Theory of Hadronic 2-body B Decays

Ringberg Phenomenology
Workshop on Heavy Flavours,
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1) Introduction

2) $\bar{B} \to$ heavy light

3) $\bar{B} \to$ light light
in particular: phenomenology of $\bar{B} \to \eta^{(\prime)} K^{(*)}$ decays

electroweak interaction

$$\langle M_1 M_2 | \mathcal{O} | \bar{B} \rangle = \; ??$$

scales m_b, (m_c), Λ_{QCD}

(Naive) Factorisation (BSW, ...)

– Working assumption

$\downarrow$

QCD Factorisation (BBNS)

heavy quark limit
effective field theory

NEW: collinear degrees of freedom, "light-cone physics"

$p^2 \ll m_b^2$
$n_+ \cdot p \sim m_b$

perturbative hard-scattering factorisation

NEW: soft external lines (spectator quark), "soft physics"

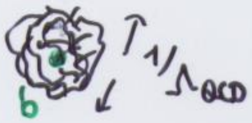
$p^2 \ll m_b^2$
$p \ll m_b$

Space-time picture of $\bar{B} \to D^+ \pi^-$ (Bjorken)

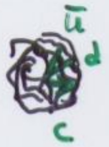
$b\bar{d}$ $\quad c\bar{d}$ $\quad d\bar{u}$

must go to D^+ (or annihilate)

$t \leq 0$



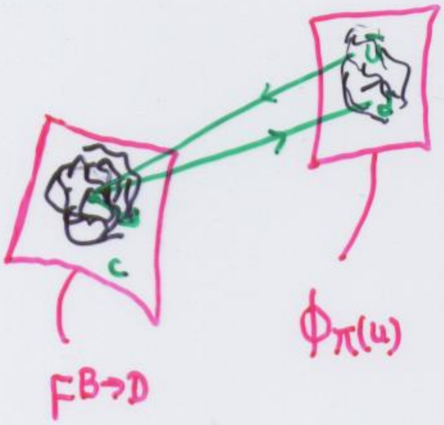
$t \sim 1/m_b$



$c\bar{u}d$ produced locally

hard, perturbative interactions with the cloud for $t < 1/\Lambda_{QCD}$

$t \sim 1/\Lambda_{QCD}$



$\bar{u}d$ has left the cloud after its proper time

$[c\bar{d} + \text{remnant}] \longrightarrow$ D meson $\leftarrow$ $F^{B\to D}$ [soft]

$[\bar{u}d]$ can only interact with soft gluons over long distance

$$\bar{u}d \longrightarrow \bar{u}SS^\dagger d = \bar{u}d \quad \leftarrow \quad \phi_\pi \text{ [collinear]}$$

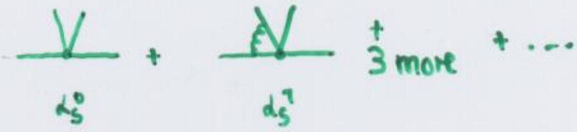
$$\bar{u}T^A d \to \bar{u}\underbrace{ST^AS}d$$

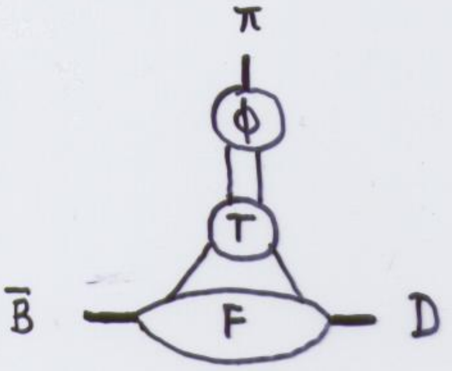
pure octet : does not form a pion

In formulae

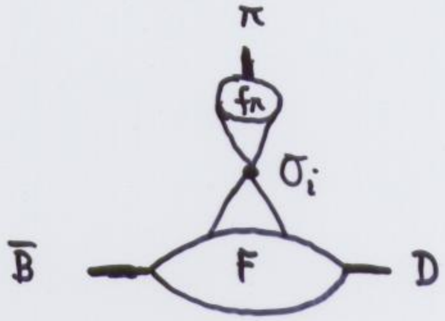
(BBNS ; Politzer, Wise)

$$\langle \pi^- D^+ | \mathcal{O}_i | \bar{B} \rangle = F^{B\to D}_{(0)} \cdot \int_0^1 du\, T^{I}_i(u)\, \phi_\pi(u) + \mathcal{O}(1/m_b)$$





