Explicit Duality Violations in B-Meson Hadronic Widths in QCD

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Why

- Why care?
  - Beauty (and Charm) lifetimes
    - $\tau(B^{\pm}) = 1.674 \pm 0.018$ ps
    - $\tau(B^{0}) = 1.542 \pm 0.016$ ps (PDG 2002)
    - $\tau(\Lambda_b) = 1.229 \pm 0.080$ ps
    - Duality:

      $$\tau(H_b) = \tau(b)\left(1 + \mathcal{O}\left(\frac{\Lambda}{M}\right)^2 + 16\pi^2\mathcal{O}\left(\frac{\Lambda}{M}\right)^3\right)$$
    - But $\frac{\Lambda}{M} \sim 5\%$ so dispersion in lifetimes larger than few percent hard to understand
  - Nierste (previous talk):

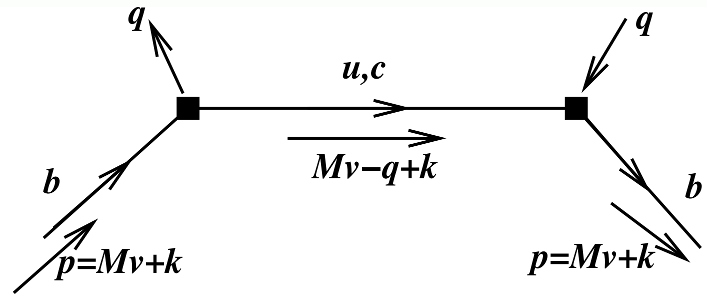
    $\tau(B^{\pm})/\tau(B^{0}) - 1 = 1.053 \pm 0.016 \pm 0.017$
  - Do we know what we are doing?

Duality

A la Shifman ("Quark-hadron duality," hep-ph/0009131):

- Inclusive: Relate rate to cut in Green function
- Large E: expand with OPE
- OPE is
  - In deep Euclidean region (Imaginargy energy E)
  - Asymptotic in E
- "Local:" point by point analytic continuation
- "Global:" integrated over a few resonances
- Violations:
  - Due to analytic continuation to real E
  - Never in full contour integrals

Example 1

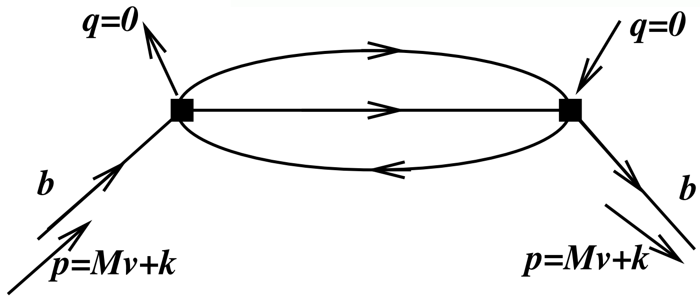


- Semileptonic B decay
- Width as cut in physical region

$$\frac{d\Gamma}{dq} \sim \mathrm{Im} \int e^{iqx} \langle \mathsf{T} J_\mu^\dagger(x) J_\nu(0) \rangle \ell^{\mu\nu}(q)$$

- At complex q do OPE $\Rightarrow$ no $1/M$ terms
- Analysis relates contour integral (complex-q) to integral across part of cut $\Rightarrow$ global duality

Example 2



B Hadronic Width

$$\Gamma \sim \text{Im} \int e^{iqx} \langle \mathsf{T} H_W(x) H_W(0) \rangle$$

- At complex q do OPE $\Rightarrow$ no $1/M$ terms
- Set $q = 0$ at end of calculation
- No “smearing” $\Rightarrow$ local duality

Some references

Based on:

B. Grinstein, Phys Lett B529 (2002) 99
Phys Rev D64(2001) 094004

B. Grinstein and R. Lebed Phys.Rev.D57:1366(1998)
Phys.Rev.D59:054022(1999)

Critique:

I. Bigi et al, Phys.Rev.D59:054011(1999)

I.Bigi and N.Uraltsev Phys.Lett.B457:163(1999)
Phys.Rev.D60:114034(1999)

Other evidence:

G. Altarelli et al Phys.Lett.B382:409(1996)

P. Colangelo et al Phys.Lett.B409:417-424(1997)

Duality in “B” Decays In The ’t Hooft Model

- ’t Hooft Model: Large N QCD in $1+1$ dims
- Why?
  - Asymptotically Free
  - Confining
  - QCD-like spectrum
  - Soluble $\Rightarrow$ Lab for Theory
- Used previously for analogues of
  - $e^+e^- \to$ hadrons
  - DIS ($ep \to eX$)
  - Form factors
  - $+\cdots$

Skip technical detail

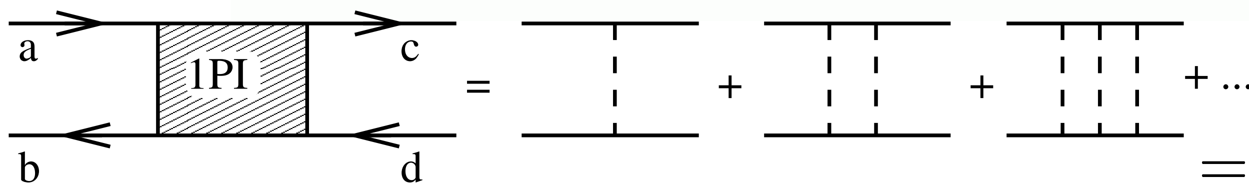
Elements of ’t Hooft Model

- Lagrangian as in QCD

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \sum_a \bar{\psi}_a \left(\gamma^\mu (i\partial_\mu - gA_\mu) - m_a\right)\psi_a$$

- Coupling g has mass dimension 1
  - Super-renormalizable $\Rightarrow$ asymptotic freedom
  - $g\sqrt{N}$ analogue of Λ_{QCD}
- Large N, g^2N fixed $\Rightarrow$ planar diagrams

(cont'd)



$$= \sum_n \frac{\phi_n(x)\phi_n(y)}{s-\mu_n^2},$$

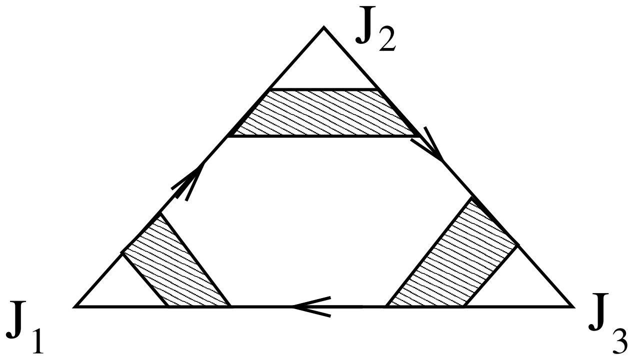
$$\begin{aligned} s &= (p_a+p_b)^2 \\ x &= (p_a)_-/(p_a+p_b)_- \\ y &= (p_c)_-/(p_a+p_b)_- \end{aligned}$$

and ϕ_n from 'tHooft equation

$$\mu_n^2\phi_n(x) = \left(\frac{m_1^2}{x}+\frac{m_2^2}{1-x}\right)\phi_n - \int_0^1 dy\,\phi_n(y)\ \mathrm{Pr}\,\frac{1}{(y-x)^2}$$

(cont'd)

