NNLL QCD Corrections to $B \to X_s \ell^+ \ell^-$ and New Physics

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NNLL Corrections to $B \to X_s \ell^+ \ell^-$

- Flavour Physics
- Perturbative Corrections in Rare B Decays
- The Decay $B \to X_s \ell^+ \ell^-$
- Experimental Data
- Two-loop Calculation:
  The Dilepton-Mass Spectrum
- Bremsstrahlung Calculation:
  Forward-Backward Asymmetry

Flavour Physics

- Two main issues:
  - Mechanism of CP violation in the B system
  - Indirect effects of new physics in rare B decays

- New physics:
  - No strict argument that new flavour physics must appear at the electroweak scale
  - Flavour sector leads to severe contraints for new physics
  - Flavour structure model-dependent issue !

Caveat

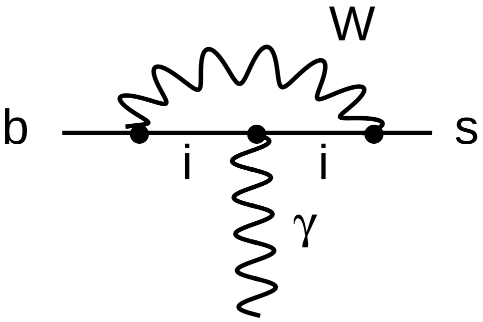
- Problem of long-distance strong interactions restricts opportunities in flavour physics significantly

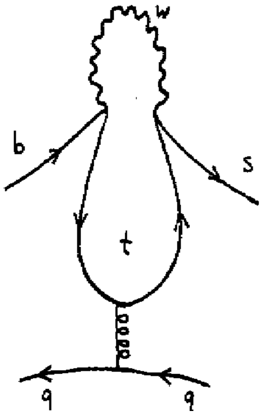
- b quark system $\Rightarrow$ heavy mass expansion : $\Lambda_{QCD} << m_b$

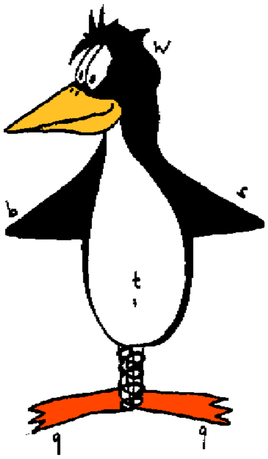
- Look for the theoretically clean variables !!

Inclusive Rare B Decays

- Rare B decays like $B \to X_{s,d}\,\gamma$ or $B \to X_s\,\ell^+\ell^-$ directly probe the SM at the one-loop level.





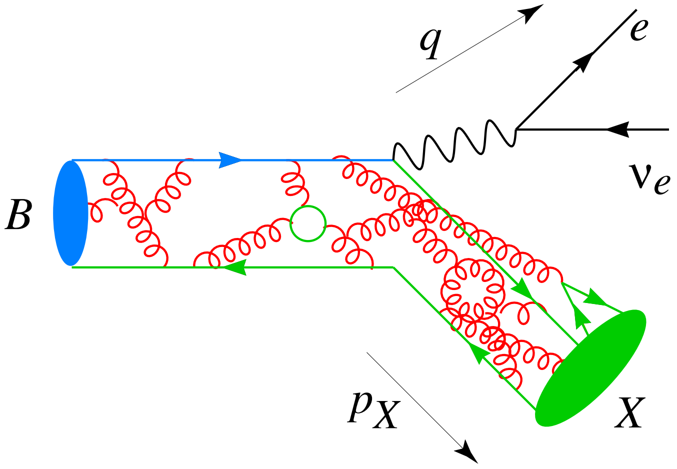


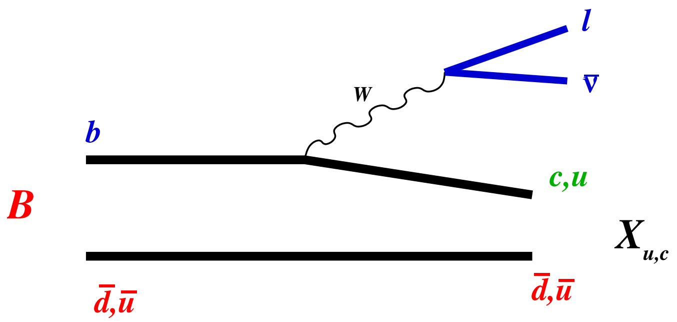
$\Rightarrow$ Indirect search for new physics

$\Rightarrow$ The measured $B \to X_s\gamma$ decay rate already plays an important role in restricting the parameter space of many extensions of the SM.

- Importance also for CKM phenomenology:
  $b \to d$ transitions $\equiv [V_{ub}/V_{cb},\ \Delta M_{B_d}\ ,\ \Delta M_{B_d}/\Delta M_{B_s}]$

Inclusive versus Exclusive Decays





Heavy Mass Expansion

$\Gamma(B \to X_s\gamma) \xrightarrow{m_b \to \infty} \Gamma(b \to X_s^{parton}\gamma)$; $\Delta^{nonpert.} \sim \Lambda^2_{QCD}/m_b^2$

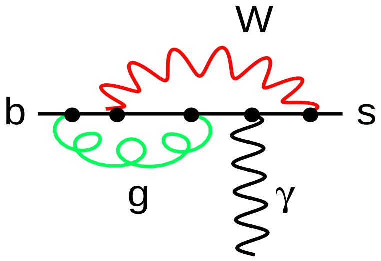
Perturbatively calculable contribution dominant

- Inclusive decay modes (like $B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$)
  - theoretically clean, valuable information for new physics search
  - experimentally more challenging
- Exclusive decay modes (like $B \to K^*\gamma$, $B \to K^*\ell^+\ell^-$)
  - large theoretical uncertainties from hadronic form factors, QCD-tests
  - cleaner experimental signals (especially at hadronic machines)
  - Exceptions: ratios like asymmetries and specific decays like $B \to \ell^+\ell^-$)

Perturbative Corrections in Rare B Decays

- Electroweak two-loop corrections play a subdominant role
- QCD corrections in inclusive rare B decays are important

$\alpha_s(M_W) Log(\frac{m_b^2}{M_W^2})$



$\rightarrow$ Resummation of Logs necessary:

LL	Leading Logs	$G_F\,(\alpha_s\,Log\,)^N$	$(N = 0, 1..)$
NLL	Next-to-Leading Logs	$G_F\,\alpha_s\,(\alpha_s\,Log\,)^N$	

- Appropriate theoretical framework

  $H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{bt} V_{st}^* \sum_{i=1} C_i(\mu)\ \mathcal{O}_i(\mu)$

- Perturbative QCD corrections to the decay rate twofold:

  I. QCD corrections to the matrix elements at the scale $\mu \simeq m_b$:

  $$\prec s\ell^+\ell^- \mid \mathcal{O}_i(\mu \simeq m_b) \mid b \succ$$

  II. QCD corrections to the Wilson coefficients $C_i(\mu \simeq m_b)$

  Only sum is renormalization group invariant

- ad I. QCD corrections to matrix elements:

$$\prec s\ell^+\ell^- \mid \mathcal{O}_i(\mu \simeq m_b) \mid b \succ$$

- ad II. QCD corrections to Wilson coefficients:

  μ-independence of H_{eff} $\Rightarrow$ $\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij} C_j(\mu)$
  Renormalization group equation (anomalous dimens. γ_{ij})

  * matching the effective theory with complete SM
  $\Rightarrow$ initial conditions of RGE $C_i(\mu \simeq M_W)$

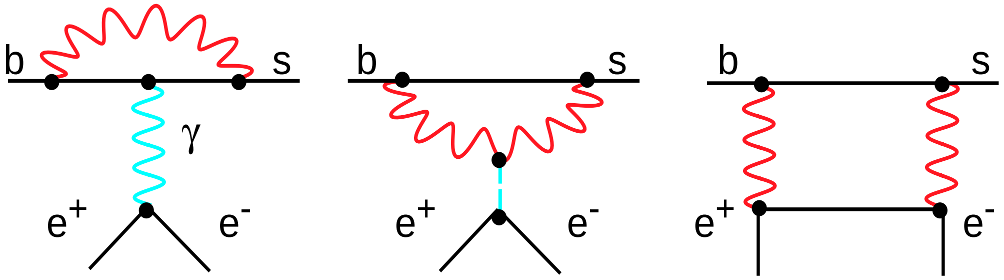
  * solving RGE : $\mu \simeq m_W \Rightarrow \mu \simeq m_b$
  $\Rightarrow$ large logs resummed in coefficients $C_i(\mu \simeq m_b)$.

