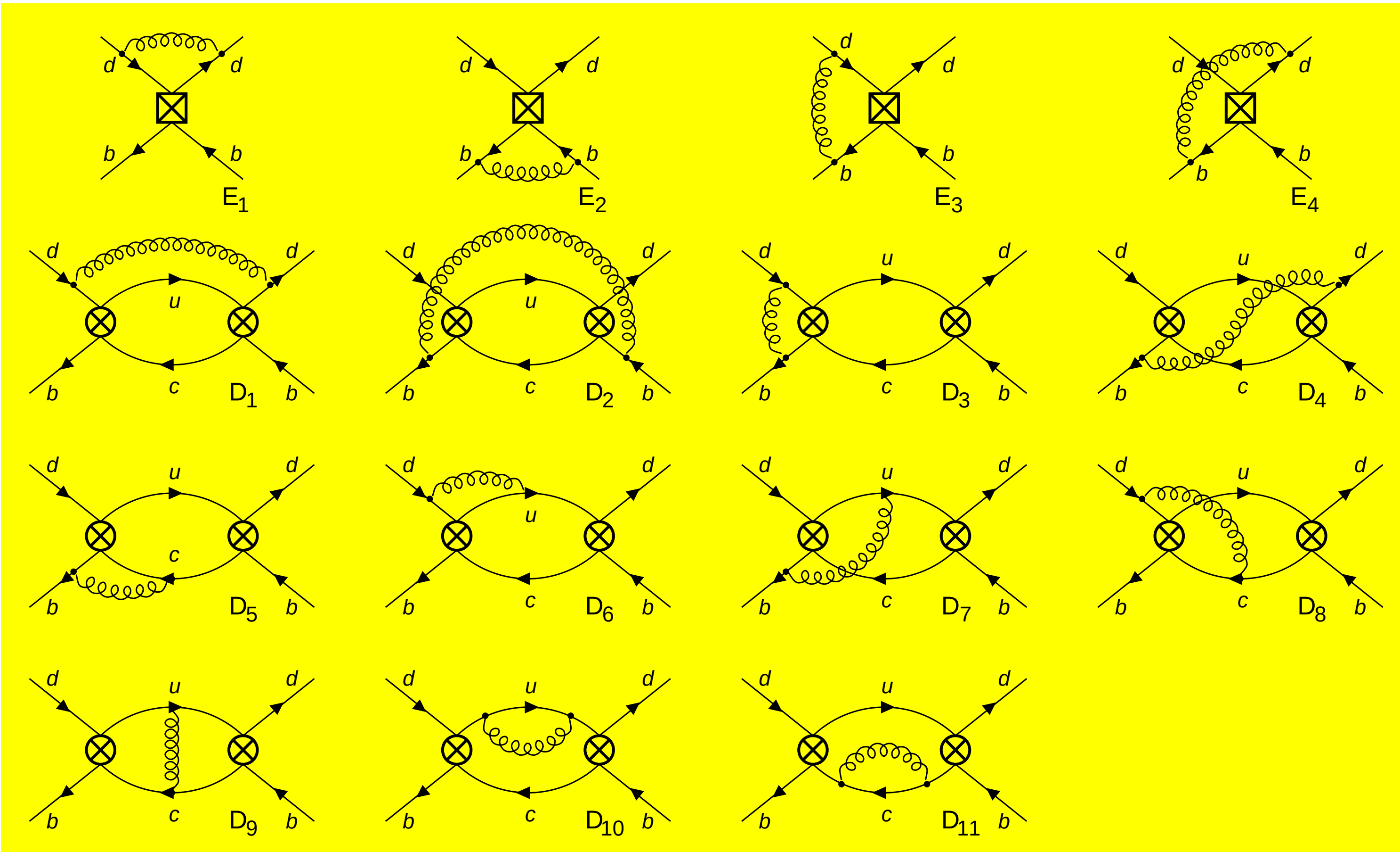
Beneke,Buchalla,Greub,Lenz,U.N., Phys. Lett. B **459** (1999) 631.

2. Lifetime splitting within the isodoublets (B^+,B_d^0) and (Ξ_b^0,Ξ_b^-): Beneke,Buchalla,Greub,Lenz,U.N., hep-ph/0202106.

The B_d^0–$B^\pm$ lifetime difference has also been calculated in

Franco, Lubicz, Mescia, Tarantino, hep-ph/0203089.

