Calculations of BR$[\bar{B} \to X_s \gamma]$

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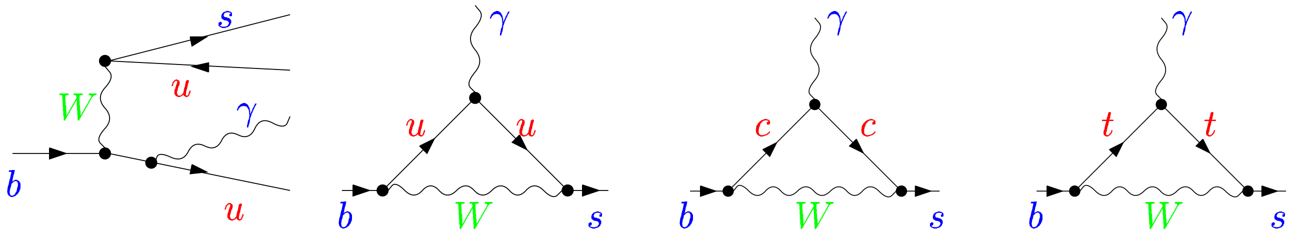
1. **SM predictions for BR$[\bar{B} \to X_s \gamma]$ vs. experiment**
2. **Perturbative calculations of $b \to X_s^{\rm parton} \gamma$**
   - **(i) completed (NLO)**
   - **(ii) ongoing (NNLO)**
3. **Non-perturbative effects**

Starting point:

$$\Gamma[\bar{B} \to X_s \gamma] \simeq \Gamma[b \to X_s^{\rm parton} \gamma]$$
$$\equiv \Gamma[b \to s\gamma] \; + \; \Gamma[b \to s\gamma g] \; + \; \ldots$$

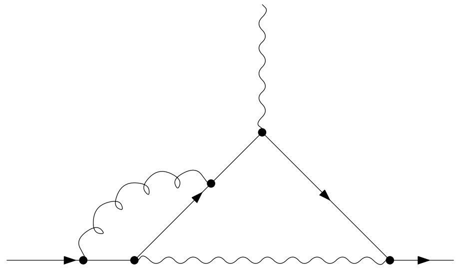
Examples of the leading electroweak diagrams for $\bar{B} \to X_s \gamma$:



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

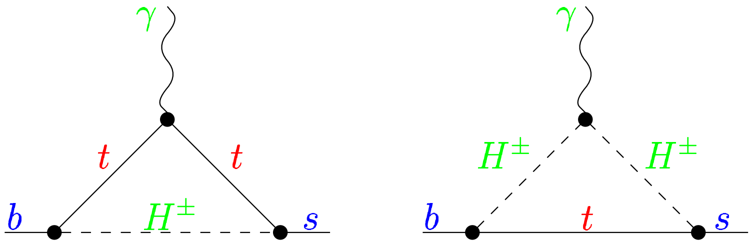
In the amplitude, after including LO QCD effects.



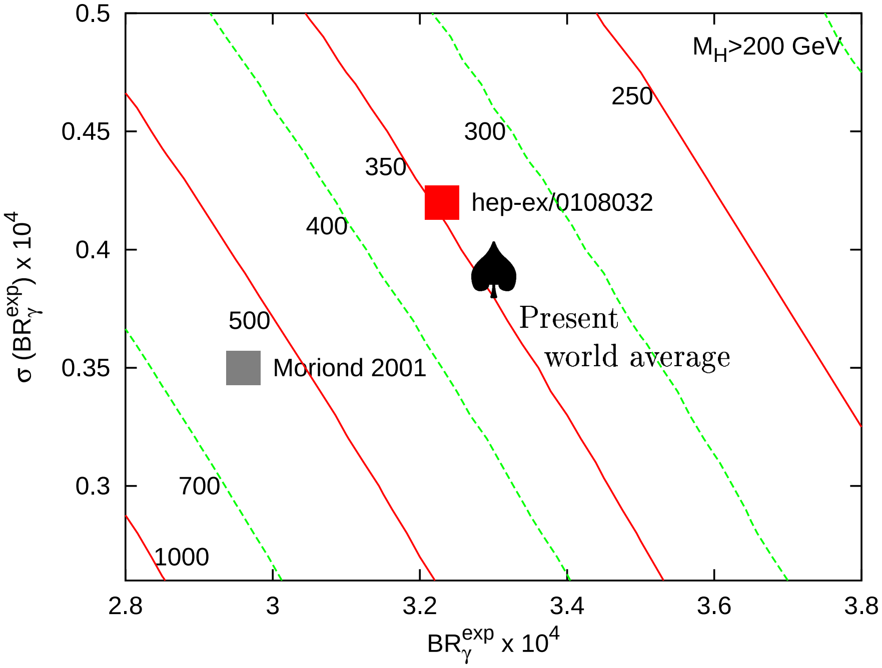
QCD logs $\alpha_s \ln \frac{M_W^2}{m_b^2}$ **enhance the BR by more than a factor of 2.**