Leading order:

$$\tilde{C}_i(\mu) = \left[\frac{\alpha_s(\mu_W)}{\alpha_s(\mu)}\right]^{\frac{\tilde{\gamma}_{ii}^{(0)}}{2\beta_0}} \tilde{C}_i(\mu_W) = \left[\frac{1}{1 + \beta_0 \frac{\alpha_s(\mu)}{4\pi} \ln \frac{\mu_W}{\mu^2}}\right]^{\frac{\tilde{\gamma}_{ii}^{(0)}}{2\beta_0}} \tilde{C}_i(\mu_W)$$

The Decay $B \to X_s \ell^+ \ell^-$

A more complex SM test than $B \to X_s \gamma$:



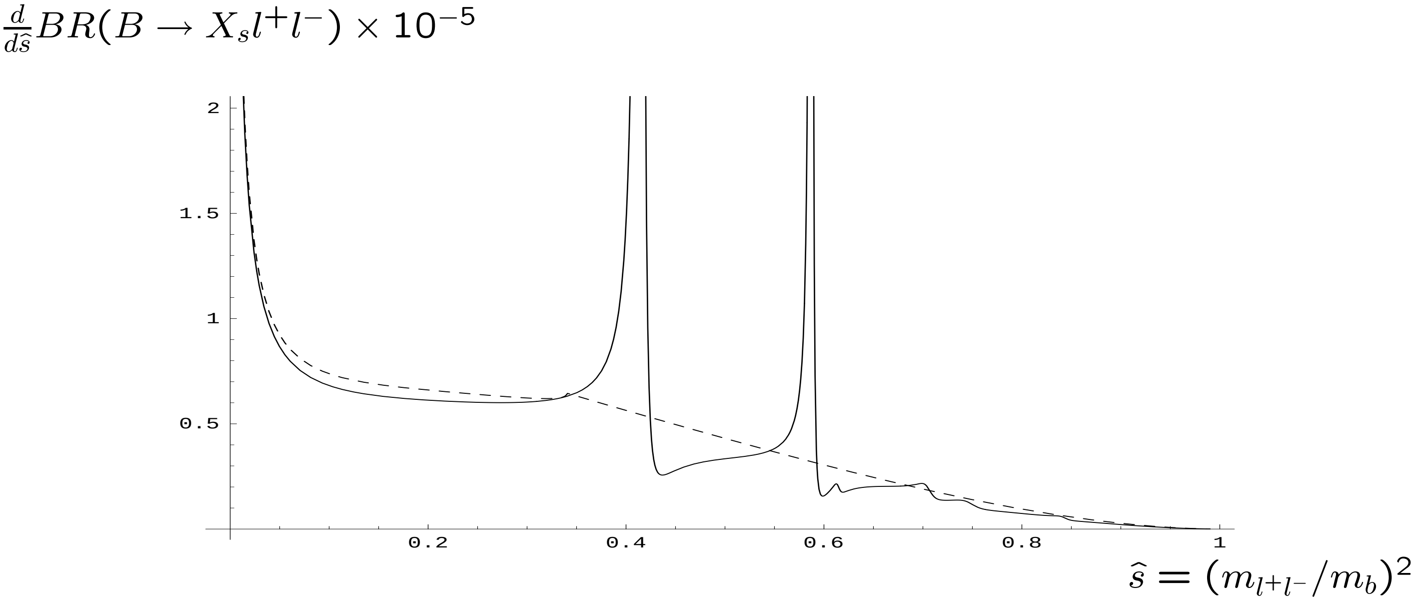
There are two perturbative windows separated by the on-shell $c\bar{c}$ resonances

Nonperturbative contributions:

- On-shell $c\bar{c}$ resonances:

  $\Rightarrow$ kinematical cuts in the dilepton mass spectrum:

  $0.05 < (m_{l^+l^-}/m_b)^2 < 0.25$ and $0.65 < (m_{l^+l^-}/m_b)^2$



- Power corrections in $1/m_b^2$ and $1/m_c^2$ as in $B \to X_s \gamma$:

  subdominant contribution, well under control due to HME and HQET

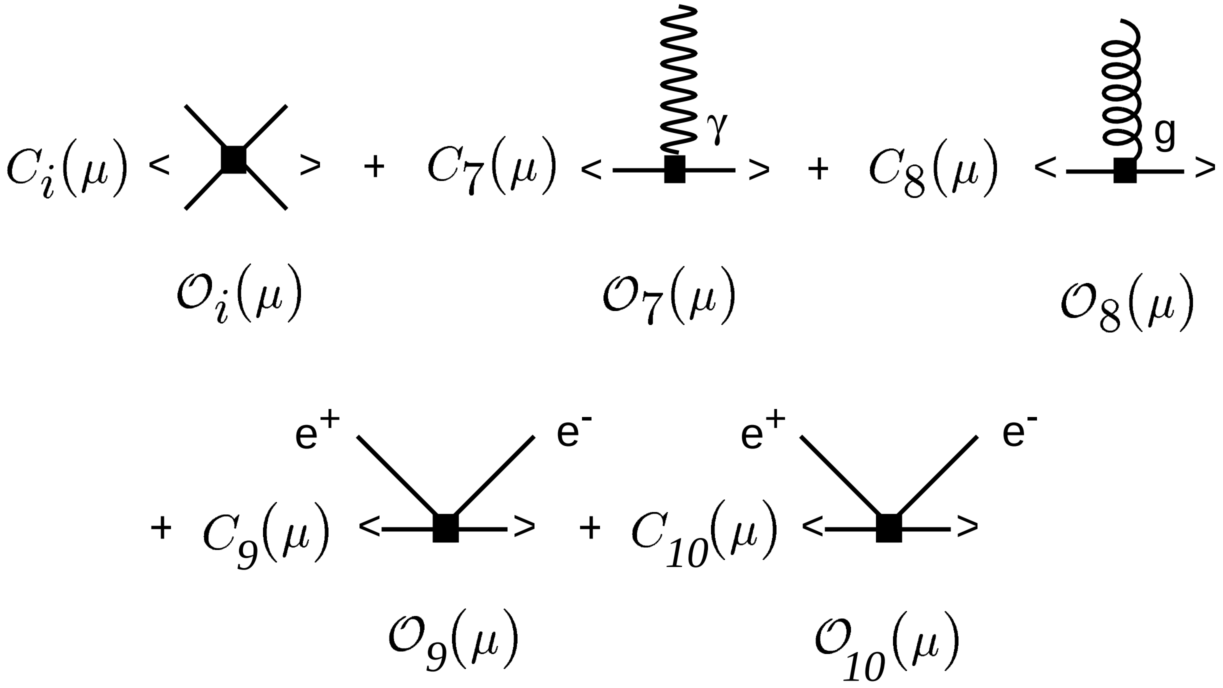
Dominating short distance contribution:

Effective field theory approach

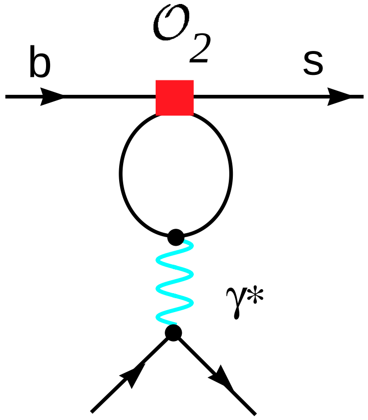
$$H_{eff}(b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{bt}V_{st}^* \sum_{i=1}^{10} C_i(\mu)\, \mathcal{O}_i(\mu)$$

Two additional operators induced :

$\mathcal{O}_1 - \mathcal{O}_6$	vectorial four-quark operators (Type II)
$\mathcal{O}_7, \mathcal{O}_8$	dipole operators
$\mathcal{O}_9, \mathcal{O}_{10}$	semileptonic operators



$\mathcal{O}_2$ mixes into $\mathcal{O}_9$ at one loop:



large logarithm $Log = \log(m_b/M_W)$
without the exchange of gluons

$G_F\ (\alpha_s)^{N-1}\, Log^N\quad (LL)$
$G_F\ (\alpha_s)^{N}\quad Log^N\quad (NLL)$
$G_F\ (\alpha_s)^{N+1}\, Log^N\quad (NNLL)$

The LL term is accidentally quite small

⇒ only NNLL term leads to ±10% accuracy

NNLL QCD Calculation

- QCD corrections to Wilson coefficients:

known to NNLL (Bobeth, Urban, Misiak '99)

$$C_9(\mu) = \frac{1}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)} + \alpha_s(\mu) C_9^{(1)} + \ldots$$

- QCD corrections to matrix elements:

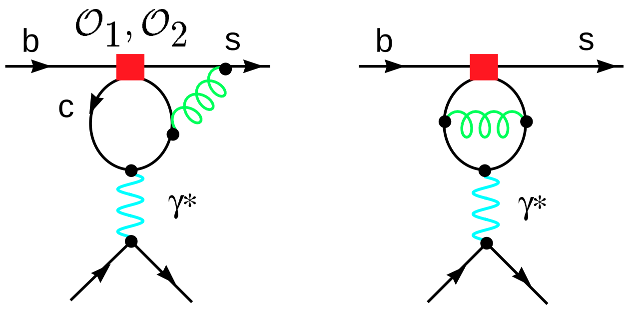
LL : $\langle \mathcal{O}_9 \rangle$ (tree level)

NLL: $\langle \mathcal{O}_9 \rangle$ (QCD one-loop)
$\langle \mathcal{O}_k \rangle$, $k = 7, 10$ (tree)
$\langle \mathcal{O}_i \rangle$, $i = 1...6$ (electroweak one-loop)

NNLL: $\langle \mathcal{O}_9 \rangle$ (QCD two-loop)
$\langle \mathcal{O}_k \rangle$, $k = 7, 10$ (QCD one loop)
$\langle \mathcal{O}_i \rangle$, $i = 1...6$ (QCD-electroweak two-loop)

Most difficult part:

NNLL matrix elements of the four-quark operators



Asatrian, Asatryan, Greub, Walker '01: $q^2/m_b^2 < 0.25$

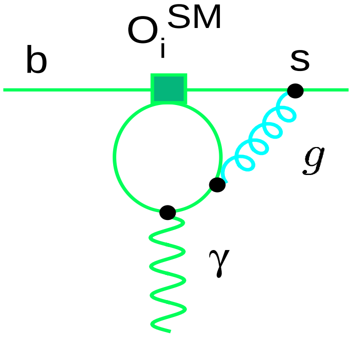
Ghinculov, Hurth, Isidori, Yao '02: for all q^2

Details of the Two-Loop QCD Calculation

Two-loop matrix elements of the four-quark operators:

- $B \to X_s \gamma$ on-shell photon, $q^2 = 0$

Greub, Hurth, Wyler



$*$ After Feynman parametrization

$$\Rightarrow \int_0^1 dxdydudv \, \frac{[x(1-x)]^{1-\epsilon} \, y^{\epsilon-1} \, [1-v]^{\epsilon} \, v}{C^{2\epsilon}} \, Poly(x,y,v,u)$$

$$C = m_b^2 v(1-v)u - m_c^2/(x(1-x))(1-v)y + i\delta$$

$*$ Use Mellin Barnes representation of the propagator for C

$$\frac{1}{(k^2-M^2)^\lambda} = \frac{1}{(k^2)^\lambda}\frac{1}{\Gamma(\lambda)}\frac{1}{2\pi i}\int_\gamma (-\frac{M^2}{k^2})^s \Gamma(\lambda+s)\Gamma(-s)ds$$

($\lambda > 0$, γ parallel to the imaginary axis)

$$k^2 \leftrightarrow m_b^2 v(1-v)u \quad ; \quad M^2 \leftrightarrow \frac{m_c^2}{x(1-x)}(1-v)y \quad .$$

$$\sum_{n,m} c_{nm} \, z^n \, \log^m z \quad , \quad z = \frac{m_c^2}{m_b^2}$$

$*$ We retained all terms up to $m = 3$; truncating at $n = 2$ differs only by 1%

- $B \to X_s \ell^+ \ell^-$ off-shell photon, $q^2 \neq 0$:

$*$ Three mass scales: m_c^2, m_b^2 and q^2

$*$ Asatrian et al. '01 used the same Mellin Barnes trick twice :

$$\sum_{i,j,n,m} c_{ijnm} s^i \, log^j(s) \, z^n \, \log^m z \quad , \quad z = \frac{m_c^2}{m_b^2} \quad s = \frac{q^2}{m_b^2}$$

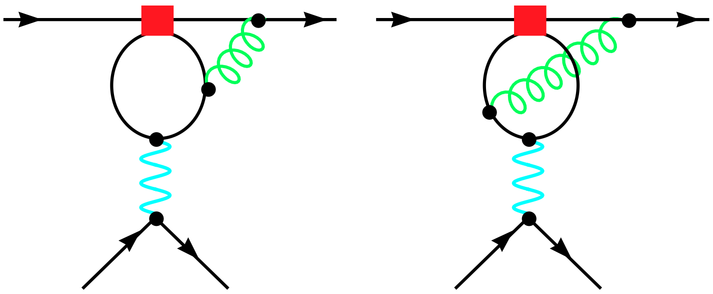
$*$ Validity of the results is restricted to the range $s < 0.25$!

Details of the Two-Loop QCD Calculation II

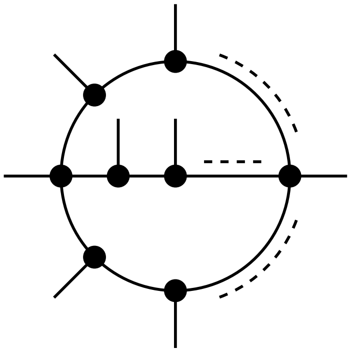
Two-loop matrix elements of the four-quark operators:

$B \to X_s \ell^+ \ell^-$ off-shell photon, $q^2 \neq 0$

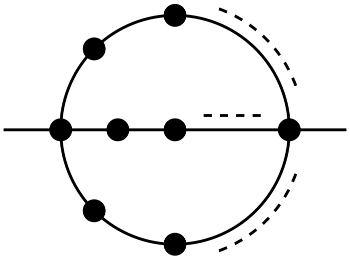
Ghinculov, Hurth, Isidori, Yao '02



- Expressing generic massive two-loop Feynman diagrams as integrals over sunset-type functions



$= \int_0^1 \int_0^1 \ldots \int_0^1 dX$



$$\int d^n p \, d^n q \, \frac{p^{\mu_1} \ldots p^{\mu_i} q^{\mu_{i+1}} \ldots q^{\mu_j}}{[(p+k)^2 + m_1^2]^{\alpha_1} (q^2 + m_2^2)^{\alpha_2} (r^2 + m_3^2)^{\alpha_3}} \quad .$$

- After tensor reduction we have a set of ten scalar integrals:

$$\begin{aligned}
\mathcal{H}_1 &= \int d^n p \, d^n q \, \frac{1}{[(p+k)^2 + m_1^2]^2 (q^2 + m_2^2)(r^2 + m_3^2)} \\
\mathcal{H}_2 &= \int d^n p \, d^n q \, \frac{p \cdot k}{[(p+k)^2 + m_1^2]^2 (q^2 + m_2^2)(r^2 + m_3^2)} \\
\ldots \quad & \ldots \quad \ldots \\
\ldots \quad & \ldots \quad \ldots \\
\mathcal{H}_{10} &= \int d^n p \, d^n q \, \frac{(q \cdot k)^3 - \frac{3}{n+2} k^2 q^2 (q \cdot k)}{[(p+k)^2 + m_1^2]^2 (q^2 + m_2^2)(r^2 + m_3^2)}
\end{aligned}$$

Construction such that the $\mathcal{H}_i$ logarithmically divergent only !

- Symmetric integral representations of the $\mathcal{H}_i$ can be found

$$\begin{array}{lcl}
\mathcal{H}_1 & = & \pi^4 \left[\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(1 - 2\gamma_{m_1}) - \frac{1}{2} + \frac{\pi^2}{12} - \gamma_{m_1} + \gamma_{m_1}^2 + h_1 \right] \\
\mathcal{H}_2 & = & \pi^4 k^2 \left[-\frac{2}{\epsilon^2} + \frac{1}{\epsilon}(\frac{1}{2} - 2\gamma_{m_1}) + \frac{13}{8} - \frac{\pi^2}{12} + \frac{\gamma_{m_1}}{2} - \gamma_{m_1}^2 - h_2 \right] \\
\ldots & .. & \ldots \\
\ldots & .. & \ldots
\end{array}$$

- Reduction to four building blocks: $\tilde{g}(x)$, $\tilde{f}_1(x)$, $\tilde{f}_2(x)$,$\tilde{f}_3(x)$

$$\begin{array}{rcc}
h_1(m_1, m_2, m_3; k^2) & = & \int_0^1 dx\, \tilde{g}(x) \\
h_2(m_1, m_2, m_3; k^2) & = & \int_0^1 dx\, [\tilde{g}(x) + \tilde{f}_1(x)] \\
h_3(m_1, m_2, m_3; k^2) & = & \int_0^1 dx\, [\tilde{g}(x) + \tilde{f}_1(x)]\,(1 - x) \\
 & \ldots & \ldots \\
 & \ldots & \ldots \\
h_9(m_1, m_2, m_3; k^2) & = & \int_0^1 dx [\tilde{g}(x) + \tilde{f}_1(x) + \tilde{f}_2(x) + \tilde{f}_3(x)](1 - x)^2 \quad . \\
h_{10}(m_1, m_2, m_3; k^2) & = & \int_0^1 dx [\tilde{g}(x) + \tilde{f}_1(x) + \tilde{f}_2(x) + \tilde{f}_3(x)](1 - x)^3 \quad .
\end{array}$$

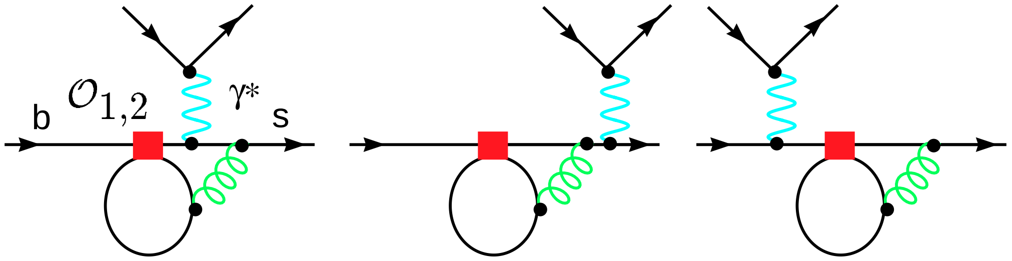
- Completeness of the $\{\mathcal{H}_i\}_{i=\overline{1,10}}$ special functions
for renormalizable theories: power counting arguments

- Caveat: IR singularities have to be isolated.

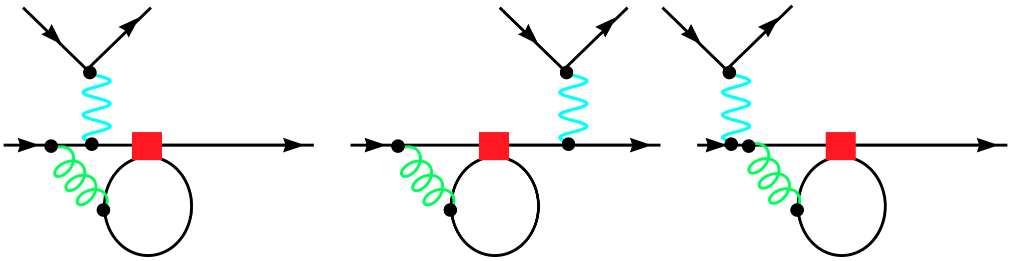
However: in the present application all the relevant two-loop integrals are IR finite !