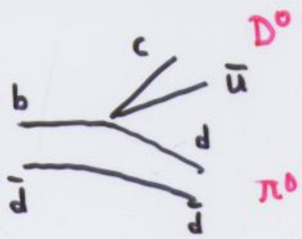
vs



QCD factorisation
$m_b \to \infty$
α_s corrections calculable

naive factorisation
$m_b \to \infty$
$\alpha_s \to 0$

- calculations done to $\mathcal{O}(\alpha_s)$
  small correction $\approx 5\%$
- factorisation is probably obvious here
  explicit demonstration to 2 loops
  trivialized in SCET (Bauer et al.)
- NOT true for $\bar{B} \to \pi^0 D^0$

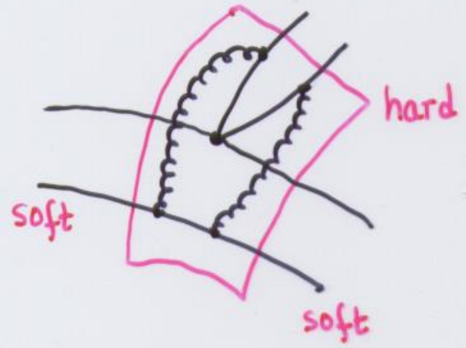


FORMALLY $1/m_{c,b}^2$ relative to $D^+\pi^-$

but

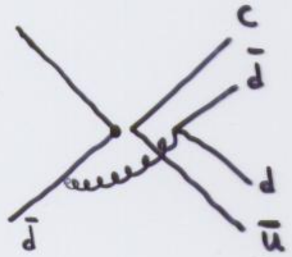
$$\frac{(m_B^2-m_\pi^2)\, f_D\, F^{B\to\pi}(m_D^2)}{(m_B^2-m_D^2)\, f_\pi\, F^{B\to D}(0)} \approx 0.9 \quad \text{not} \quad \left(\frac{\Lambda_{QCD}}{m_{c,b}}\right)^2$$

- hard spectator interactions are suppressed



$\sim \quad \alpha_s^2(\sqrt{m_b \Lambda}) \cdot \dfrac{\Lambda_{QCD}}{m_b}$

suppressed



$\sim \quad \alpha_s \cdot \dfrac{\Lambda_{QCD}}{m_b}$

Phenomenology

- good agreement for $D^{(*)+} L^-$ ($L = \pi, \rho, K^{(*)}$) final states
  nearly universal correction to naive factorisation
  data not precise enough to test the correction or the predicted non-universality

  note : factorisation does not work for $D \to K\pi$, $K \to \pi\pi$
  $\rightarrow$ heavy quark limit is important

- $D\pi$ system

$$c_1 + \frac{c_2}{N_c} + O(\alpha_s)$$

			BR
$A(\bar{B} \to D^+\pi^-)$	$= T + A$	$\propto a_1$	$\approx 3 \cdot 10^{-3}$
$A(B^- \to D^0\pi^-)$	$= T + C$	$\propto a_1 + x a_2$	$\approx 5.3 \cdot 10^{-3}$
$\sqrt{2}\, A(\bar{B} \to D^0\pi^0)$	$= C - A$	$\propto x a_2$	$\approx 0.25 \cdot 10^{-3}$

$$c_2 + \frac{c_1}{N_c} + ??$$ "colour-suppressed"

3 Br's + 3 unknowns ($|a_1|$, $x a_2/a_1$)

$$\Rightarrow \quad x\frac{a_2}{a_1} \approx 0.42\, e^{\pm i\, 55°}$$

sizeable rescattering

(e.g. Neubert, Petrov)

naive factorisation $\quad a_2 \approx 0.12$

estimate treating D as light $\quad a_2 \approx 0.25\, e^{-41^\circ}$ $\quad$ (both uncertain)

$\longrightarrow$ clear evidence for breakdown of factorization in $\pi^0 D^0$ as expected

$\longrightarrow$ clear evidence (when $\pi^0 D^0$ is combined with $K\, J/\psi$) for non-universality of a_2 as expected

- Annihilation

<u>Crude</u> estimate

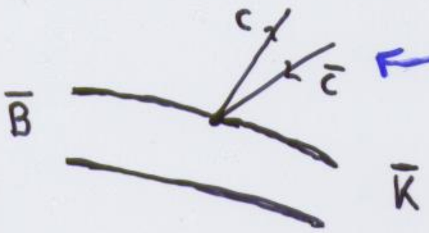
$$A/T \approx \frac{2\pi \alpha_s}{3} \frac{C_+ + C_-}{2C_+ + C_-} \frac{f_D f_B}{m_B^2 F^{B\to D}_{(0)}} \int dv \frac{\phi_D(v)}{v^2} \approx 0.04$$