- All Green functions in terms of this, eg, $\langle T(J_1 J_2 J_3)\rangle =$



- No cuts, only poles $\Rightarrow \Gamma$ only from two body decays

$$\Gamma = \sum_{n,m} \Gamma(B \to \pi_n \pi_m)$$

B: lightest $\bar{Q}q$ state
π_n: tower of $\bar{q}q$ states

(cont'd)

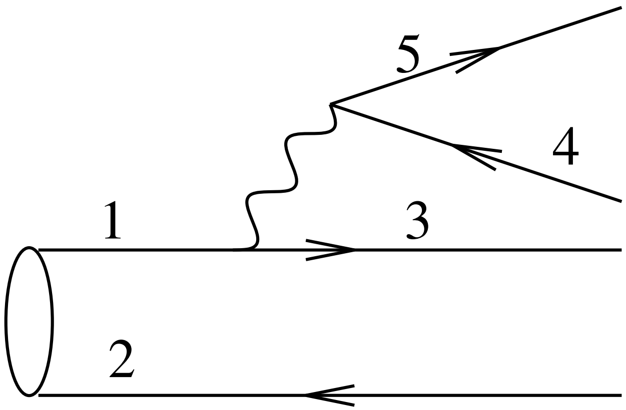
- In D dims

$$d\Gamma = \frac{|\mathbf{p}|^{D-3}}{(2\pi)^{D-2}\, 8M^2} |\mathcal{M}|^2 d\Omega$$

$\Rightarrow$ for $D = 2 = 1 + 1$ phase space diverges at threshold

Computing Widths

- Parton model:



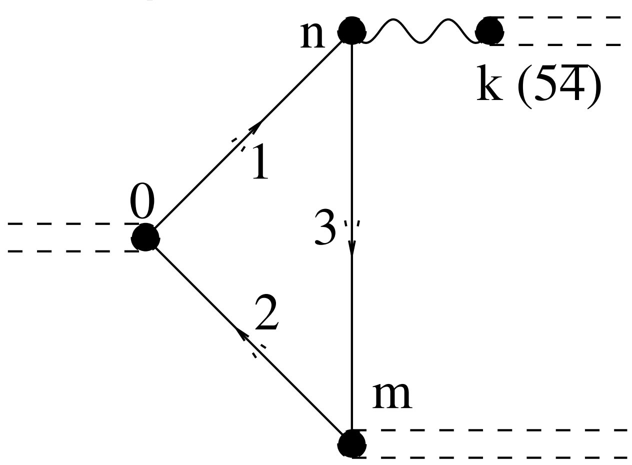
$$m_1 = m_Q = M$$

$$m_2 = m_3 = m_4 = m_5 = m_q$$

$$m_q \leq \Lambda_{\mathrm{QCD}} = g\sqrt{N} \ll m_Q$$

Computing Widths cont'd

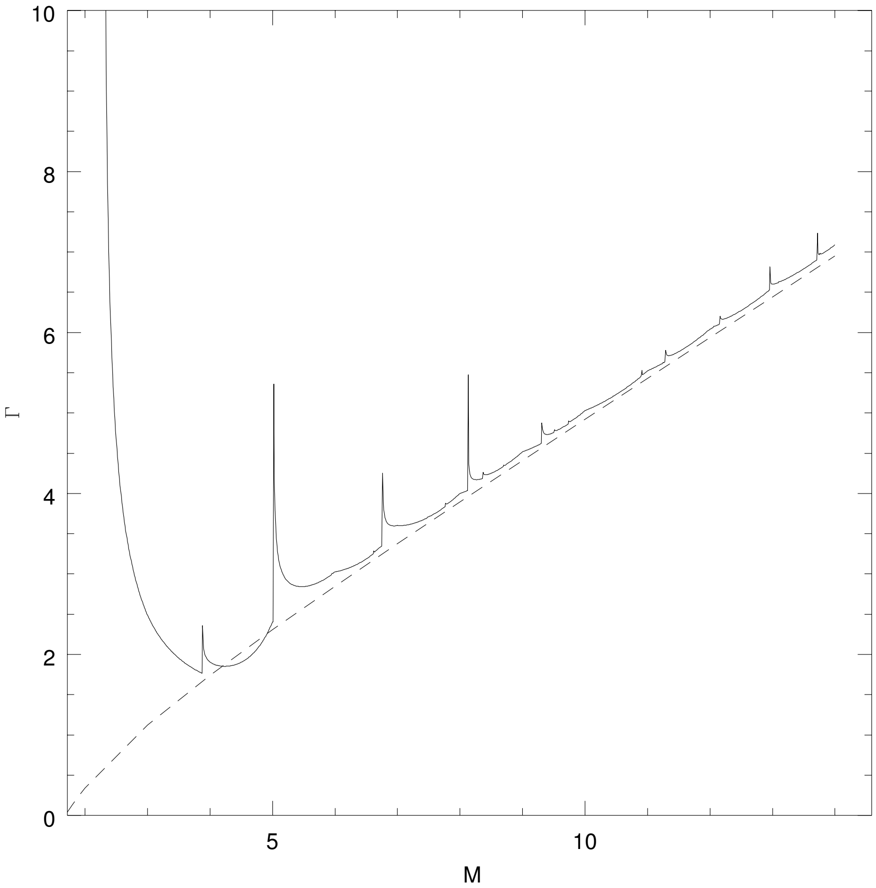
- Exact:
  - Compute heavy-light ϕ_0 (B) and mass M_B
  - ... and light-light ϕ_n's and masses μ_n
  - Given m_Q determine (n, m) so $\mu_n + \mu_m < M$
  - Compute



Results

- ϕ_n's and integrals computed numerically
- $g^2N/\pi = 1$ sets units
- $m_q = 0.56$, $M = 2.28 \to 15.00$
- Decay amplitudes $\mathcal{M}$ smooth in M figure
- Each partial Γ diverges at threshold figure
- Sum over final states
- Compare with parton (dual rate)

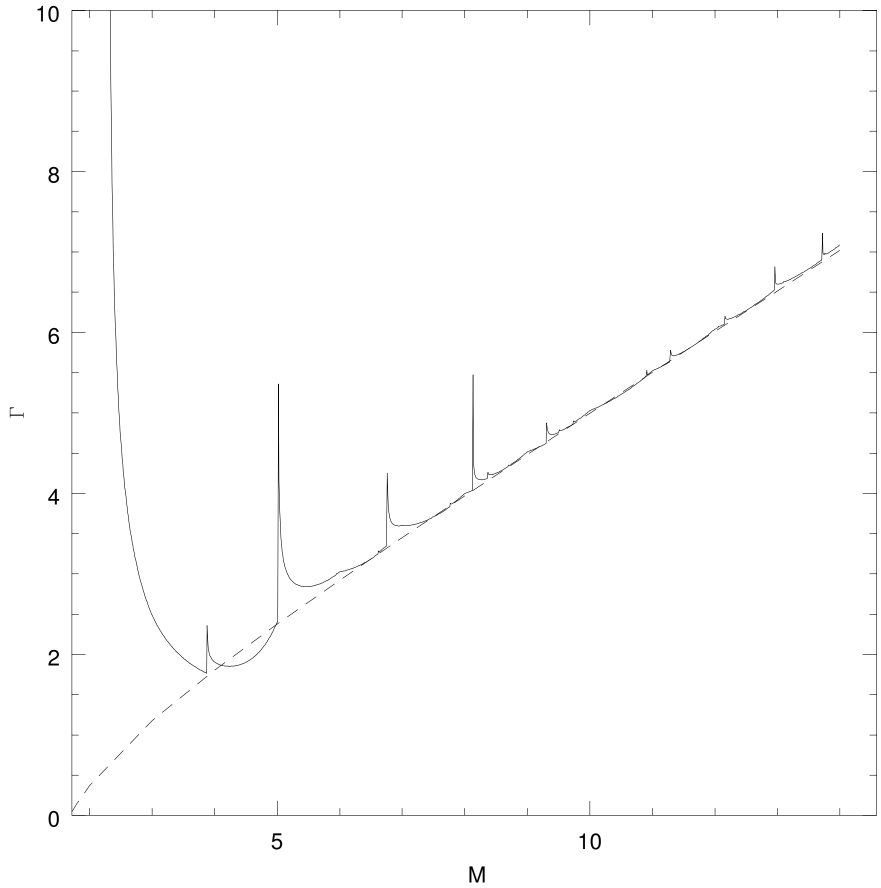
$\sum \Gamma(B \to \pi_n \pi_m)(M)$ ***vs*** $\Gamma_{\text{dual}}(M)$