A_c is receives very small logarithmic QCD corrections ($\sim 1\%$).
$|A_t|$ gets suppressed by around 30% $\leftrightarrow$ evolution of $m_b(\mu)$.

Example of a non-SM contribution to $b \to s\gamma$:

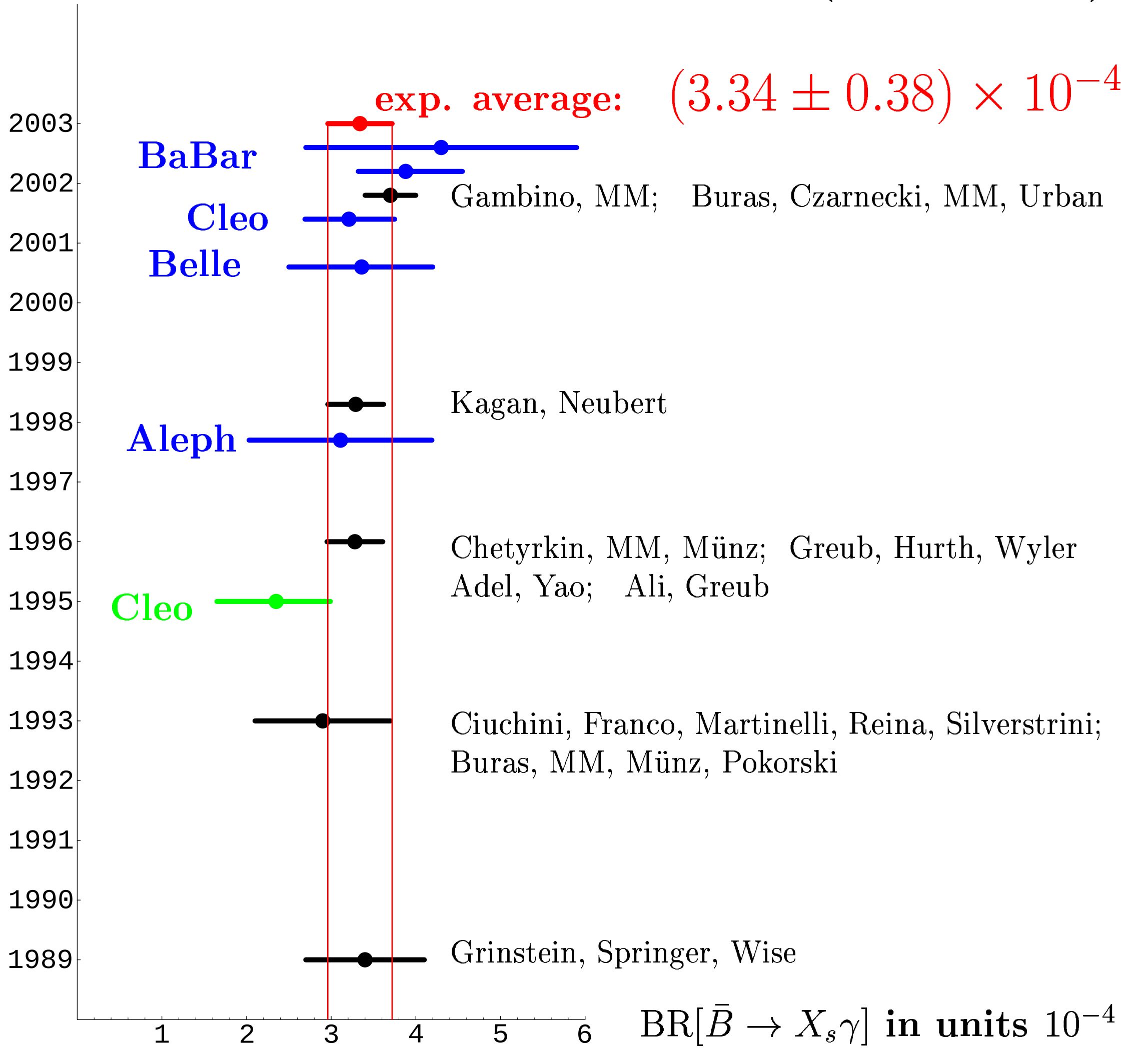


$\bar{B} \to X_s\,\gamma$ **constraints on the charged scalar mass in the Two-Higgs-Doublet Model (II):**