(Λ/m_b suppressed)

Apply to the pure annihilation decay $\bar{B}_d \to D_s^+ K^-$

$Br \approx 1.2 \cdot 10^{-5}$ $\quad$ [observed : $(3.8 \pm 1.1) \cdot 10^{-5}$]

Charmonium modes



not energetic, but compact as $m_b \to \infty$, m_c/m_b fixed

charmonium (formally) a non-relativistic bound state

- soft gluon suppression $\Lambda_{QCD}/m_c \alpha_s$ not sufficient

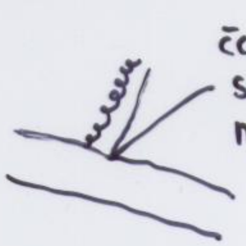
  indeed $|a_2^{exp}| / |a_2^{LO}| \approx 2$

  NLO correction not large enough (Cheng, Yang; Chay; Kim) (see Melic, 2003: QCD sum rules)

- P-wave production

  $\chi_{c0,c2}$ not produced at LO, but not really suppressed phenomenologically

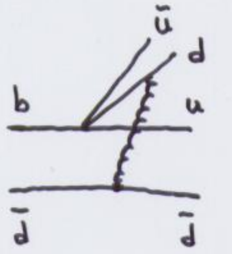
  combine with NRQCD factorisation (Bodwin et al.)



- phenomenology discouraging similar to $\bar{B} \to$ charmonium $+ X$ (MB, Maltoni, Rothstein)

$B \to 2$ light mesons (e.g. $\pi\pi$ - non-singlet)

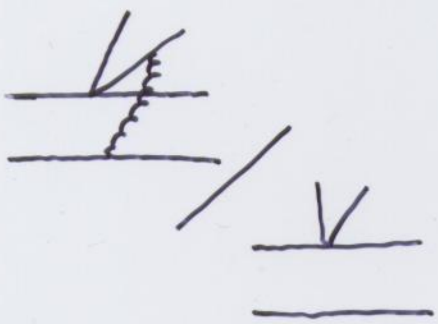
New effect : hard spectator scattering



soft scattering Λ_{QCD}/m_b suppressed
(colour transparency)

hard exchange involves virtuality
$m_b\Lambda \to \infty$

in fact hard $\hat{=}$ <u>hard-collinear</u>



$$\approx \frac{\pi\alpha_s(\sqrt{m_b\Lambda})\, \overset{\Lambda^2 \frac{\Lambda^{3/2}}{m_b^{1/2}}}{f_\pi^2 f_B}\, \overset{m_b/\Lambda}{\int \frac{d\xi}{\xi}\phi_B(\xi)} \left(\int_0^1 \frac{du}{u}\phi_\pi(u)\right)^2}{\underset{\Lambda\,(\Lambda/m_b)^{3/2}}{m_b^2 f_\pi F^{B\to\pi}}}$$

$$\approx \quad \alpha_s(\sqrt{m_b\Lambda}) \qquad \text{LEADING POWER}$$

so now:

$$\langle \pi^+\pi^- | O_i | \bar{B} \rangle = F_{(0)}^{B\to\pi} \cdot T_i^{I} * \phi_\pi + \phi_B * T_i^{II} * \phi_\pi * \phi_\pi + O(1/m_b)$$

as before + 
 etc.

spectator interaction

- explicit calculations done to $O(\alpha_s)$

- no formal proof of the factorisation formula
interesting part is the hard spectator interaction

    - two collinear directions + soft physics
      $\longrightarrow$ many modes in EFT treatment

    - ϕ_B - light-cone distribution of the B meson
      but evolution due to soft modes (Lange et al., 2003)

    - need theoretical discussion of endpoint behaviour / convergence of convolution integrals

- Power corrections

    - some are $\propto \frac{2 m_\pi^2}{m_b (m_u + m_d)}$ and not all of them can be computed

    - Penguin - weak - annihilation
      $\longrightarrow$ 10-30% effect on penguin amplitude ?

penguin op.

+ others

$1/m_b$ suppressed

(Keum, Li, Sanda)

    - some power corrections estimated with QCD sum rules, e.g. " charming penguin "
      not large, similar in size to perturbative correction (Khodjamirian et al.)