$m_q = 0.56.$

Dashed line is the tree-level parton result

$$\Gamma_{\text{dual}}(M) \to \Gamma_{\text{dual}}(M) \cdot (1 + 0.15/M)$$



- Looks BETTER!

Global vs Local Duality?

- We saw local duality violation: at $1/M$
- Is Global duality any better?
- What is global duality here? (what smearing variable?)
- *Idea: Smearing*
  - *Smear over* M

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  - *Larger weight under peaks at low* $M \Rightarrow 1/M$ *into* $1/M^2$

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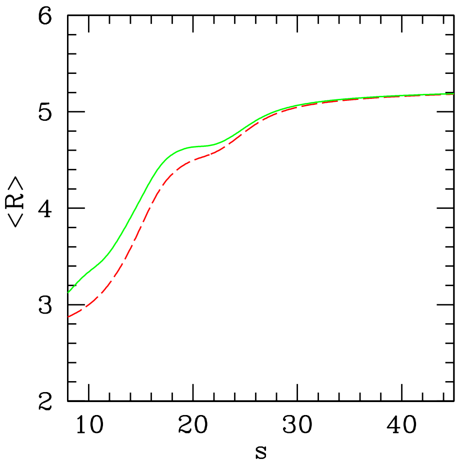
  - *Smear over M*
  - *Larger weight under peaks at low $M \Rightarrow 1/M$ into $1/M^2$*



$\langle \sigma(e^+e^- \to X) \rangle$ *(a la Poggio, et al) with and without ψ^n included.*

Gaussian Smearing

- Define

$$\langle f(M) \rangle = \frac{\int_{M_{\min}}^{M_{\max}} dx\, x^{n} e^{-(x-M)^2} f(x)}{\int_{M_{\min}}^{M_{\max}} dx\, x^{n} e^{-(x-M)^2}}$$

with $M_{\min} = 2.28$ and $M_{\max} = 15.00$

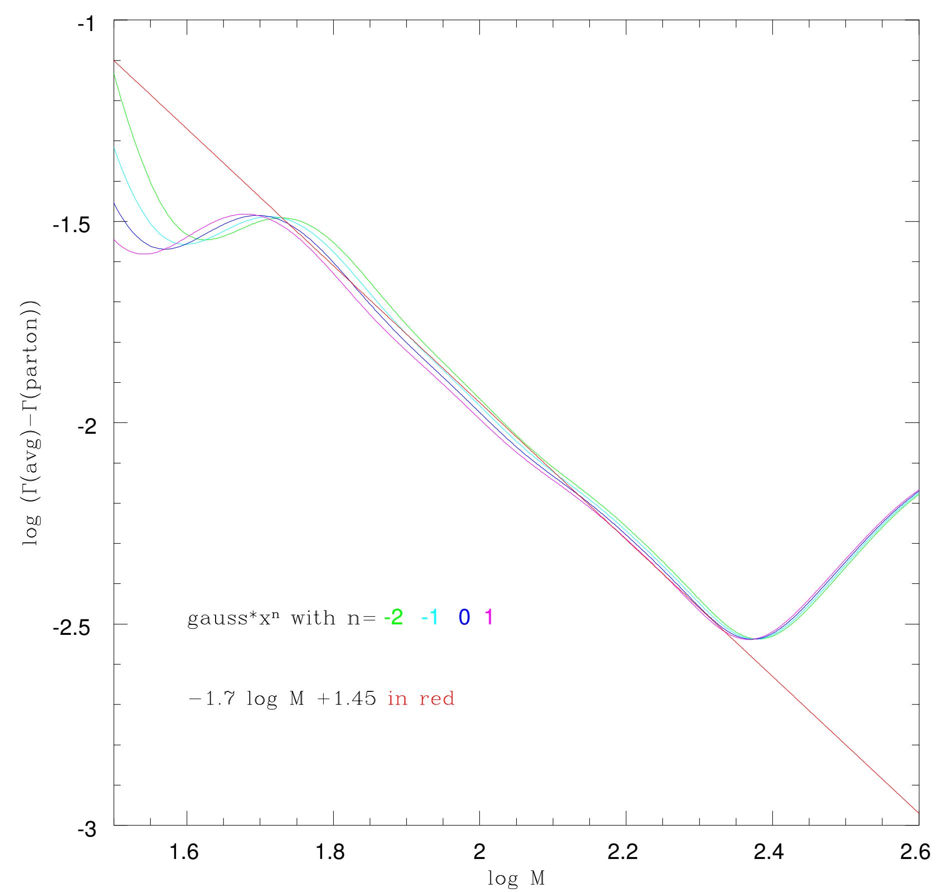
- We can emphasize low or high M by varying n

Smearing: numerical results

- Linear eye-fit
  $\ln(\Gamma - \Gamma_{\mathrm{part}}) =$
  $-1.7 \ln M + 1.45$
- Fit $\exp(-1.7 \ln M + 1.45)$
  in $\ln M \in [1.8, 2.3]$ to

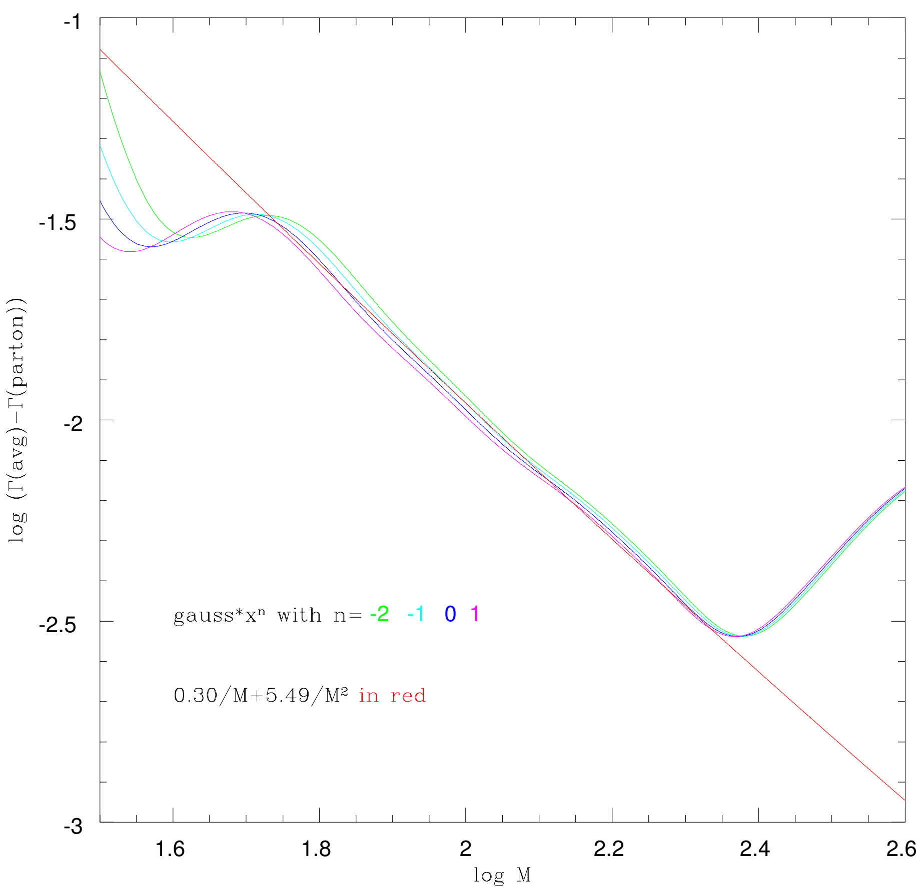
$$M \left[\frac{a}{M} + \frac{b}{M^2} + \frac{c}{M^3} \right]$$