Measurements and the SM calculations

(some of them)



The effective Lagrangian:

$$\mathcal{L} \;=\; \mathcal{L}_{QCD\times QED}(u,d,s,c,b) \;+\frac{4G_F}{\sqrt{2}}V_{ts}^{*}V_{tb}\sum_{i=1}^{8}C_i(\mu)O_i$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i=1,2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\boldsymbol{\Sigma}_q(\bar{q}\Gamma'_i q), & i=3,4,5,6, & |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}, & i=7, & C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}, & i=8, & C_8(m_b) \sim -0.15 \end{cases}$$

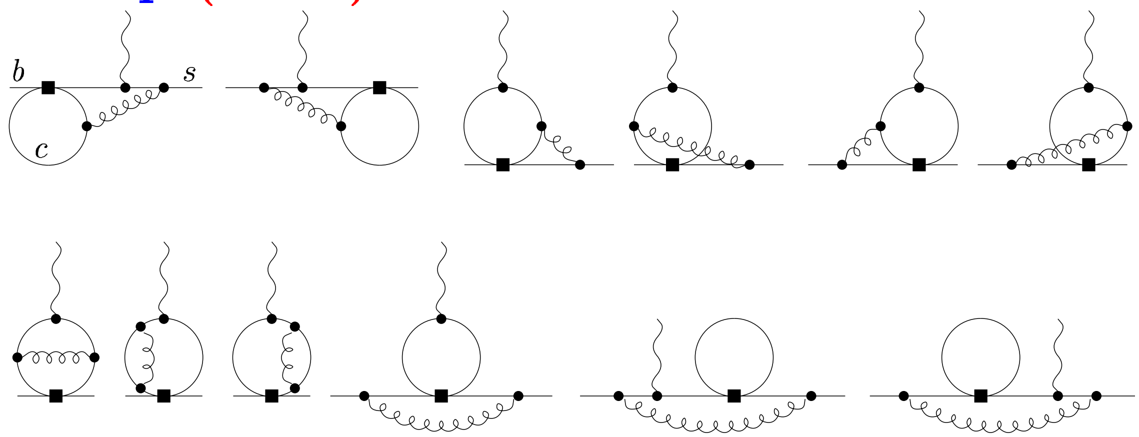
Three steps of the calculation:

Matching: **Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and eff. theory Green functions.**

Mixing: **Deriving the eff. theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$.**

Matrix elements: **Evaluating the on-shell amplitudes at $\mu_b \sim m_b$.**

Two-loop (NLO) matrix elements of the four-quark operators:



Three methods have been used for calculating these on-shell diagrams:

1. **Mellin-Barnes transform ⇒ Expansion in m_c/m_b** [Greub, Hurth, Wyler, 1996]
2. **Asymptotic expansions ⇒ Expansion in m_c/m_b** [Buras, Czarnecki, MM, Urban, 2001]
3. **"Brute force" ⇒ No expansion in m_c/m_b** [Buras, Czarnecki, MM, Urban, 2002]

The $b \to s\gamma$ amplitude becomes dependent on m_c only via these very diagrams. Thus, the question whether one should use:

$$\frac{m_c^{\rm pole}}{m_b^{\rm pole}} = 0.29 \pm 0.02 \qquad \textbf{or} \qquad \frac{m_c^{\overline{\rm MS}}(\mu)}{m_b^{\rm pole}} = 0.22 \pm 0.04 \qquad (\mu \in [\mathbf{m_c}, \mathbf{m_b}])$$

can, in principle, be answered only at the NNLO. However, changing m_c/m_b from 0.29 to 0.22 enhances BR$[\bar{B} \to X_s\gamma]$ by 10% ! ⇒ Accuracy ↘

Way out: NNLO calculation

Matching ($\mu_0 \sim M_W, m_t$):

	LO	NLO	NNLO	
$C_i(\mu_0)\ =$	$C_i^{(0)}(\mu_0)$	$+\ \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0)$	$+\ \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2 C_i^{(2)}(\mu_0)$	
4-quark ($i = 1, \ldots, 6$):	**tree**	**1-loop**	**2-loop**	[Bobeth, MM, Urban, NPB 574 (2000) 291]
"magn. mom." ($i = 7, 8$):	**1-loop**	**2-loop**	**3-loop**	[Steinhauser, MM, in progress]