Phenomenology (with a broad brush)

1) $Br(\pi^\pm\pi^0) \Longrightarrow$ $|T|$ ok within form factor / CKM uncertainties +

2) $\dfrac{Br(\pi^\pm \overset{(-)}{K}{}^0)}{Br(\pi^\pm\pi^0)} \Longrightarrow \left|\frac{P}{T}\right|$ ok for PP ++

accounts for sizeable πK Br's
difficult in naive factorisation

3) $\dfrac{Br(\pi^\pm \overset{(-)}{K}{}^{0*})}{Br(\pi^\pm\pi^0)} \Longrightarrow \left|\frac{P}{T}\right|$ too small for PV ($\approx 50\%$) −

$[\bar{d}s]_{\substack{V-A\\S-P}} \to K^{0*}$ S-P suppressed

4) $\dfrac{Br(\pi^+\pi^-)}{Br(\pi^\pm\pi^0)} \Longrightarrow$ Interference of T and P not ok ?

$\longrightarrow$ $\gamma \gtrsim 90^\circ$?
strong phases very wrong ?

5) $A_{CP}(\pi^-K^+) \Longrightarrow$ strong phase small ok, but wrong sign o

$\longrightarrow$ power corrections
NLO corrections ?

(MB, Neubert)

$\bar{B} \to \eta^{(\prime)} \bar{K}^{(*)}$

$Br(\eta' K) \approx 65 \gg Br(\pi^0 K) \approx 10 > Br(\eta K) \approx 4 \quad \cdot 10^{-6}$

$Br(\eta' K^*) < 15 \qquad Br(\pi^0 K^*) \approx 10 \qquad Br(\eta K^*) \approx 20$

Anything special about $\eta^{(\prime)}$?

$\longrightarrow$ singlet component $u\bar{u}+d\bar{d}+s\bar{s}$, gg

Amplitude

$$A(\bar{B}\to\bar{K}P) = i\,\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V_{ps}^{*}\,\mathcal{A}_p(KP)$$

$$P = \eta \text{ or } \eta', \quad K = K \text{ or } K^*$$

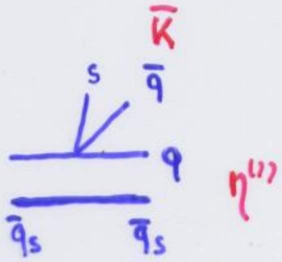
neglect EW penguins + annihilat.
(included in the analysis)

$$\mathcal{A}_p(KP) =$$

$$m_B^2\, F_{(0)}^{B\to P}\, \frac{f_K}{\sqrt{2}} \left\{ \delta_{pu}\,\delta_{q_s u}\, \alpha_1(PK) + \alpha_4^p(PK) \right\}$$

CKM-suppressed tree

QCD penguin



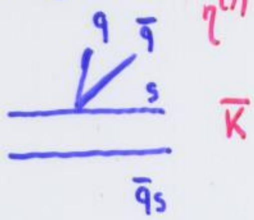
$+$

$$m_B^2\, F_{(0)}^{B\to K} \left\{ \frac{f_P^q}{\sqrt{2}} \left[\delta_{pu}\,\alpha_2(KP_q) + 2\,\alpha_3^p(KP_q) \right] \right.$$

$$+\, f_P^s \left[\alpha_3^p(KP_s) + \alpha_4^p(KP_s) \right]$$

$$\left. +\, f_P^c \left[\delta_{pc}\,\alpha_2(KP_c) + \alpha_3^p(KP_c) \right] \right\}$$

QCD singlet penguin



$\langle P | \bar{s}\gamma^\mu\gamma_5 s | 0 \rangle$

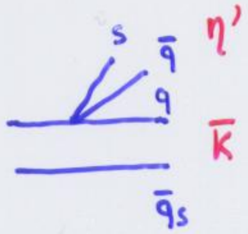
$>0 \quad \eta'$

$<0 \quad \eta$

once phase of f_P^q fixed

charm

QCD penguin

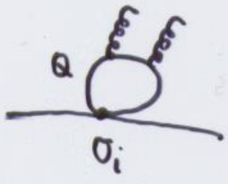


The flavour singlet amplitude

1) $b \to s \underset{\sim}{gg}$

$\quad\quad \to \eta^{(\prime)}$

(Simma, Wyler [general quark amplitude])



has to be computed for heavy quarks only (c,b)