end point effect



numerical results cont'd

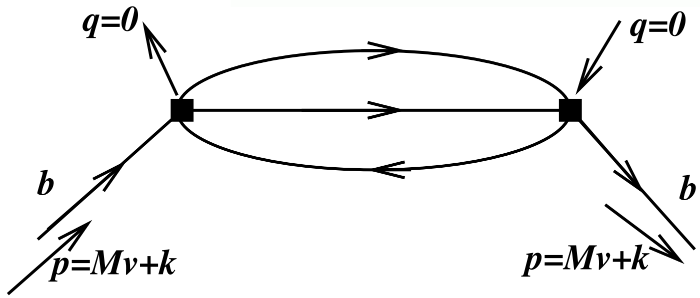
$$\Rightarrow a \ll 1 \qquad b \approx 0.3 \qquad c \approx 5.5$$



Lessons

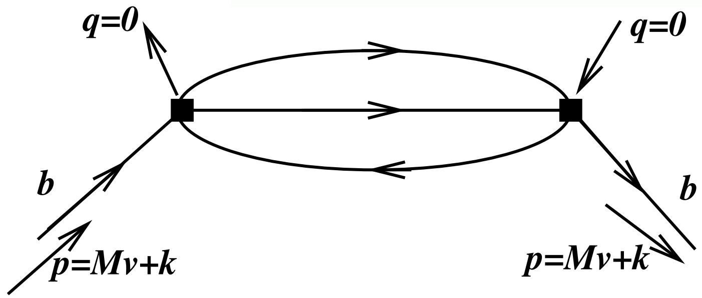
- There is a $1/M$ correction to local duality (unlikely to be "numerical" systematic error since it integrates to $1/M^2$)
- There is no $1/M$ correction to global duality (mass smearing)
- Smear before expanding in $1/M$

4D



Questions

- Can one make sense of smearing over M?
- If so, are there no “global duality” violations at $1/M$?
- Are there “local duality” violations at $1/M$



Questions

- Can one make sense of smearing over M?
- If so, are there no “global duality” violations at $1/M$?
- Are there “local duality” violations at $1/M$

Yes, yes, and don’t know

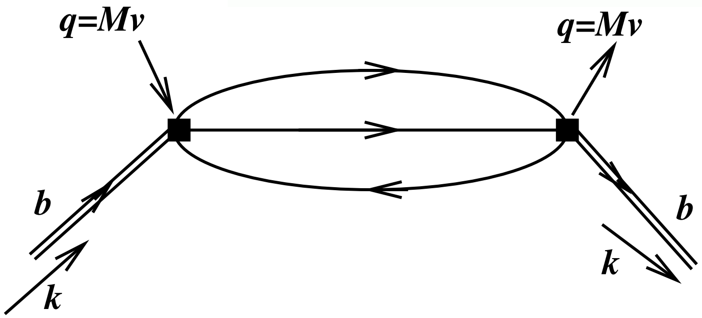
Smearing?: Idea

In a heavy quark effective theory the four quark operator describing a weak B-meson hadronic decay, is

$$(\bar{u}_L \gamma^\mu b_L \bar{d}_L \gamma_\mu u_L)(x) \rightarrow e^{-iMv\cdot x}(\bar{u}_L \gamma^\mu h_v \bar{d}_L \gamma_\mu u_L)(x),$$

Explicit factor of $\exp(-iq\cdot x)$ but with $q = Mv$

Width in HQET



$$\Gamma \sim \text{Im} \int e^{iqx} \langle \mathsf{T} H_W(x) H_W(0) \rangle$$

- has $q = Mv \neq 0$
- Analytic continuation over q is analytic continuation over M
- M can be manipulated: it only enters as above (contrast original theory)

Technical Issues

- Analytic structure mismatch
- Additional M dependence (not just in q);
  - Wilson coef's of Weak Hamiltonian $C(m_b/M_W)$
  - $1/M$ expansion of HQET lagrangian
  - $1/M$ expansion of matched H_W

details

Smearing: result

The smeared dual rate has no $1/M$ (I'll spare you the technical details):

$$\langle\Gamma(M)\rangle = \langle\Gamma_{\rm dual}(M)\rangle \left(1 + \mathcal{O}(\frac{1}{M^2})\right)$$

Now, let's ask about $1/M$ corrections to local duality.

Shifman's model

M. Shifman, Nucl. Phys. B **388**, 346 (1992)
Not realistic. Used only to study the presence/absence of $1/M$ duality violations in QCD

- $b \to c\bar{u}d$, with Hamiltonian

$$\mathcal{H}_W = \kappa_1 \, \bar{c}\gamma^\mu(1-\gamma_5)b \, \bar{d}\gamma_\mu(1-\gamma_5)u + \kappa_2 \, \bar{d}\gamma^\mu(1-\gamma_5)b \, \bar{c}\gamma_\mu(1-\gamma_5)u$$

- $m_b = 2M + \Delta \qquad m_c = m_d = M \qquad m_u = m_q = 0$
- $M \gg \Delta \gg \Lambda_{\rm QCD}$
- $1/N_c$ and $1/M$ exapnsions