WA contributions in the next-to-leading order of QCD:



As expected all infrared divergences properly factorize between the diagrams D_{1-4} and diagrams E_{1-4}, so that they cancel from the Wilson coefficients.

Typical result for the α_s–part of a coefficient function:

$$F_{11}^{u,(1)} - F_{11}^{d,(1)} =$$

$$\left[\frac{16\ (1-z)\ \left(-4-3\,z+3\,z^2\right)}{3}\right]\ \left[\mathrm{Li}_2(z)+\frac{\ln(1-z)\ \ln(z)}{2}\right]\ +$$

$$\left[\frac{4\,(1-z)^2\ (16+19\,z)}{3}\right]\ \ln(1-z)\ +$$

$$\left[\frac{4\,z\ \left(93+40\,z-57\,z^2\right)}{9}\right]\ \ln(z)\ +$$

$$\left[32\,(1-z)^2\right]\ \ln\frac{\mu_1}{m_b}\ -\ \left[16\,(1-z)^2\right]\ \ln\frac{\mu_0}{m_b}\ +$$

$$\left[\frac{32\ (1-z)}{9}\right]\ \pi^2\ +\ \frac{2\ (1-z)\ \left(152+149\,z+155\,z^2\right)}{27}$$

with $z = m_c/m_b$.

. . . and 11 more coefficients.

Phenomenology

We find:

$$\frac{\tau(B^+)}{\tau(B_d^0)} - 1 \;=\; \tau(B^+)\,\left[\Gamma(B_d^0) - \Gamma(B^+)\right]$$

$$= \quad 0.0325\,\left(\frac{|V_{cb}|}{0.04}\right)^2 \left(\frac{m_b}{4.8\,\text{GeV}}\right)^2 \left(\frac{f_B}{200\,\text{MeV}}\right)^2 \times$$

$$\Big[\,(1.0\pm 0.2)\,B_1 \;+\; (0.1\pm 0.1)\,B_2$$

$$-\,(18.4\pm 0.9)\,\epsilon_1 \;+\; (4.0\pm 0.2)\,\epsilon_2 \;+\; 0.2\pm 0.2\Big].$$

The last number is the $1/m_b$ correction. The numbers multiplying $B_1 \ldots \epsilon_2$ (normalized at $\mu_0 = m_b$) correspond to a certain renormalization scheme for the $\Delta B = 0$ operators defined by

- dimensional regularization with $\overline{\text{MS}}$ subtraction,
- anti-commuting γ_5,
- choice of the evanescent operators such that Fierz-invariance is maintained at the loop level.

The hadronic parameters must be calculated in the same scheme.

Parametrically one has $B_{1,2} = \mathcal{O}(1)$ and $\epsilon_1 = \mathcal{O}(1/N_c)$, where $N_c = 3$ is the number of colors. In the vacuum insertion approximation one has $\epsilon_i = 0$.
Pathological situation: The small ϵ_i's have large coefficients.

Hadronic matrix elements calculated with lattice QCD:

$$(B_1, B_2, \epsilon_1, \epsilon_2) \ = \ (1.10 \pm 0.20,\ 0.79 \pm 0.10,\ -0.02 \pm 0.02,\ 0.03 \pm 0.01).$$

Becirevic, hep-ph/0110124.

NLO corrections: $\mathcal{O}(30\%)$ increase,
$1/m_b$ corrections: $\mathcal{O}(-10\%)$ decrease:
Result:

$$\frac{\tau(B^+)}{\tau(B_d^0)} \ = \ 1.047 \pm 0.016 \pm 0.017 \pm 0.007,$$

$$\left[\frac{\tau(B^+)}{\tau(B_d^0)}\right]_{\rm LO, no\ 1/m} \ = \ 1.041 \pm 0.040 \pm 0.013,$$