The NNLO matching has a -1.5% **effect on BR**$[\bar{B} \to X_s \gamma]$ **(in** $\overline{\rm MS}$**, for** $m_t = M_W$**).**

Mixing:

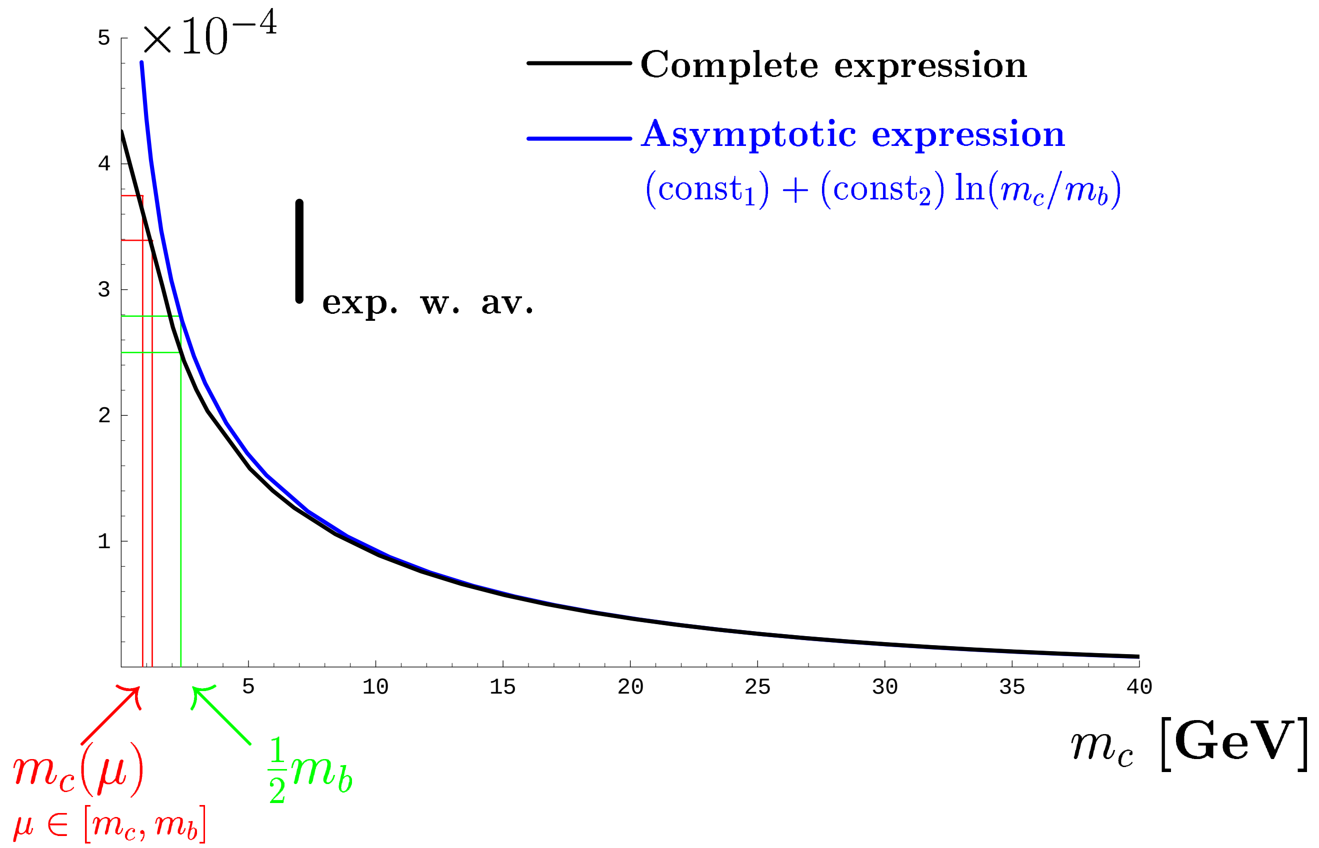
$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \begin{pmatrix} 1L & 2L \\ 0 & 1L \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} 2L & 3L \\ 0 & 2L \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^3 \begin{pmatrix} 3L & 4L \\ 0 & 3L \end{pmatrix}$$

Haisch, Gorbahn, Gambino, Schroeder

Matrix elements ($\mu_b \sim m_b$):