Δ/M

- One can show that to order Δ/M there are no duality violations

The End

The End

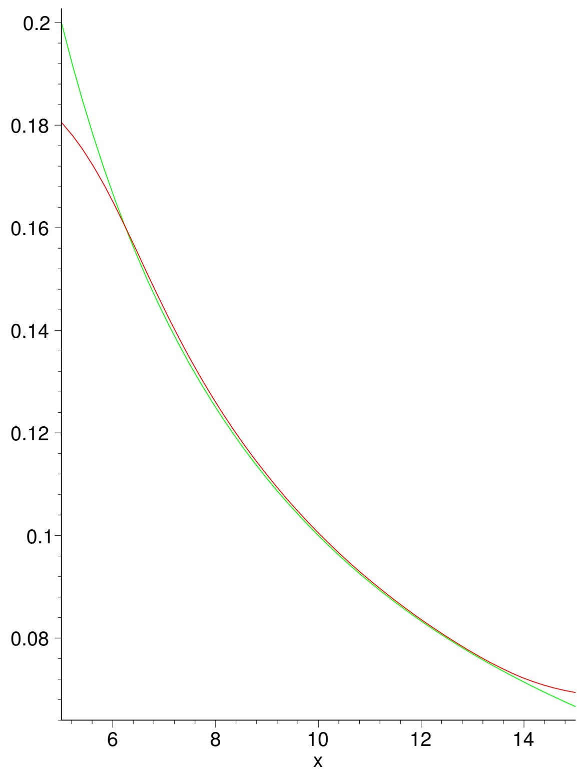
Smearing distorts endpoints

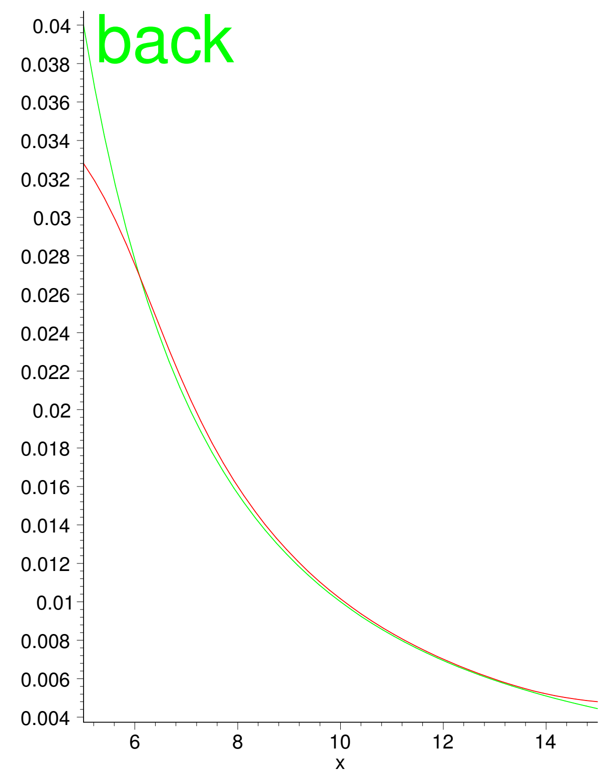
$1/x$

$\langle 1/x \rangle$

$1/x^2$

$\langle 1/x^2 \rangle$





back

Analytic Proof: Idea

In a heavy quark effective theory the four quark operator describing a weak B-meson hadronic decay, is

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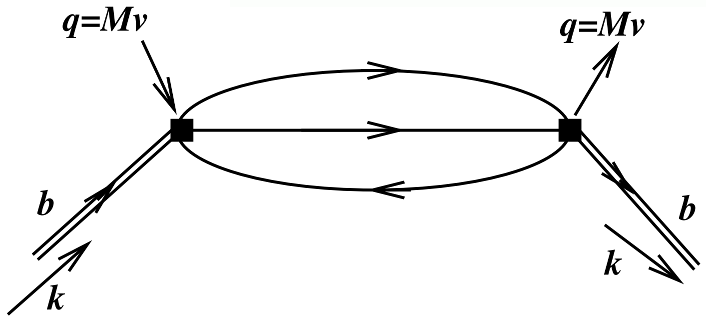
Width in HQET



$$\Gamma \sim \text{Im} \int e^{iqx} \langle \mathsf{T} H_W(x) H_W(0) \rangle$$

- has $q = Mv \neq 0$
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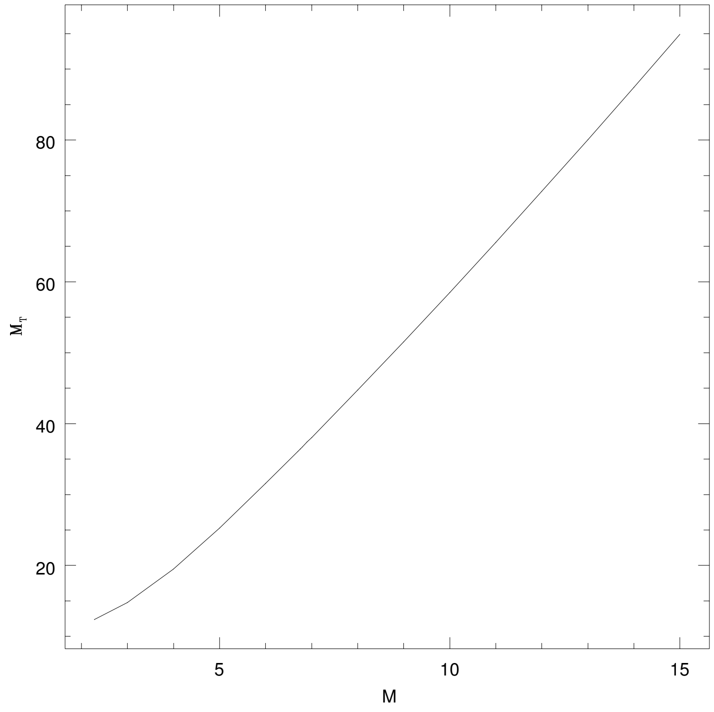
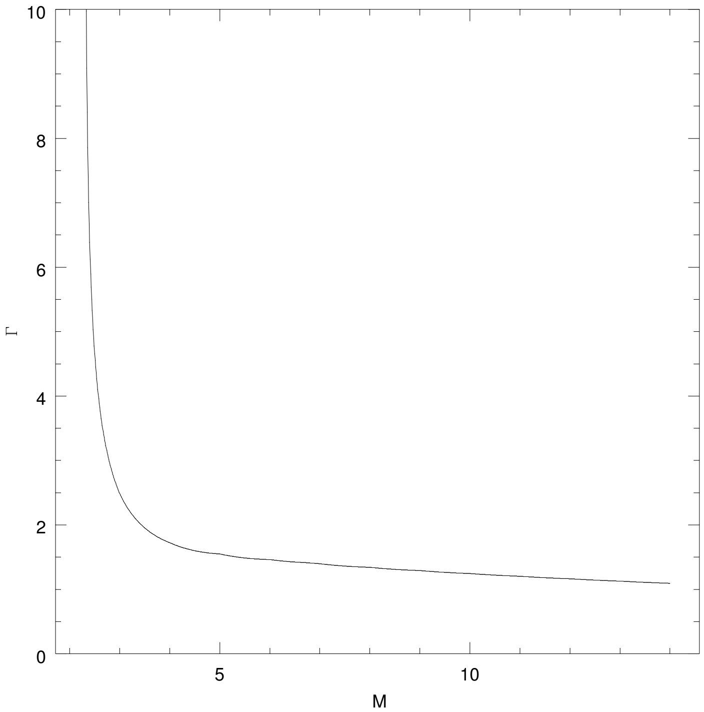
$\mathcal{M}_T(B \to \pi_0\pi_0)$