Gauge invariant subgroups:

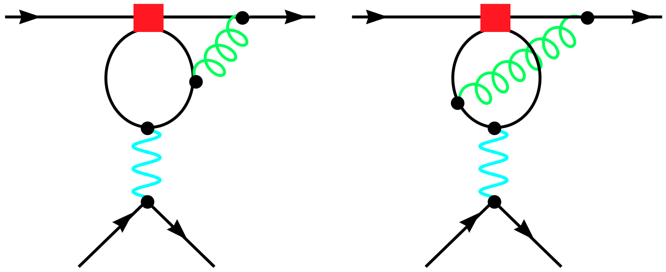
a)



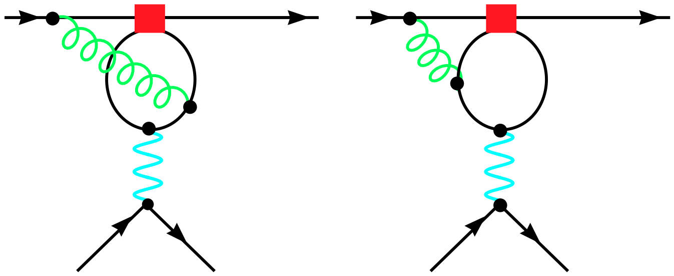
b)



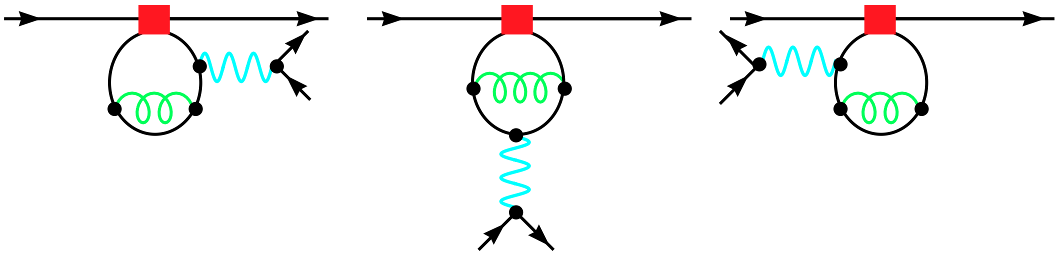
c)



d)



e)



Plots of the unrenormalized diagrams

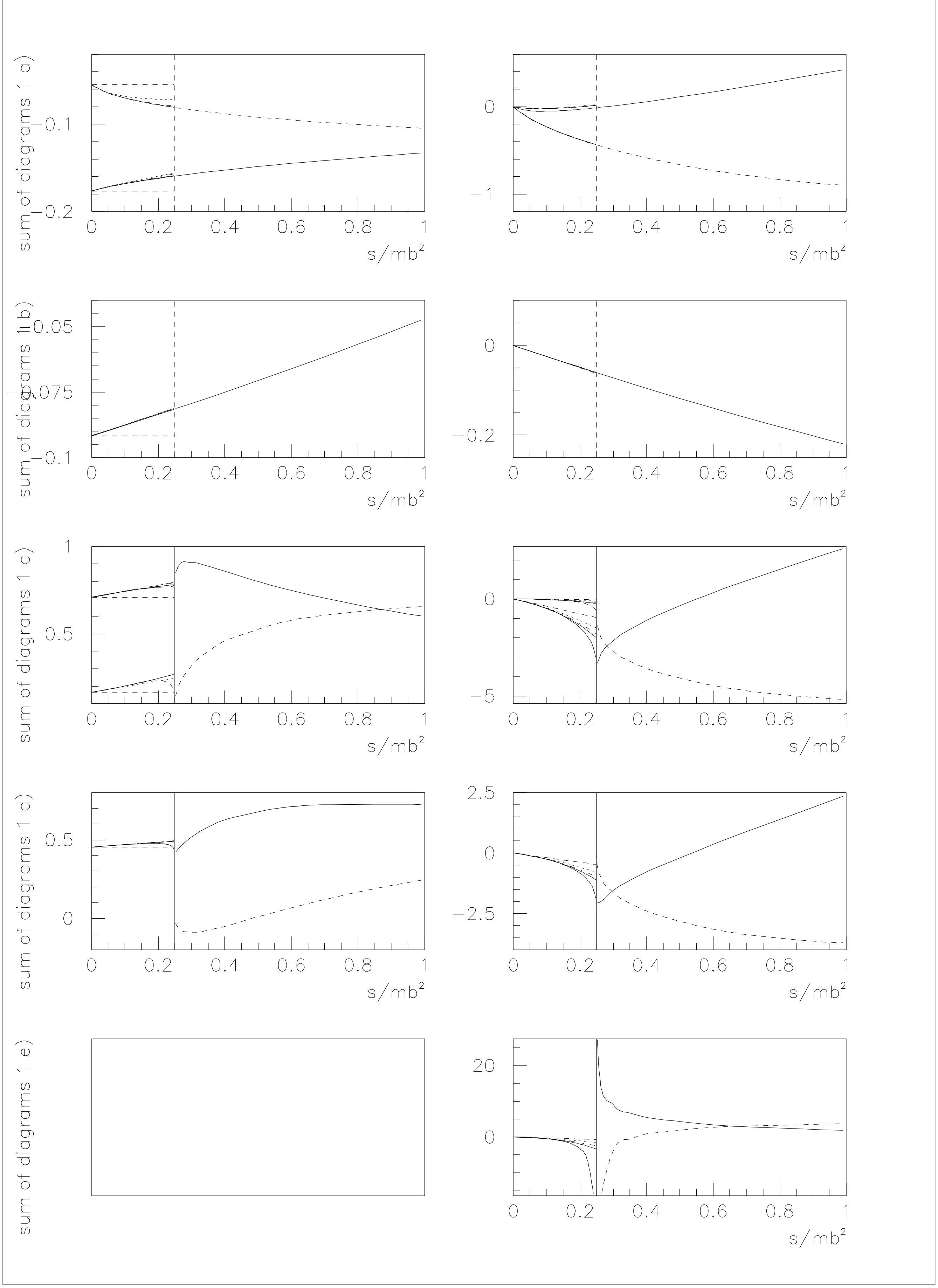
(1) Threshold behaviour

(2) Comparison with Asatrian et al. in low s region:
Convergence of the expanded result

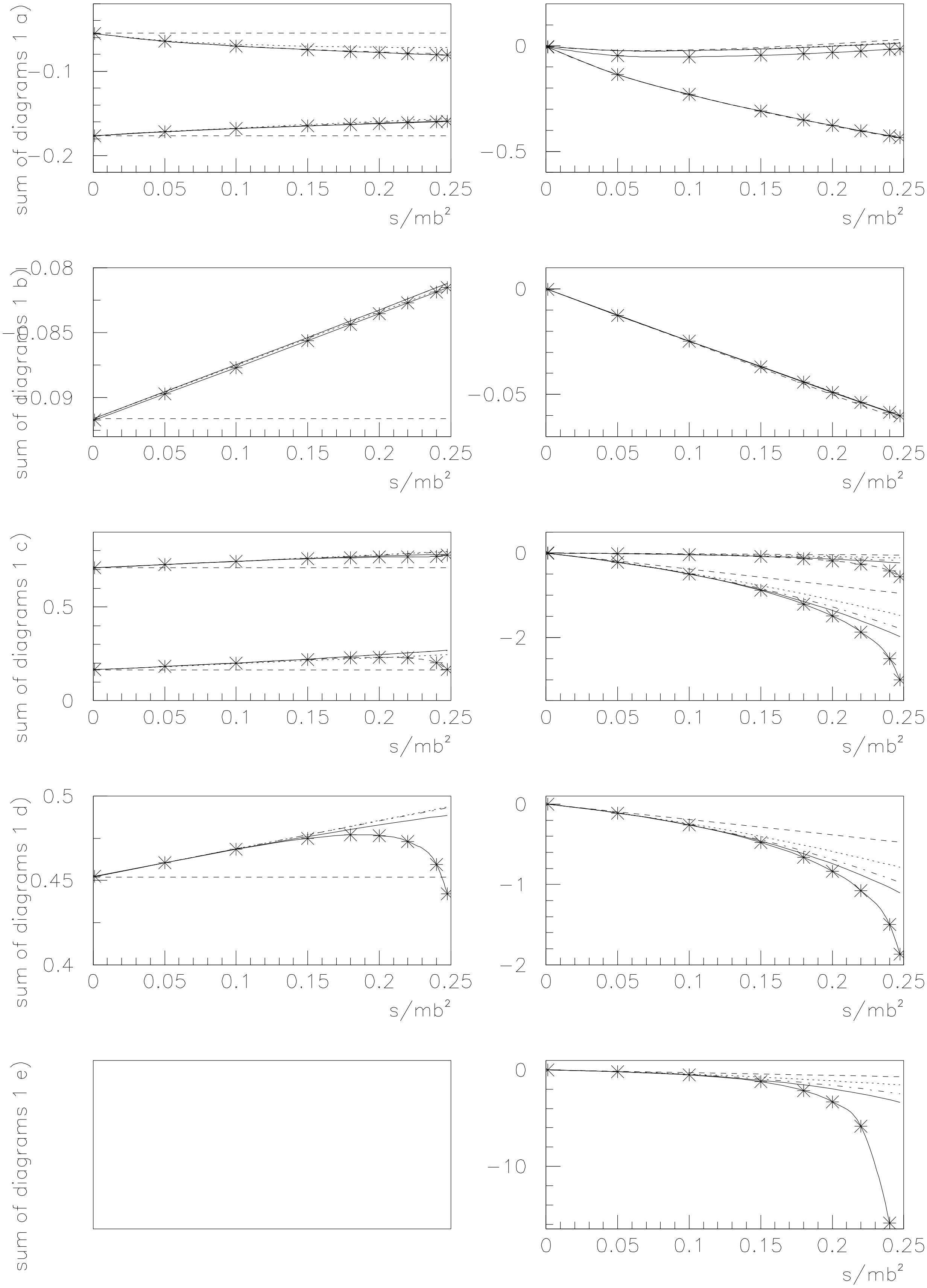
Two structures: $< \mathcal{O}_7 >_{tree}$, $< \mathcal{O}_9 >_{tree}$

In the low s region, we have a strong test of our results !