	LO	NLO	NNLO	
$\langle O_i \rangle(\mu_b)\ =$	$\langle O_i \rangle^{(0)}(\mu_b)$	$+\ \frac{\alpha_s(\mu_b)}{4\pi} \langle O_i \rangle^{(1)}(\mu_b)$	$+\ \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \langle O_i \rangle^{(2)}(\mu_b)$	
4-quark ($i = 1, \ldots, 6$):	**1-loop**	**2-loop**	**3-loop**	[Bieri, Greub, Steinhauser, hep-ph/0302051] $\mathcal{O}(\alpha_s^2 n_f)$, Steinhauser, MM (extrapol. in m_c)
"magn. mom." ($i = 7, 8$):	**tree**	**1-loop**	**2-loop**	Greub, Hurth, Asatrian

Charm mass dependence of $\mathbf{BR}[\bar{B} \to X_s \gamma \;\; (E_\gamma > 1.6 \text{ GeV})]$:



$\Rightarrow$ **Hint for the NNLO: large m_c expansion + extrapolation.**

Non-perturbative effects in $\bar{B} \to X_s \gamma$

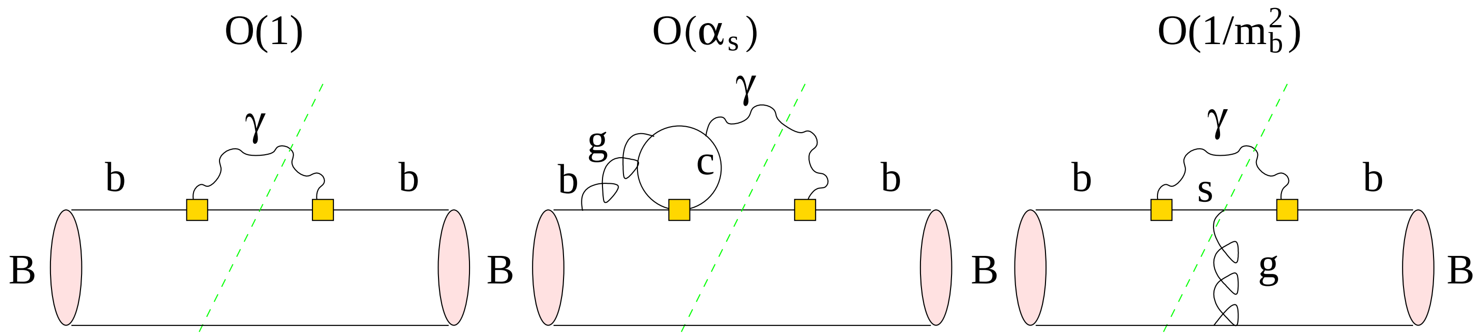


Fig. 2 from hep-ph/0212360 by M. Neubert

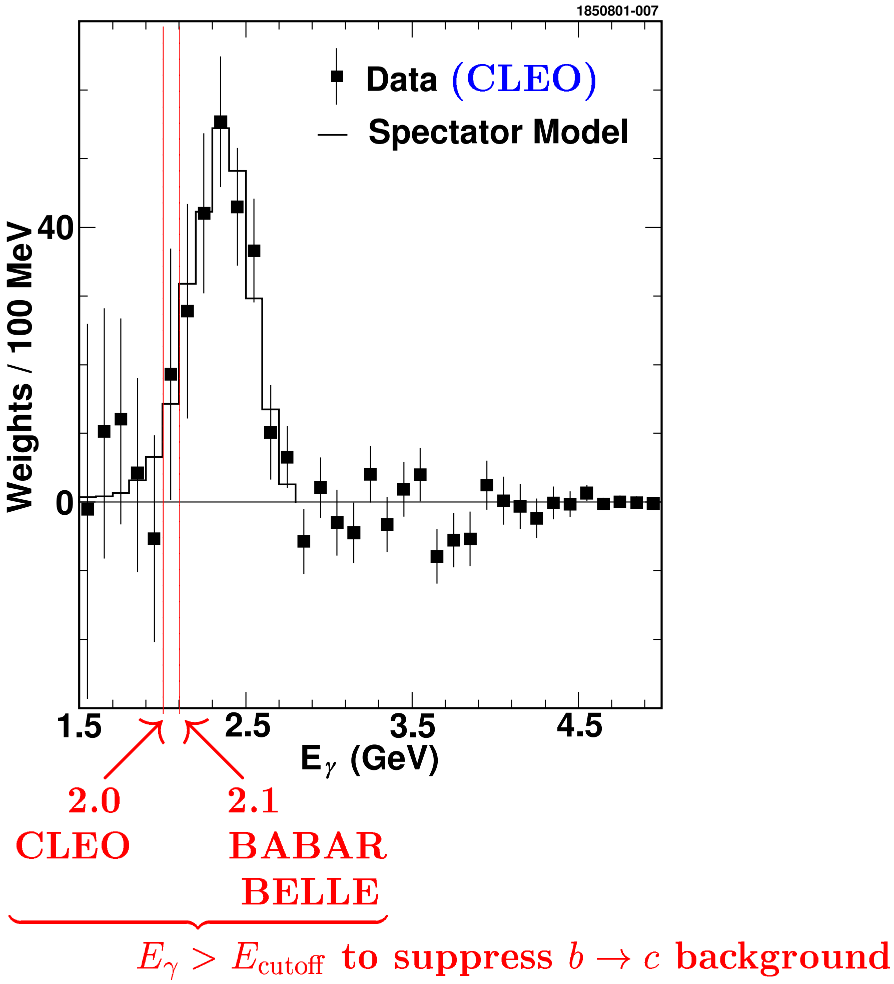
- **The first and third diagrams give rise to the $\mathcal{O}(\Lambda^2/m_b^2)$ corrections ($\sim -3\%$). Coefficients at λ_1 and λ_2 turn out to be identical to those in $\bar{B} \to X_u l \nu$.**
  [Falk, Luke, Savage, 1994; Bigi *et al*, 1992]