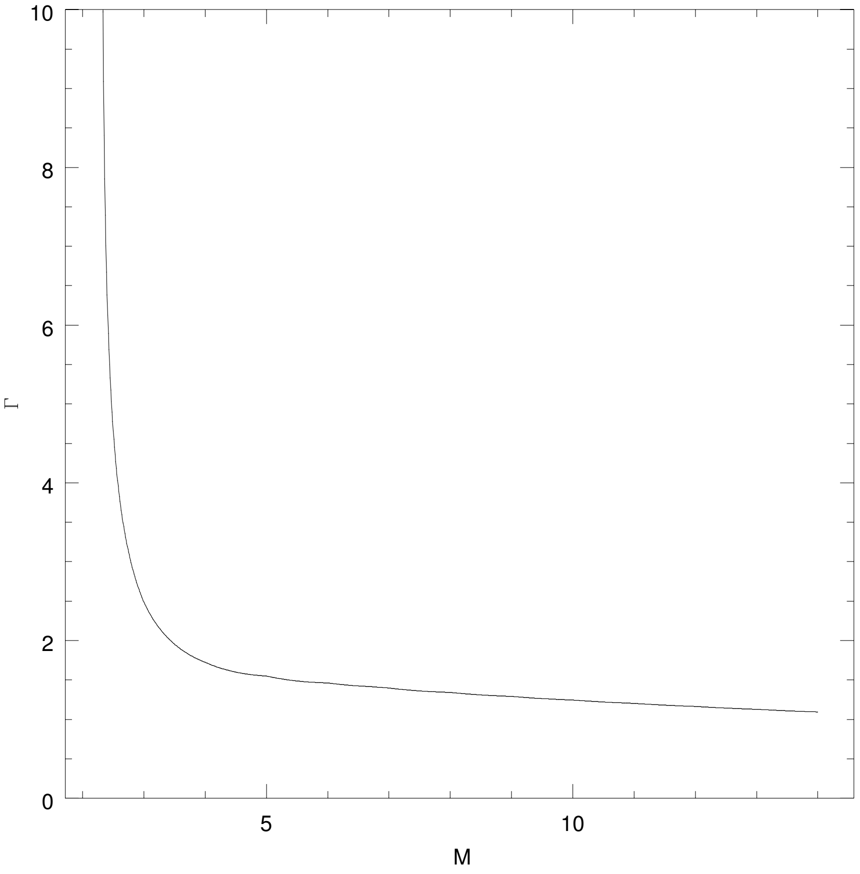
Weak decay amplitude $\mathcal{M}_T$ for the exclusive decay to the lowest mode $B \to \pi_0\pi_0$, as a function of heavy quark mass M, with light quark mass $m_q = 0.56$. The overall factor $2\sqrt{2/\pi}\,G_F V_{31} V_{45}^*(c_V^2 - c_A^2)$ in the amplitude is suppressed for convenience. back

$\Gamma(B \to \pi_0\pi_0)$



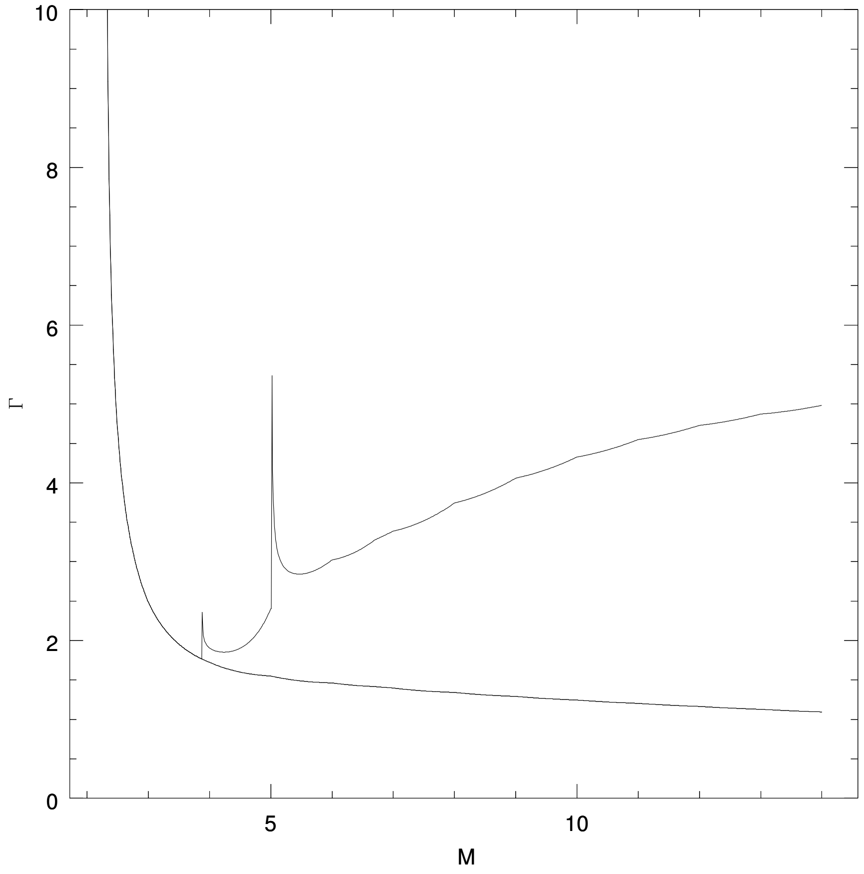
The full decay width as a function of heavy quark mass M, with light quark mass $m_q = 0.56$, including only the exclusive mode with the lowest threshold value $(\pi_0\pi_0)$. The scale is the same as in the previous transparency

How resonances stack up



Contributions of first 1, 3, 5 and 11 channels to Γ back

How resonances stack up



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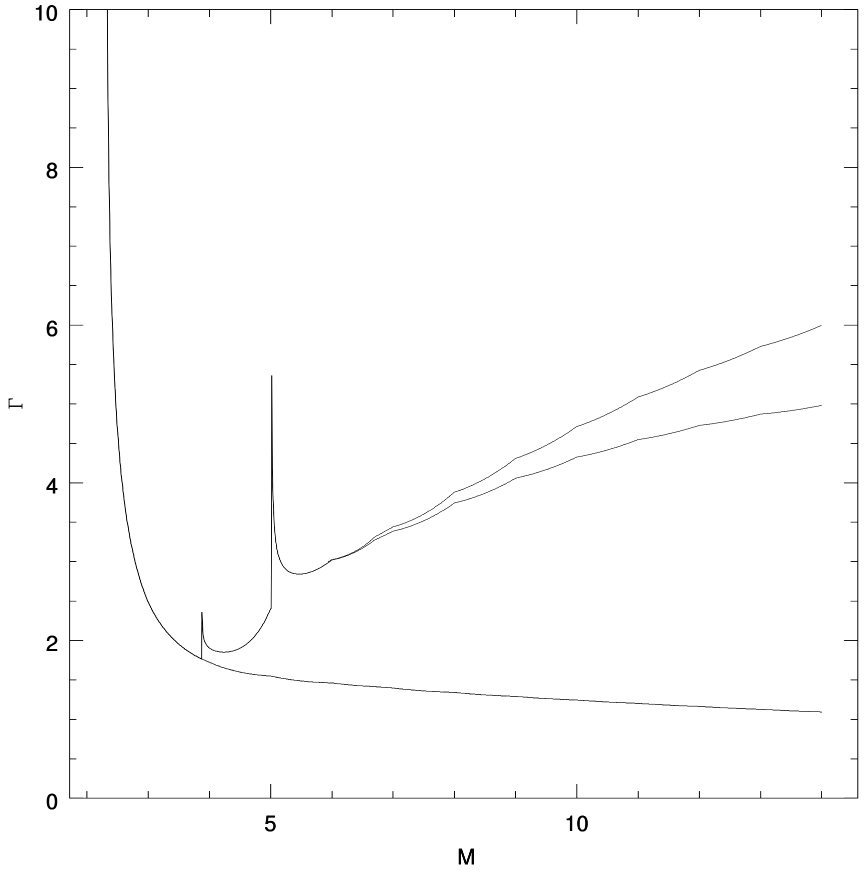