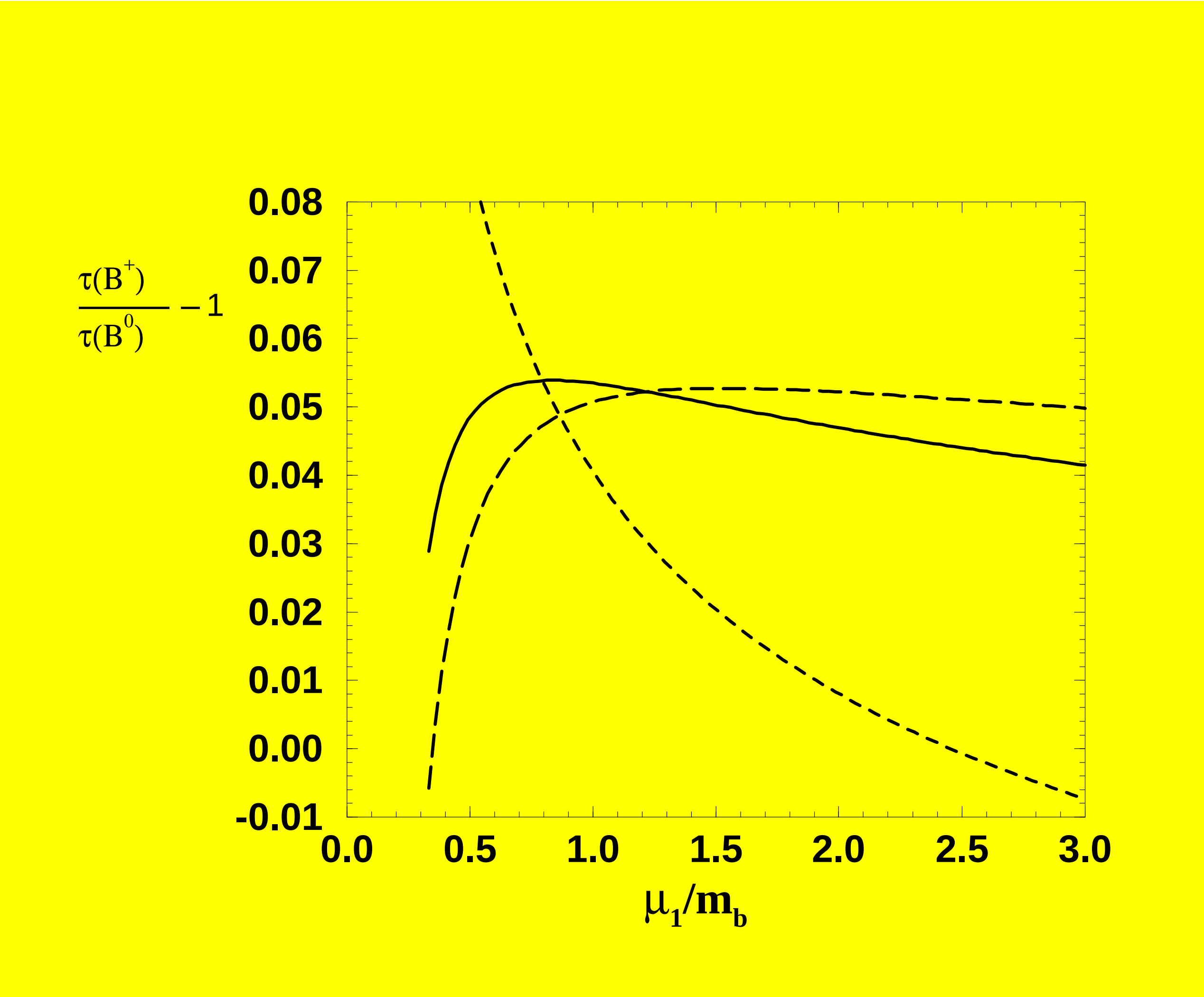
First error: uncertainty in $B_{1,2}$, $\epsilon_{1,2}$
and (small) residual scale dependence.
Second error: uncertainty from m_b, $|V_{cb}|$ and f_B.
Third error: from uncertainty in $1/m_b$ term.

Good agreement with

$$\frac{\tau(B^+)}{\tau(B_d^0)} \ = \ 1.073 \pm 0.014 \qquad \text{2002 world average}$$