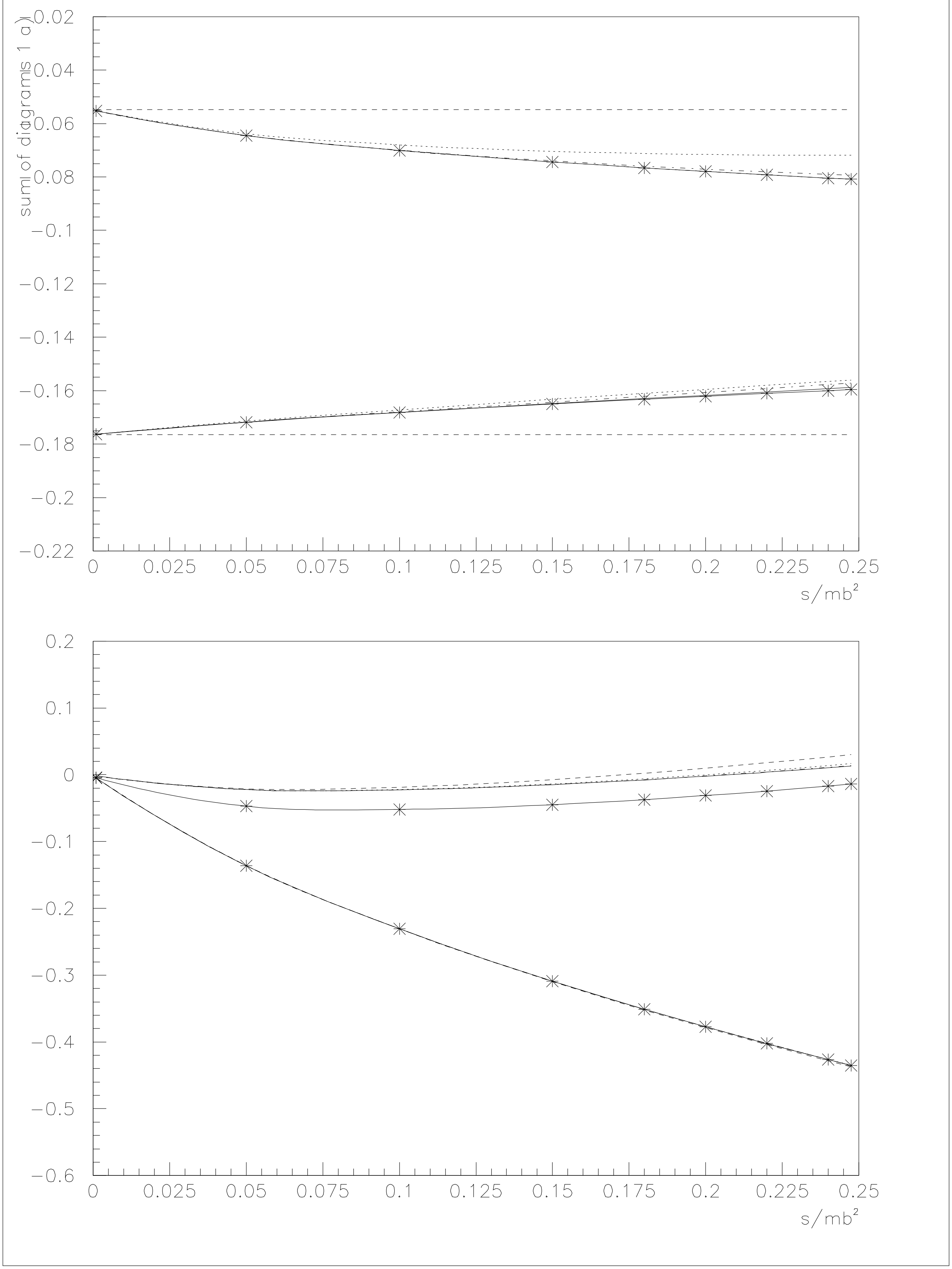
ad 1


sum of diagrams 1 a)
−0.1
−0.2
0
−1
sum of diagrams 1 b)
−0.05
−0.075
−0.1
0
−0.2
sum of diagrams 1 c)
1
0.5
0
−5
sum of diagrams 1 d)
0.5
0
2.5
0
−2.5
sum of diagrams 1 e)
20
0
0
0.2
0.4
0.6
0.8
1
s/mb²

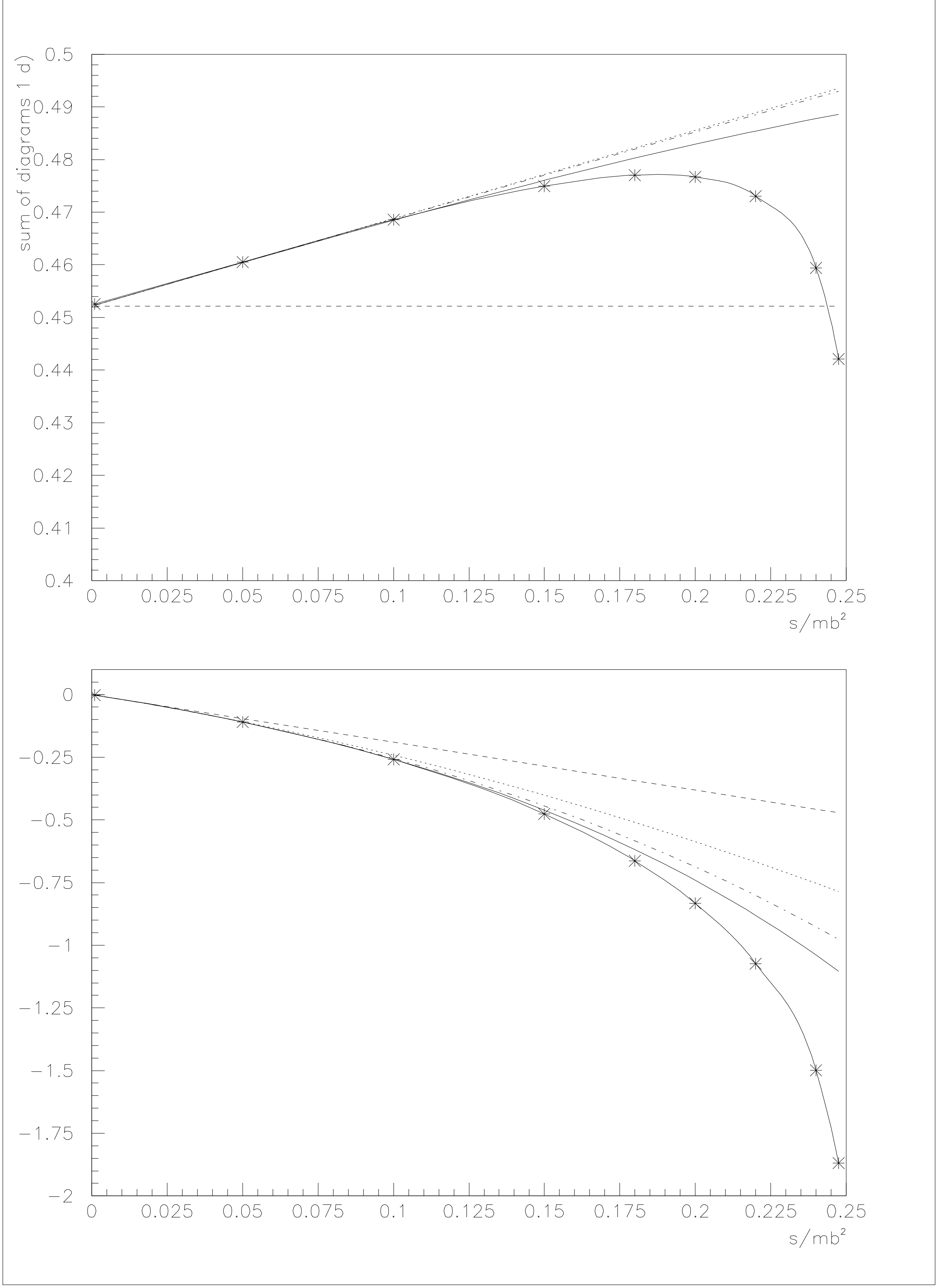
ad 2


sum of diagrams 1 a)
sum of diagrams 1 b)
sum of diagrams 1 c)
sum of diagrams 1 d)
sum of diagrams 1 e)
−0.1
−0.2
0
−0.5
−0.08
−0.085
−0.09
0
−0.05
0.5
0
0
−2
0.5
0.45
0.4
0
−1
−2
0
−10
0
0.05
0.1
0.15
0.2
0.25
s/mb²

ad 2

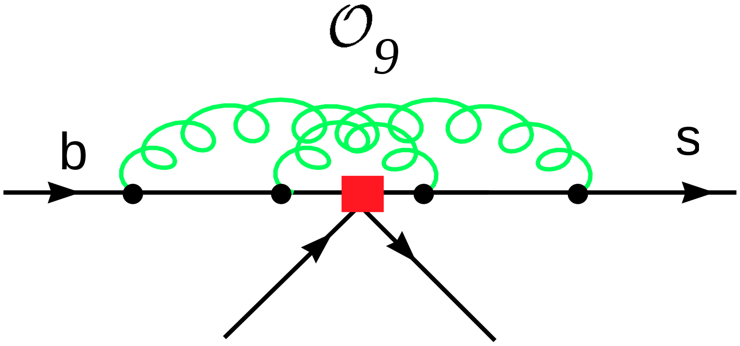

sum of diagrams 1 a)
−0.02
−0.04
−0.06
−0.08
−0.1
−0.12
−0.14
−0.16
−0.18
−0.2
−0.22
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
s/mb²
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
s/mb²

ad 2


sum of diagrams 1 d)
0.5
0.49
0.48
0.47
0.46
0.45
0.44
0.43
0.42
0.41
0.4
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
s/mb²
0
−0.25
−0.5
−0.75
−1
−1.25
−1.5
−1.75
−2
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
s/mb²