- **Diagrams like the middle one are not tractable with the usual OPE because one may cut through the c-quark loop rather than the photon.
  On the other hand, the corresponding perturbative effects are very large:
  BR$[\bar{B} \to X_s \gamma]$ changes by 35% (!!!) when m_c is varied between $0.2 m_b$ and $0.5 m_b$.**

- **In the absence of hard gluons, charm quark decoupling is applicable here even for $m_c < m_b$ because the resulting expansion parameter is $\Lambda m_b / m_c^2$ and the coefficients of the expansion decrease fast. The leading $\mathcal{O}(\Lambda^2/m_c^2)$ term enhances BR$[\bar{B} \to X_s \gamma]$ by $\sim +2.5\%$.**
  [Voloshin, 1997; Khodjamirian *et al*, 1997; Ligeti *et al*, 1997; Grant *et al*, 1997; Buchalla *et al*, 1998];

- **The $O_{1,2}$–$O_{1,2}$ matrix elements have an overlap with the intermediate ψ contribution that is subtracted out in the experimental analyses. The product of BR$[\bar{B} \to X^{(1)} \psi]$ and BR$[\psi \to X^{(2)} \gamma]$ is larger than BR$[\bar{B} \to X_s \gamma]$, so long as no cut on E_γ is imposed.**

$\bar{B} \to X_s \gamma$ photon energy spectrum



Extrapolation from $E_{\rm cut}^{\rm exp}$ down to $E_{\rm cut}^{\rm th}$:

	2.0 GeV	**2.1 GeV**	**2.2 GeV**
1.6 GeV	$5.3^{+5.6}_{-2.8}$ %	$10^{+8.5}_{-5.6}$ %	20^{+11}_{-10} %
1.5 GeV	$5.7^{+5.6}_{-2.8}$ %	$11^{+8.5}_{-5.6}$ %	20^{+11}_{-10} %
1.4 GeV	$6.1^{+5.6}_{-2.8}$ %	$11^{+8.5}_{-5.6}$ %	20^{+11}_{-10} %

(Errors from M. Neubert, hep-ph/9809377.)

A fit to the measured spectra can be (but has not been) used to reduce uncertainties in the extrapolation.

Conclusions:

- **The current agreement of the SM predictions for BR$[\bar{B} \to X_s\gamma]$ with experiment implies, that there is little chance for observing clear signals of new physics in this observable. However, the possibility of deriving tight constraints on extensions of the SM is a sufficient motivation for efforts to reduce the perturbative and experimental uncertainties. It is realistic to expect that all such uncertainties will soon decrease from the present $\sim 10\%$ to or below $\sim 5\%$.**

- **Since the calculation of the $\mathcal{O}(\Lambda^2/m_b^2)$ and $\mathcal{O}(\Lambda^2/m_c^2)$ corrections, it has been widely believed that the unknown non-perturbative effects in BR$[\bar{B} \to X_s\gamma]$ are well below 5%. However, non-perturbative phenomena that may arise on the top of the huge $\mathcal{O}(\alpha_s)$ correction call for better understanding.**