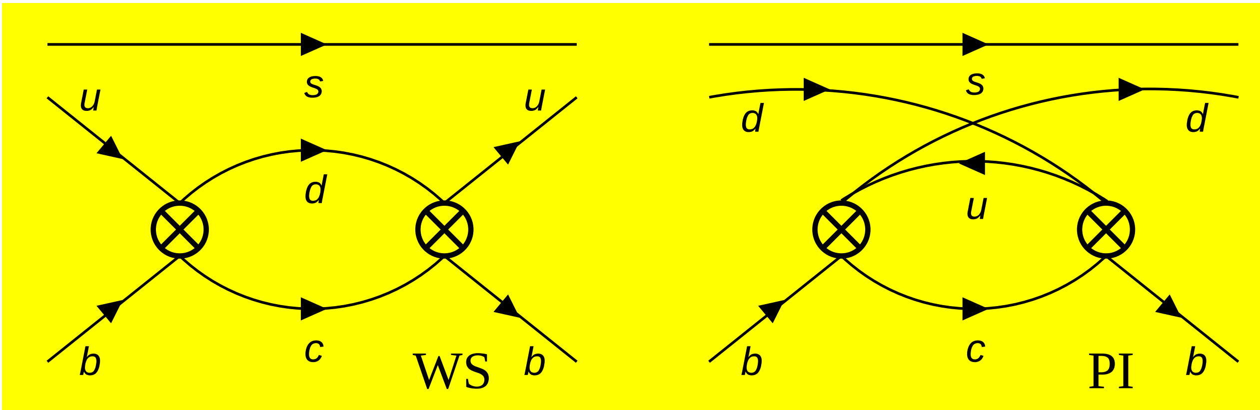
CERN B Lifetime Working Group

Scale uncertainty



Short-dashed line: LO result
Solid line: NLO result
Long-dashed line: NLO result with $m_c = 0$ in the α_s–terms

The Ξ_b^0–Ξ_b^- lifetime difference



Two differences compared to the B^+–B_d^0 lifetime splitting:

1. There are only two hadronic parameters L_1 and L_2.
2. The decay of the spectator s quark triggering $\Xi_b \to \Lambda_b X$ can be relevant.

Voloshin, Phys. Lett. B **476** (2000) 297.

To correct for 2. define

$$\begin{aligned}\overline{\Gamma}(\Xi_b) &\equiv \Gamma(\Xi_b) - \Gamma(\Xi_b \to \Lambda_b X) \\ &= \frac{1 - B(\Xi_b \to \Lambda_b X)}{\tau(\Xi_b)} \equiv \frac{1}{\overline{\tau}(\Xi_b)} \quad \text{for } \Xi_b = \Xi_b^0, \Xi_b^- .\end{aligned}$$

We find

$$\frac{\overline{\tau}(\Xi_b^0)}{\overline{\tau}(\Xi_b^-)} - 1 = \overline{\tau}(\Xi_b^0)\left[\overline{\Gamma}(\Xi_b^-) - \overline{\Gamma}(\Xi_b^0)\right]$$

$$= 0.59 \left(\frac{|V_{cb}|}{0.04}\right)^2 \left(\frac{m_b}{4.8\,\text{GeV}}\right)^2 \left(\frac{f_B}{200\,\text{MeV}}\right)^2 \frac{\overline{\tau}(\Xi_b^0)}{1.5\,\text{ps}} \times$$

$$\left[(0.04 \pm 0.01)\, L_1 \; - \; (1.00 \pm 0.04)\, L_2\right].$$

$\Rightarrow$ good opportunity for the lattice community to calculate L_2 and predict the Ξ_b^0–Ξ_b^- lifetime splitting prior to its measurement.

Exploratory lattice result:

$$L_1 \sim \; -0.3 \qquad\qquad L_2 \sim \; 0.2$$

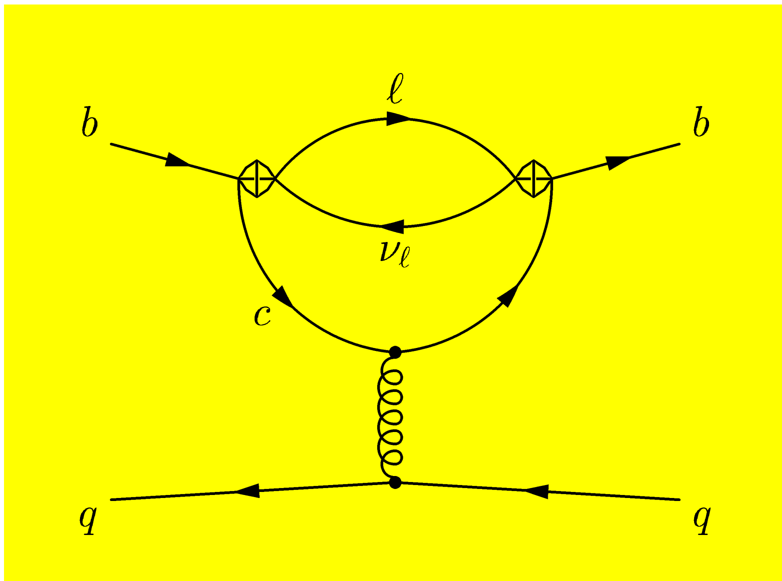
Di Pierro, Sachrajda, Michael (UKQCD)

So Ξ_b^0 lives shorter than Ξ_b^- by roughly 10%.