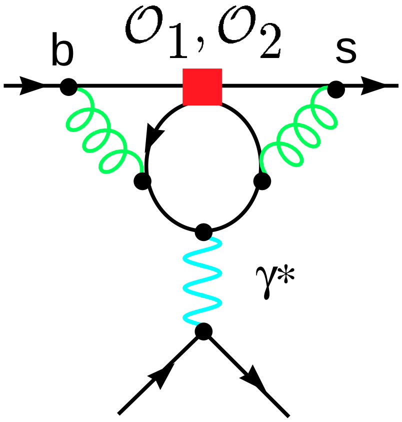
Missing pieces for a complete NNLL calculation of the rate : both are estimated to be numerically small

* Two-loop matrixelement of operator $\mathcal{O}_9$:



* Three-loop mixing of four-quark operators into operator $\mathcal{O}_9$:

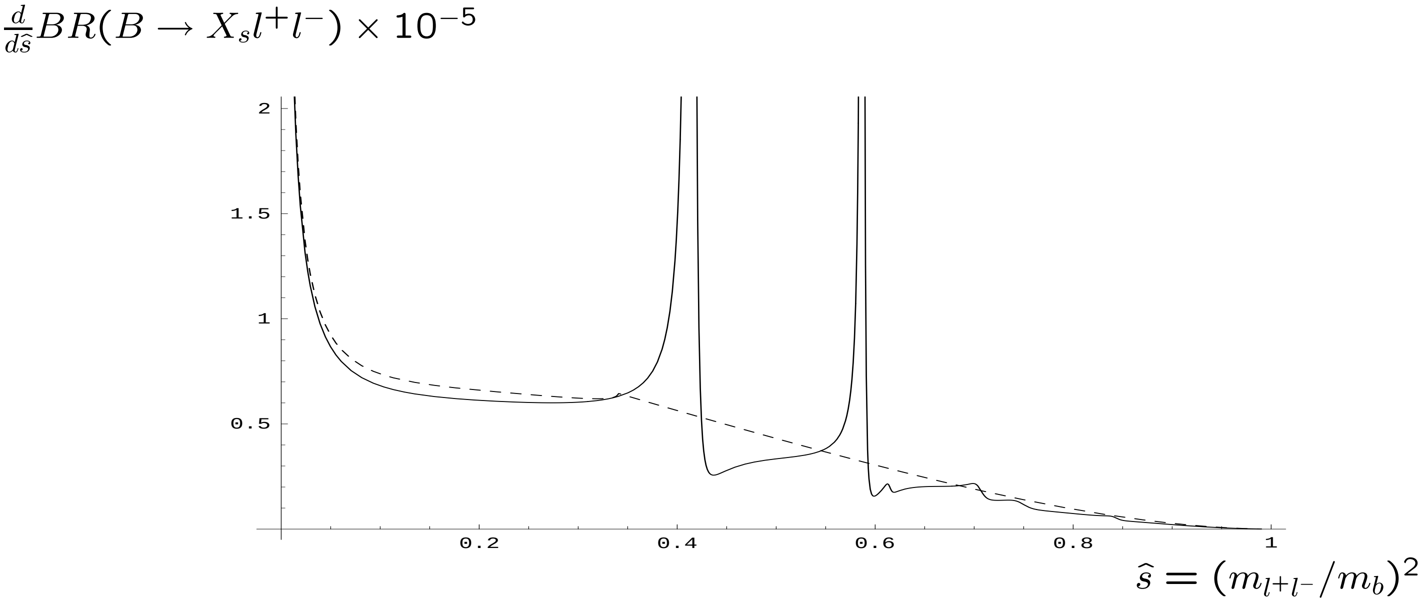
Done, talk by Haisch



⇒ Introduce another counting Regard formally leading $O(1/\alpha_s)$ term in C_9 as an $O(1)$ term

However: NNLL calculation of forward-backward-asymmetry already complete !

Dilepton Mass Spectrum



- Theory: Asatrian et al.; Ghinculov,Hurth,Isidori,Yao

$$BR(B \to X_s l^+ l^-)_{Cut:\ \widehat{s}\in[0.05,0.25]} =$$

$$= BR(B \to X_c e\overline{\nu}) \int_{Cut} d\widehat{s} \left[R^{l^+l^-}_{quark}(\widehat{s}) + \delta_{1/m_b^2}R(\widehat{s}) + \delta_{1/m_c^2}R(\widehat{s})\right]$$

$$= (1.25 \pm 0.08\ \text{(only scale)}\ )\ 10^{-6}$$

$$m_c^{\text{pole}}/m_b^{\text{pole}} = 0.29 \pm 0.04\ :$$

$\pm 0.02 \Rightarrow \pm 7\%$, $\pm 0.04 \Rightarrow \pm 15\%$

Impact of the NNLL matrix elements:
central value: -14%, scale uncertainty: $13\% \to 6.5\%$

- Sensitivity to New Physics:

  If $C_7^{eff}(m_b)$ changes sign: $BR_{\ cut}$: 1.5 10^{-6} $\to$ 3 10^{-6}

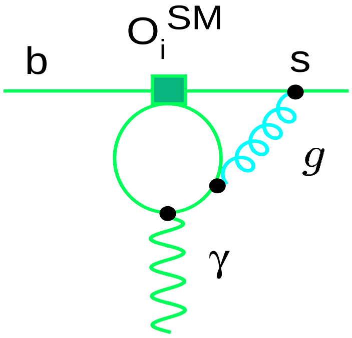
- Comparison with exclusive modes (see Ball et al.):

  $\Delta BR(B \to K^* \mu^+ \mu^-) = (^{+26}_{-17}, \pm 6, ^{+6}_{-4}, ^{-0.7}_{+0.4}, \pm 2)\%$

Analysis of m_c dependence at NLL, NNLL:

- $B \to X_s\gamma$:

$$< O_2 > \equiv < X_s\gamma|(\bar{s}c)_{V-A}(\bar{c}b)_{V-A}|b >$$



Matrixelement starts at two-loop
⇒ renormalization scheme for m_c is a three-loop issue.

$$m_c^{\text{pole}}/m_b^{\text{pole}} = 0.29 \pm 0.02 \Rightarrow m_c^{\overline{\text{MS}}}(\mu)/m_b^{\text{pole}} = 0.22 \pm 0.04$$

. (with $\mu \in [m_c, m_b]$)

⇒ Increase of BR$_\gamma$ by around 11%.

Gambino, Misiak

Educated guess of NNLL effect clearly favors the $\overline{MS}$ scheme (off-shell quark mass).

- $B \to X_s\ell\ell$:

Matrixelement starts already at one-loop:

problem of mass renormalization scheme less severe

No optimal choice: MS mass $m_c^{\overline{\text{MS}}}(\mu)$ would be best far away from $c\bar{c}$ threshold. However, near the threshold the pole mass scheme is better.

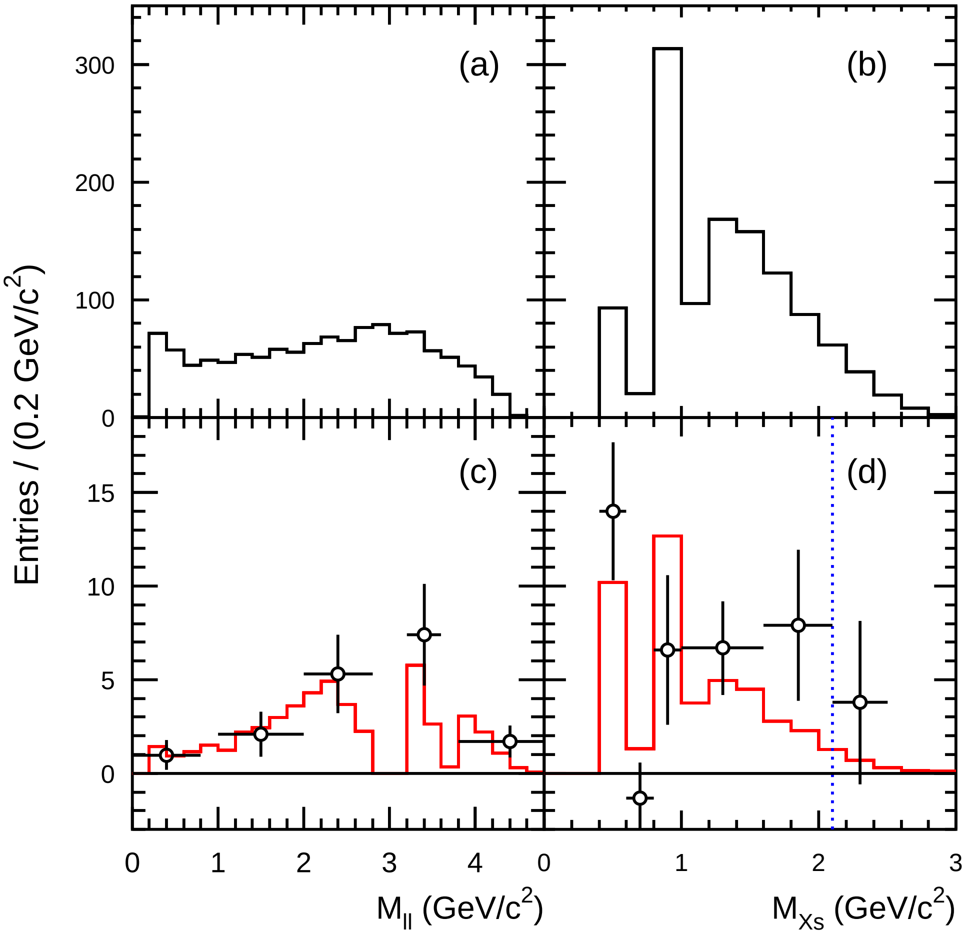
We use $m_c^{\text{pole}}/m_b^{\text{pole}} = 0.29 \pm 0.04$.