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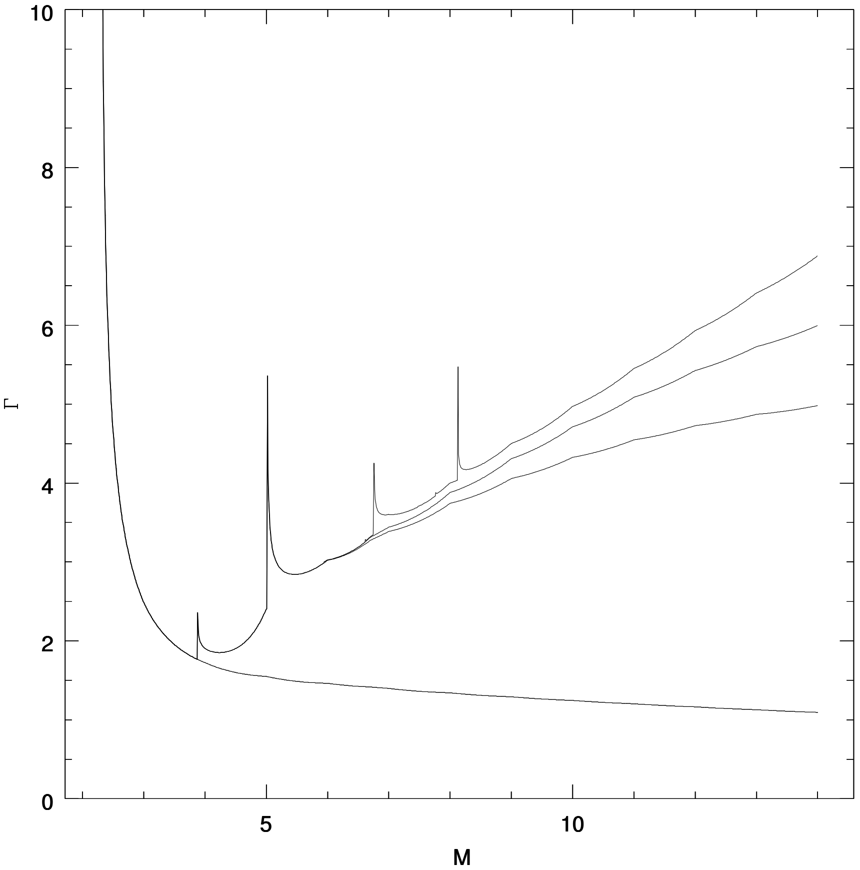


Contributions of first 1, 3, 5 and 11 channels to Γ back

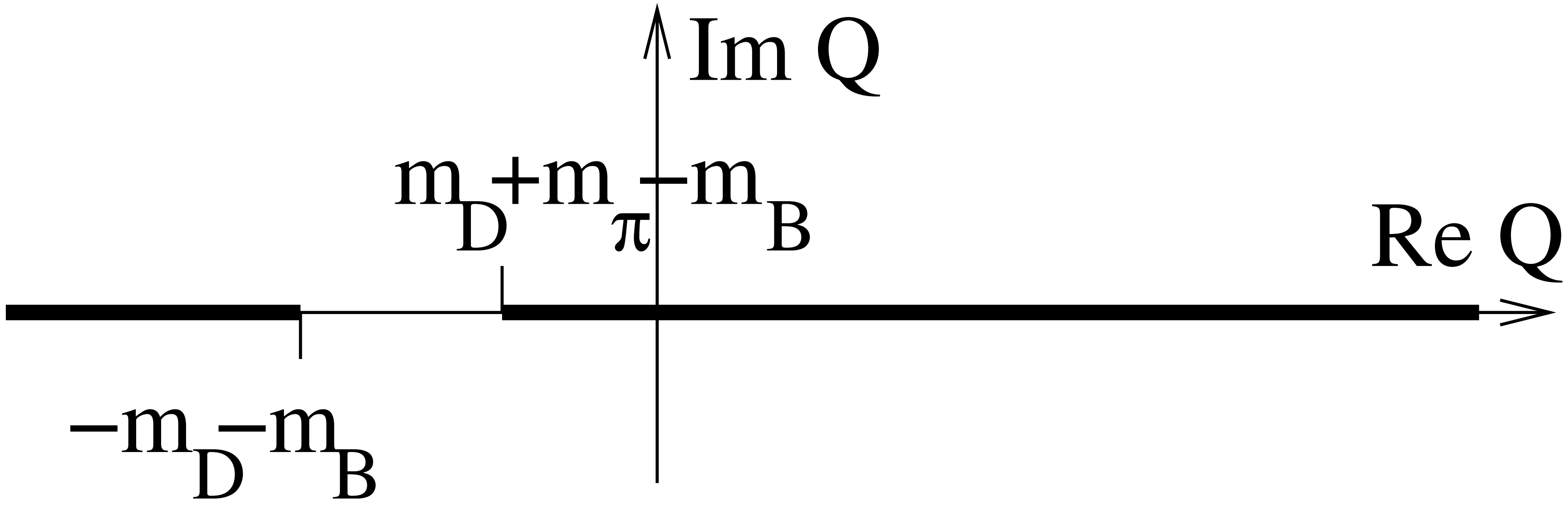
Technicalities begin: Generalities

$$T(q,p) = i\int d^4x\; e^{iq\cdot x}\langle\bar{B}(p)|T(\mathcal{H}^\dagger(x)\mathcal{H}(0))|\bar{B}(p)\rangle$$

$$\mathcal{H} = \frac{4G_F}{\sqrt{2}}V_{cb}V_{ud}^*\bar{c}_L\gamma^\mu b\bar{d}_L\gamma_\mu u_L.$$

$$\text{Im}T = \sum_X \pi(2\pi)^3\delta^4(q+p-p_X)|\langle X|\mathcal{H}(0)|\bar{B}\rangle|^2$$
$$+\sum_X \pi(2\pi)^3\delta^4(q-p+p_X)|\langle X|\mathcal{H}^\dagger(0)|\bar{B}\rangle|^2$$

Analytic Struture



$$\Gamma(B) = \frac{1}{m_B} \mathrm{Im} T|_{q=0}.$$

HQET Width

$$\hat{T}(q,v) = i\int d^4x\, e^{i(q+Mv)\cdot x}\langle H(v)|T(\hat{\mathcal{H}}^\dagger(x)\hat{\mathcal{H}}(0))|H(v)\rangle$$

$$\mathrm{Im}\hat{T} = \sum_X \pi(2\pi)^3\delta^4(q+Mv-p_X)|\langle X|\hat{\mathcal{H}}(0)|H\rangle|^2$$

$$\Gamma(B) \approx \mathrm{Im}\hat{T}|_{q=0}$$

- Only right side cut

$M \to M + \delta M$

- To leading order in $1/M$, M enters only in the combination $q + Mv$
- $\Gamma(M + \delta M) \approx \text{Im}\hat{T}|_{q=\delta M v}$
- $\hat{T}(Q) = \hat{T}(Qv, v)$ depends on M and Q only through the combination $M + Q$
  $\Rightarrow$ can use analysis to relate integral of Γ to the Green function of complex argument

Smearing Defined

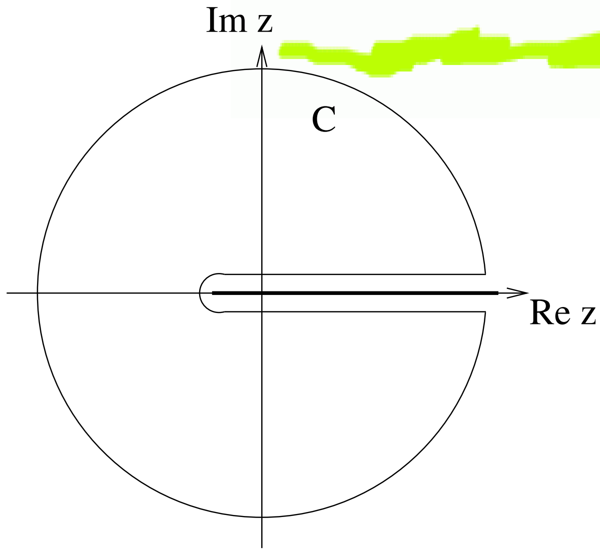
As in PQW we define smearing that can be analyzed with analytic tools

$$\langle f(M) \rangle = \int_{-\infty}^{\infty} dx \, w(x) f(x)$$

$$w(x) = \frac{(n-1)!}{(2n-3)!!} \frac{1}{2\pi\Delta} \left(\frac{2\Delta^2}{(x-M)^2 + \Delta^2} \right)^n$$