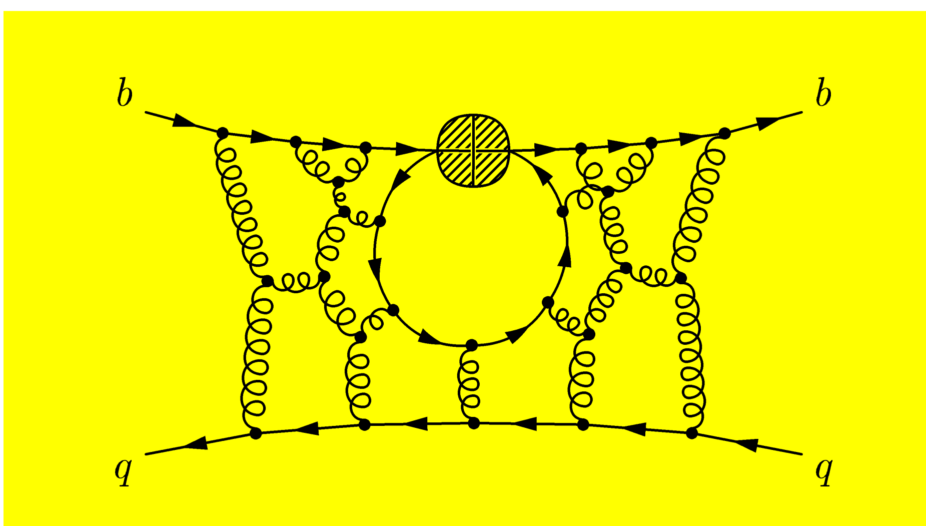
Remark: $\tau(\Lambda_b^0) = \overline{\tau}(\Xi_b^0)$ up to small $SU(3)_F$ and $\mathcal{O}(m_c^2/m_b^2)$ corrections.

The Λ_b^0 lifetime

For the prediction of the lifetime difference between B_d and Λ_b^0 still important ingredients are missing. Now strong interaction between the b and the valence quark(s) plays a role. At α_s the coefficient functions receive contributions from:



More important: At present "penguin" (or "eye") contractions of the $\Delta B = 0$ four quark operators cannot be computed with lattice QCD:



The experimental value

$$\frac{\tau_{\Lambda_b^0}}{\tau_{B_d^0}} = 0.80 \pm 0.05$$

has been regarded as a problem by many theorists. The current status of the theory prediction is

$$\frac{\tau_{\Lambda_b^0}}{\tau_{B_d^0}} = 0.90 \pm 0.05 \pm ???,$$

where “???” represents the yet unknown penguin effects.

Franco, Lubicz, Mescia, Tarantino, hep-ph/0203089.

$\Rightarrow$ Claims of duality violations are premature.

Lifetime differences in the charm system

Weakly decaying c-flavored hadrons:

$$D^+ \sim c\overline{d} \qquad D^0 \sim c\overline{u} \qquad D_s^+ \sim c\overline{s}$$

$$\Lambda_c^+ \sim cdu \qquad \Xi_c^+ \sim csu \qquad \Xi_c^0 \sim csd$$

$$\Omega_c \sim css$$

Expansion parameters are now Λ_{QCD}/m_c and $\alpha_s(m_c)$. Are they small enough?

The expansion parameter of the $1/m_c$ expansion is really (in the meson system):

$$2\,\frac{M_D - m_c}{m_c} = 0.5 - 0.7,$$

where m_c is some (properly infrared-subtracted) pole mass.