Experimental Data

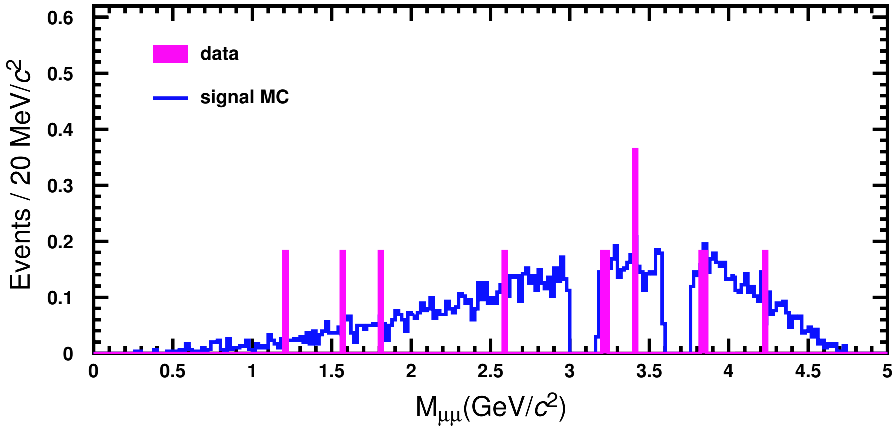
- Measurements of the exclusive modes $B \to K^*, K\,\ell^+\ell^-$ now available from Belle (hep-ex/0109026) and Babar (hep-ex/0207082)

- Inclusive Measurement of Belle (hep-ex/0208029): $BR(B \to X_s\ell^+\ell^-) = (6.1 \pm 1.4(stat) + 1.4 - 1.1(syst))10^{-6}$

SM expectations and observed spectra:
Efficiency in high s region



Dimuon mass distribution of $B \to K\mu^+\mu^-$ candidates:



Forward-Backward-Charge-Asymmetry

$$A_{\mathrm{FB}} \equiv \frac{1}{\Gamma_{semilep}} \left(\int_0^1 d(\cos\theta) \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d(\cos\theta) \frac{d^2\Gamma}{dq^2 d\cos\theta} \right)$$

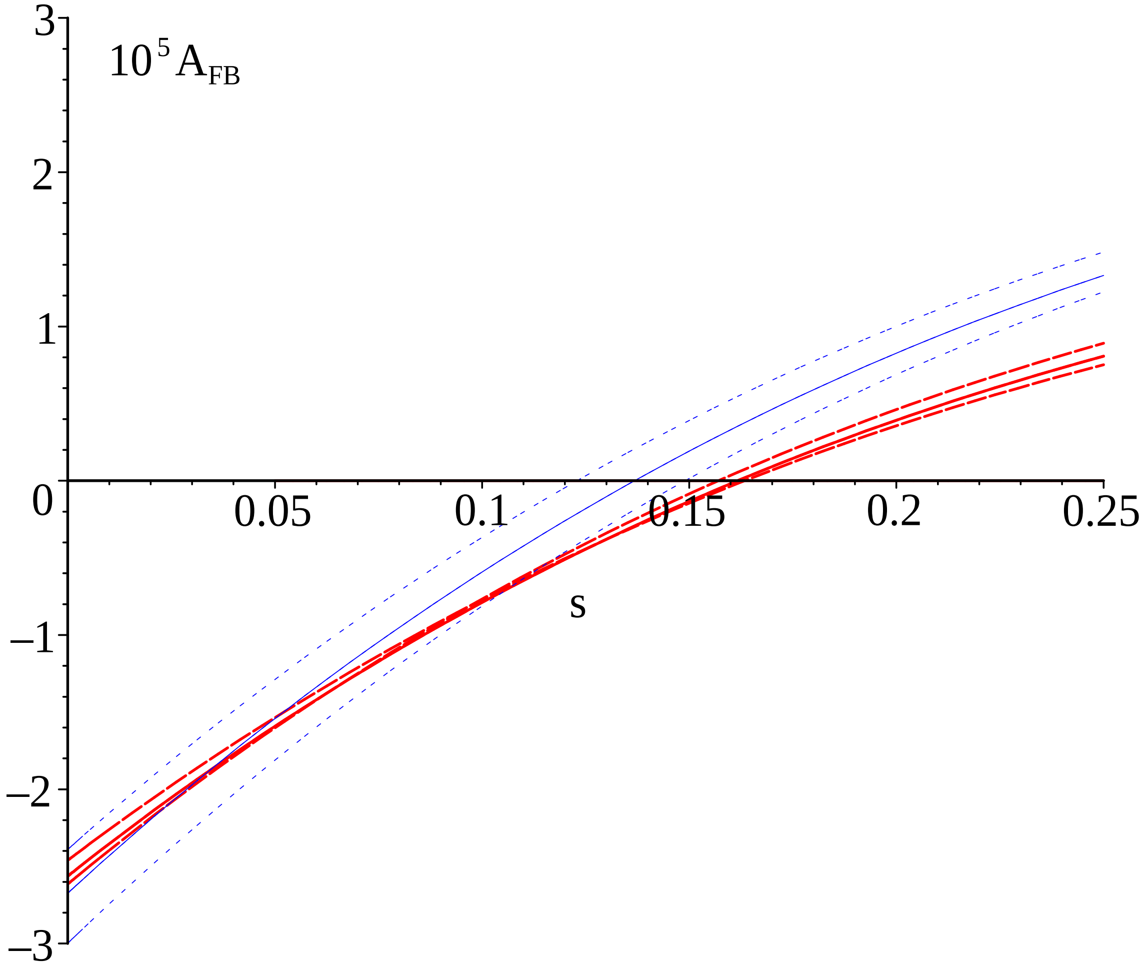
(θ angle between l^+ and B momenta
in the dilepton center-of-mass frame)

$A_{FB}(s) = 0$ for $s = q^2/m_b^2 \sim C_7/C_9$

NNLL corrections induce large $\sim$ 10% Shift of the Zero

5% theoretical error

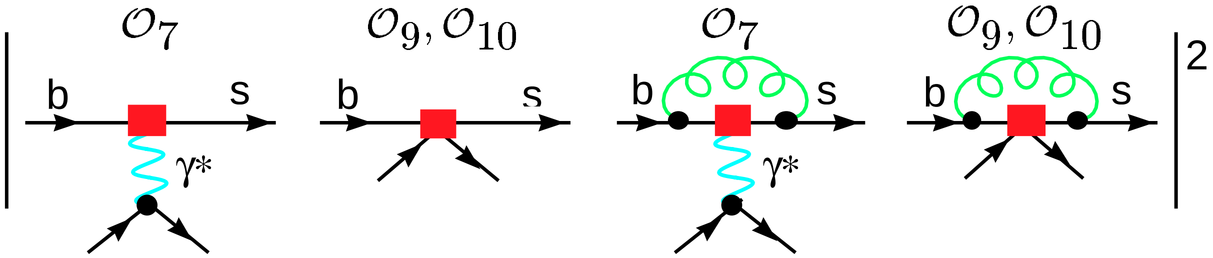
Ghinculov, Hurth, Isidori, Yao '02
Asatrian, Bieri, Greub, Hovhannisyan '02

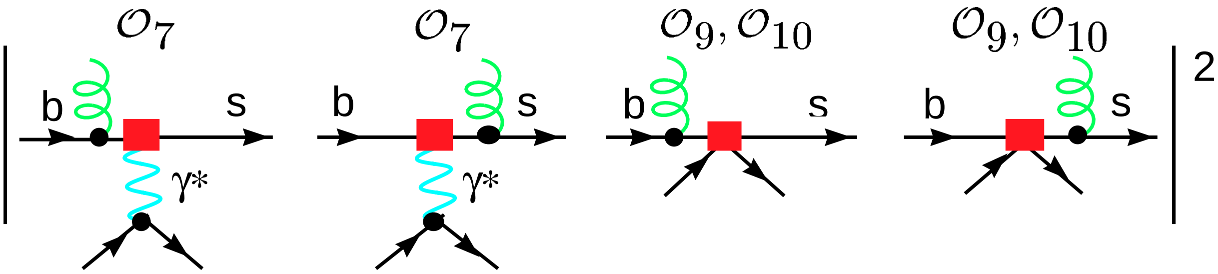


Note normalization with $\Gamma_{semilep}$

Bremsstrahlung Calculation

Ghinculov,H.,Isidori,Yao '02;   Asatrian et al. '02





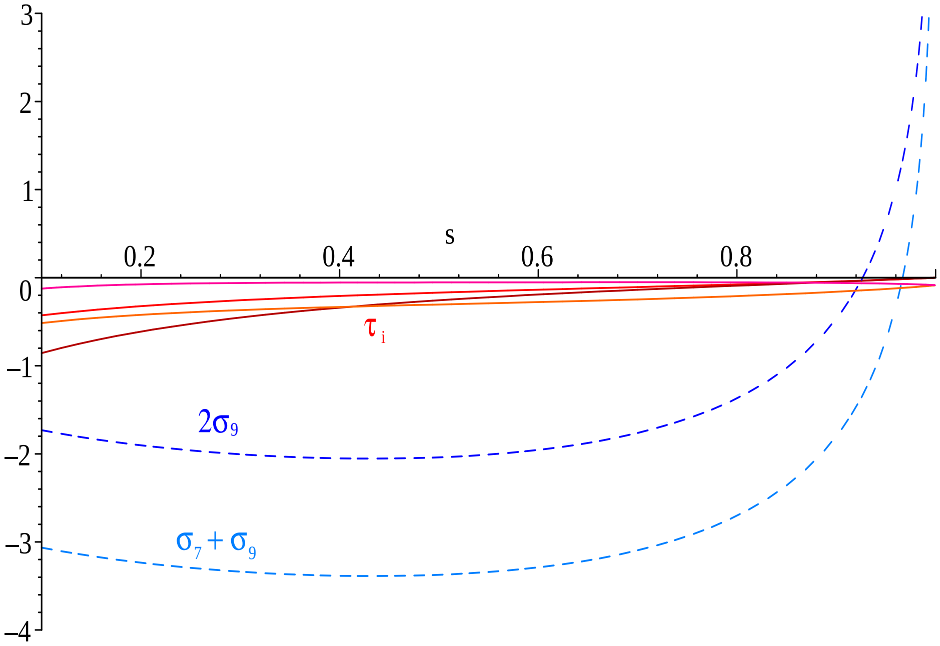
$$R(s) \sim \Big\{4\left(1+\tfrac{2}{s}\right)|C_7^{new}(s)|^2\left(1+\tfrac{\alpha_s}{\pi}\tau_{77}(s)\right) + (1+2s)\left[|C_9^{new}(s)|^2+|C_{10}^{new}(s)|^2\right]\left(1+\tfrac{\alpha_s}{\pi}\tau_{99}(s)\right) + 12\,\mathrm{Re}\left[C_7^{new}(s)C_9^{new}(s)^*\right]\left(1+\tfrac{\alpha_s}{\pi}\tau_{79}(s)\right)\Big\}\ ,$$