M center of weight function, of width Δ

Elementary Contour Integral



$$\frac{1}{2\pi i}\oint dz \frac{\hat{T}(z)}{(z^2+\Delta^2)^n} =$$

$$\frac{1}{(n-1)!}\frac{d^{n-1}}{dz^{n-1}}\frac{\hat{T}(z)}{(z+i\Delta)^n}\bigg|_{z=i\Delta} + (i\Delta \to -i\Delta)$$

Average width: first look

Use

$$\Gamma(B) \approx \text{Im}\hat{T}|_{q=0}$$

for integral above and below cut:

$$\langle\hat{\Gamma}_0(M)\rangle = \frac{2^{n-1}\Delta^{2n-1}}{(2n-3)!!}\left\{ \frac{d^{n-1}}{dz^{n-1}} \frac{\hat{T}(z)}{(z+i\Delta)^n}\bigg|_{z=i\Delta} + (i\Delta \to -i\Delta)\right\}$$

R.H.S. is $i\Delta$ away from cut $\Rightarrow$ OPE (perturbative) $+$ matrix element $\langle H(v)|\bar{h}_v h_v|H(v)\rangle = 1$, so

$$\langle\hat{\Gamma}_0(M)\rangle = \langle\Gamma_Q(M)\rangle$$

$1/M$ ***corrections?***

- Next term in OPE of $\hat{T}(i\Delta)$ is

$$\langle H(v)|\bar{h}_v \Gamma D_\mu h_v|H(v)\rangle = 0$$

$\Rightarrow$ no $1/M$ corrections

- BUT
  - $\mathcal{H}$ itself has an expansion in $1/M$
  - the relation between the weak Hamiltonian and its representation in the HQET involves Wilson coefficients that have explicit mass dependence
  - Both spoil invariance: $M \to M + \delta M$, $Q \to Q - \delta M$

M dependence from Hamiltonian

$$\mathcal{H} = c_0 \hat{\mathcal{H}}_0 + \frac{1}{M} c_1 \hat{\mathcal{H}}_1 + \cdots$$

with $c_k = c_k(M/\mu)$ (μ fixed). Then ($C_0 = c_0^2$, etc)

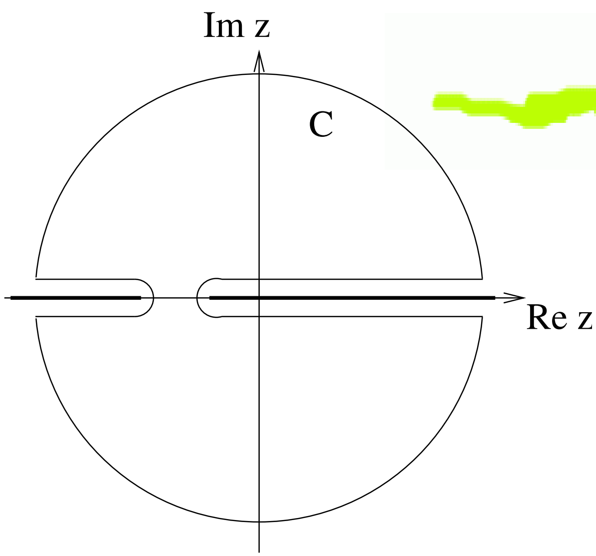
$$T = C_0 \hat{T}_0 + \frac{1}{M} C_1 \hat{T}_1 + \cdots$$

and for the width

$$\Gamma = \hat{\Gamma}_0 + \hat{\Gamma}_1 + \cdots$$

Consider the individual averages

$$\langle \hat{\Gamma}_k(M) \rangle = \int dQ \; w(Q) \hat{\Gamma}_k(Q)$$



explicit $1/M$'s give $1/(z+M)$'s, $\Rightarrow$ poles at $z=-M$

Wilson coefficients $\Rightarrow \ln(M)$ behavior

$\Rightarrow$ cuts from $z=-M$ to $-\infty$.

$$\frac{1}{2\pi i}\oint dz\;\frac{C_k(z+M)\hat{T}_k(z)}{(z+M)^k(z^2+\Delta^2)^n} =$$

$$\frac{1}{(n-1)!}\,\frac{d^{n-1}}{dz^{n-1}}\,\frac{C_k(z+M)\hat{T}_k(z)}{(z+M)^k(z+i\Delta)^n}\bigg|_{z=i\Delta} + (i\Delta \to -i\Delta)$$

Estimate wrong cut contribution

- estimate $\hat{T}(z) \sim (z+M)^p$ where $p = 2D-3$
- ($p = 5$ in $D = 4$ as in $\Gamma \sim G_F^2 M^5$)

$$\frac{1}{2\pi i} \int dz \frac{\ln(z+M)(z+M)^{p-k}}{(z^2+\Delta^2)^n}$$

$$= \frac{1}{(n-1)!}\frac{d^{n-1}}{dz^{n-1}}\left[\frac{\ln(z+M)(z+M)^{p-k}}{(z+i\Delta)^n}\right|_{z=i\Delta} + \Delta \to -\Delta$$

$$= \mathcal{O}(M^{p-k+1-2n}\ln(M))$$

For comparison, a similar estimate of the average $\langle \Gamma_k(M) \rangle$ using $\Gamma_k(x) \sim x^{p-k}$ gives

$$\int_0^\infty dx \frac{x^{p-k}}{[(x-M)^2 + \Delta^2]^n} = \frac{2\pi}{(2\Delta)^{2n-1}} \frac{(2n-2)!}{[(n-1)!]^2} M^{p-k} \left(1 + \mathcal{O}(\frac{\Delta}{M})\right)$$

Thus, we can express the width average in terms of the off-shell Green function and derivatives up to corrections suppressed by $(\Delta/M)^{2n-1} \ln(M)$.

Just like semileptonic

- we can calculate $\langle\Gamma(M)\rangle$ by computing the off-shell Green functions using an OPE.
- we can show no corrections $\sim 1/M$:
  - OPE: expansion in $\sim \bar{h}_v \Gamma D^l h_v$, $D = \partial + A$
  - $\langle H(v)|\bar{\Gamma} h_v D_\mu h_v|H(v)\rangle = 0$
  - expansion of $\hat{T}_k$ starts at order $\bar{h}_v \Gamma D^k h_v$

- $\hat{T}_k \sim \bar{h}_v \Gamma D^k h_v + \cdots$ is non-trivial
- $B \to X_c \ell \nu$: Chay et al sidestep this question by simultaneous OPE and the HQET
- Mannel: first do Green function in the HQET, only then OPE
- Mannel does not consider the effect of subleading operators, but two approaches yield the same result only if the OPE of products of subleading operators starts at the corresponding order in $\bar{h}_v D^l h_v$.
- similar indirect argument for $\Gamma_l(B)$, but no proof yet

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