The canonical size of relative rate differences is

$$16\pi^2 \left[\frac{M_D - m_c}{m_c}\right]^3 = 2 - 7$$

D^+–D^0 lifetime difference

Experiment:

$$\frac{\tau_{D^+}}{\tau_{D^0}} - 1 \quad = \quad 1.56$$

For $m_c = 1.5$ GeV, $f_D = 225$ MeV, $\alpha_s(M_Z) = 0.118$ find:

$$\frac{\tau_{D^+}}{\tau_{D^0}} - 1 \; = \; \begin{cases} 2.8^{+1.3}_{-0.6} & \text{LO} \\ 2.6 \pm 0.2 & \text{NLO} \end{cases}$$

with the uncertainty stemming from 1 GeV $\leq \mu_1 \leq 2$ GeV.

$\Rightarrow$ expansion in $\alpha_s(m_c)$ in good shape.

Yet $1/m_c$ terms reduce the result by 45 ± 15 %:

$$\frac{\tau_{D^+}}{\tau_{D^0}} - 1 \; = \; 1.4 \pm 0.4 \pm 0.4,$$

where the first error comes from the uncertainty in the $1/m_c$ corrections and the second error stems from the unknown $1/m_c^2$ terms.

$\Rightarrow$ expansion in $1/m_c$ in poor shape,
but semiquantitative agreement with the data.

D_s^+–D^0 lifetime difference

Experiment:

$$\frac{\tau_{D_s^+}}{\tau_{D^0}} - 1 \quad = \quad 0.19$$

Here only weak annihilation diagrams contribute. In the absence of QCD they vanish by helicity.

$$\frac{\tau_{D_s^+}}{\tau_{D^0}} - 1 \; = \; \begin{cases} 0 & \text{LO with } B_1 = B_2, \epsilon_1 = \epsilon_2 \\ 0.45 \pm 0.10 & \text{NLO with } B_1 = B_2 = 1, \epsilon_1 = \epsilon_2 = 0 \\ 0.66 \pm 0.10 & \text{NLO with } B_1 = 1.1, B_2 = 1, \epsilon_1 = \epsilon_2 = 0 \end{cases}$$

$\Rightarrow$ NLO corrections show semiquantitative agreement with the data, challenging for lattice community.

The $1/m_c$ corrections are the smaller problem here.

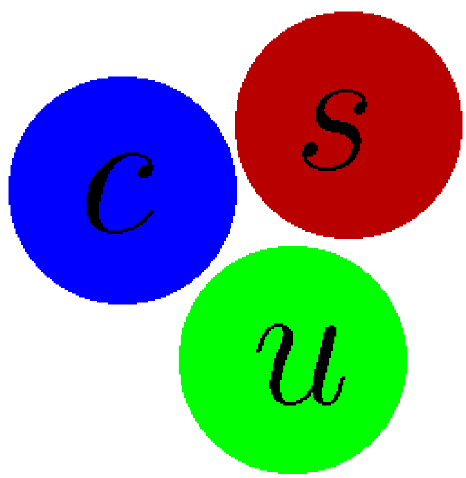
Wish list

...to our lattice colleagues:

- Compute L_2 needed for Ξ_b^+, Ξ_b^0 and Λ_b^0 to predict the b-baryon lifetime splittings before they are measured.
- Work harder on ϵ_1 to get a better handle on $\tau(B^+)/\tau(B_d^0)$.
- Develop ideas for the penguin contractions needed for $\tau(\Lambda_b^0)/\tau(B_d^0)$.
- Tackle $1/m$ corrections.
- Unquench!!!

Finally...

$$\Lambda_c^+ \sim cdu, \qquad \Xi_c^+ \sim csu$$



3. Summary

- We have calculated NLO corrections to the B^+–B_d^0 and Ξ_b^0–Ξ_b^- lifetime differences, which are necessary for a meaningful use of hadronic parameters computed with lattice gauge theory.

- We find

$$\frac{\tau(B^+)}{\tau(B_d^0)} \quad = \quad 1.047 \pm 0.024,$$

  establishing $\tau(B^+) > \tau(B_d^0)$.

- $\tau(\Xi_b^0)/\tau(\Xi_b^-)$ essentially depends on only a single hadronic parameter L_2.

- Lifetimes of charmed hadrons can be understood semiquantitatively.

- For a reliable prediction of $\tau_{\Lambda_b^0}/\tau_{B_d^0}$ still more theoretical work is necessary. Claims of violation of quark-hadron duality are premature.

- Better lattice results are needed!