$$A_{\mathrm{FB}}(s) \sim \Big\{s\,\mathrm{Re}\left[C_{10}^{new}(s)^*C_9^{new}(s)\right]\left(1+\tfrac{\alpha_s}{\pi}\tau_{910}(s)\right) + 2\,\mathrm{Re}\left[C_{10}^{new}(s)^*C_7^{new}(s)\right]\left(1+\tfrac{\alpha_s}{\pi}\tau_{710}(s)\right)\Big\}$$

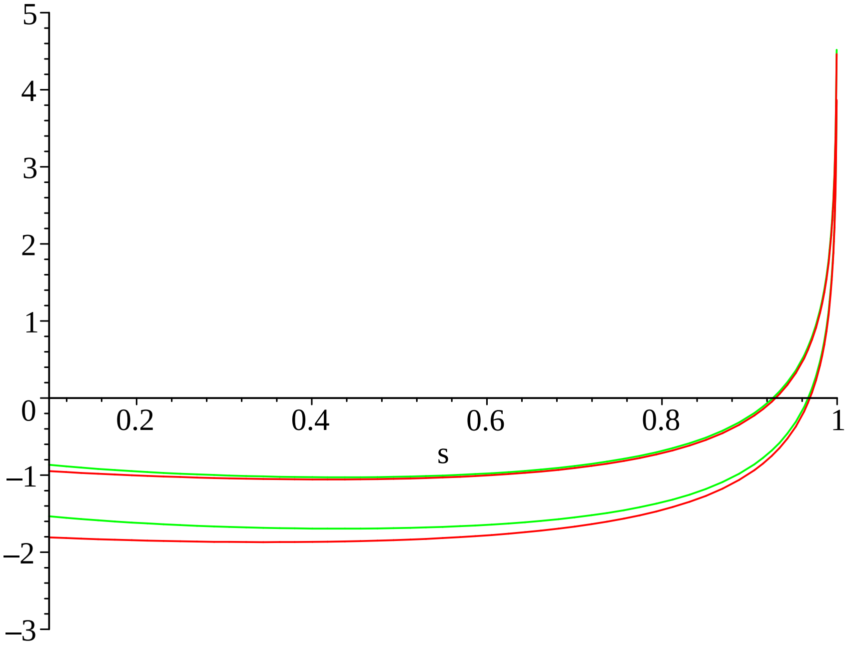
$$\begin{aligned} C_7^{new}(s) &= \left(1+\frac{\alpha_s}{\pi}\sigma_7(s)\right)\widetilde{C}_7^{\mathrm{eff}} + \ldots \\ C_9^{new}(s) &= \left(1+\frac{\alpha_s}{\pi}\sigma_9(s)\right)\widetilde{C}_9^{\mathrm{eff}} + \ldots \\ C_{10}^{new}(s) &= \left(1+\frac{\alpha_s}{\pi}\sigma_9(s)\right)\widetilde{C}_{10}^{\mathrm{eff}} \end{aligned}$$

The universal corrections $\sigma_i(s)$ defined such that in the soft-gluon limit ($s \to 1$) the nonuniversal corrections to the rate, $\tau_{77}, \tau_{99}, \tau_{79}$, vanish.

- Universal functions dominating:



- Universal versus complete corrections



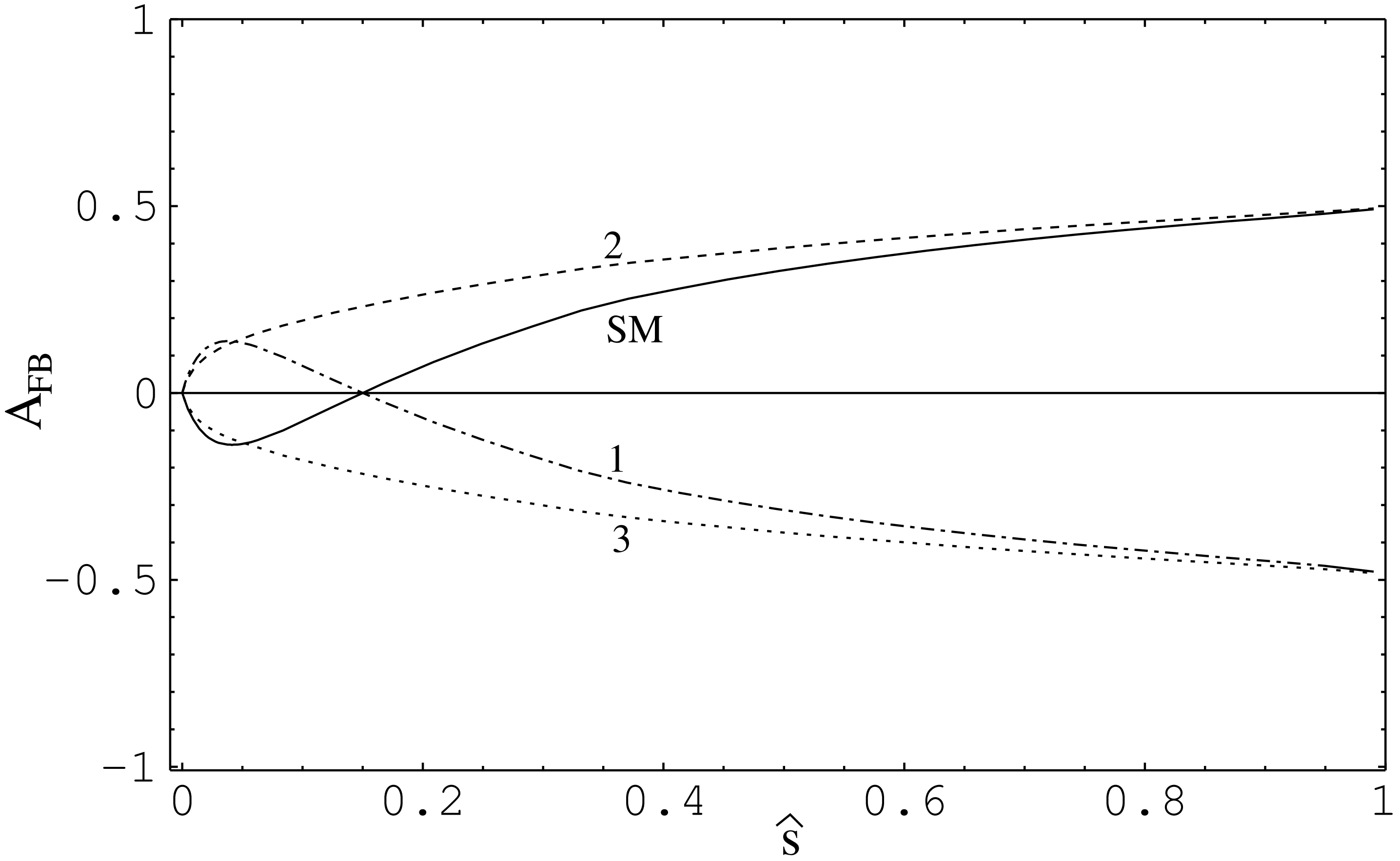
- Obvious γ_5 problem in A_{FB}: we used hybrid scheme; IR with dimensional reduction, UV with dimensional regularization

$$Z_\psi(m_s = 0) = 1 - \frac{\alpha_s}{4\pi}\frac{4}{3}\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} - 1\right)$$

New Physics Search

Four different shapes of the 'normalized' FB asymmetry $\overline{A}_{FB}$ for the decay $B \to X_s \ell^+ \ell^-$ within the MSSM compatible with the present data.

Ali et al.



Summary

Model-independent analysis of $B \to X_s \ell^+ \ell^-$ and $B \to X_s \gamma$

Global fit to the Wilson coefficients C_7, C_9, C_{10}

- $\Gamma(B \to X_s \gamma)$
- $d\Gamma(B \to X_s \ell^+ \ell^-) \, / \, d\hat{s}$
  Invariant dilepton mass distribution
- $A(s) = \int_{-1}^{1} \, dcos\theta \; d^2\Gamma(B \to X_s \ell^+ \ell^-) \, / \, dsdcos\theta \;\; sgn(cos\theta)$
  Forward-Backward Charge Asymmetry

$\Rightarrow$ Determines magnitude + sign of C_7, C_8, C_{10}

(kinematical distributions: high statistics necessary!)

Precision test of new physics !