$\Rightarrow$ Result can be interpreted as

$$f_P^c \left[\delta_{Pc}\, \alpha_2(KP_c) + \alpha_3(KP_c) \right]$$

$$\alpha_2 \leftarrow C_2 + \frac{C_1}{N_c} \qquad \alpha_3 \leftarrow C_3 - C_5 + \frac{C_4 - C_6}{N_c}$$

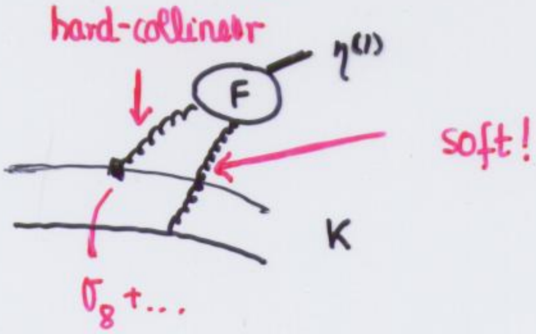
$$f_P^c \leftarrow \frac{1}{12 m_c^2} \langle P | \frac{\alpha_s}{4\pi} G\tilde{G} | 0 \rangle \approx - \frac{m_P^2}{12 m_c^2} \frac{f_P^q}{\sqrt{2}}$$

(also Franz, Polyakov, Goeke)

very small effect for b loop

not a large effect

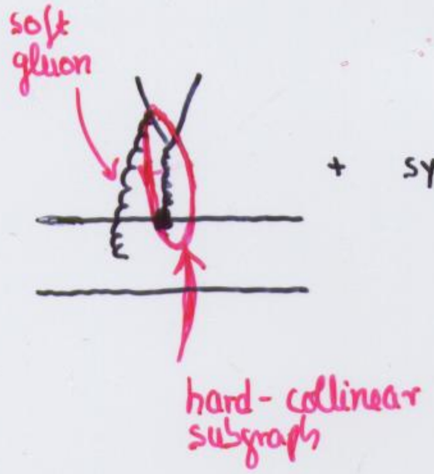
2) Spectator scattering



cannot use perturbative $\eta^{(\prime)} g^* g^*$ form factor

Soft spectator scattering at leading power in $1/m_b$

$\longrightarrow$ Breakdown of factorisation?



+ sym.

soft gluon "sees" only a compact $q\bar q$ pair

$$\alpha_3^p(K_{q,s})\Big|_{\text{soft spec}} = -\frac{3\alpha_s(\sqrt{m_b\Lambda})}{8\pi N_c}\, C_8^{\text{eff}} \left(\int_0^1 du\, \frac{\phi_{q,s}(u)}{6u\bar u} + \text{gluon}\,(gg) \right) \cdot \frac{F_g^{B\to K}(0)}{F^{B\to K}(0)}$$

$$\int_{-\infty}^{0} ds\, s\, \langle \bar K |\, s(0)\, [\not n, \gamma^\mu](1+\gamma_5)\, g_s n^\alpha \tilde G_{\mu\alpha}(sn)\, b(0)\, | \bar B\rangle \equiv \frac{m_B^2}{m_b} F_g^{B\to K}(0)$$

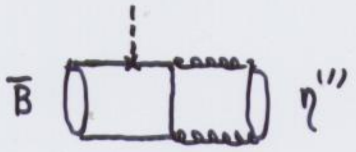
non-local "form factor"

Factorisation formula must be amended by this term for mesons with a flavour-singlet component

3) Singlet weak annihilation

$1/m_b$ suppressed
very uncertain
$\longrightarrow$ estimates as for non-singlet

Note  is leading power → unknown singlet contribution to the $B \to \eta^{(\prime)}$ form factor



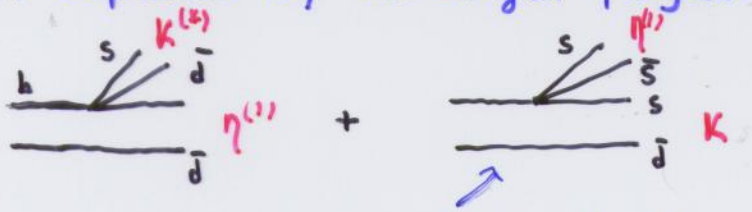
Results

- Any of the three effects might be as large as the singlet penguin in naive factorisation, but never important on an absolute scale, except perhaps for $\eta' K^*$

- VERY large uncertainties from m_s, $F^{B\to\eta}$, non-singlet annihilation

Mode	Br	Br	Mode
$\eta' K^-$	59^{+22+41}_{-16-17}	$7.7^{+7.6+8.0}_{-6.7-3.2}$	$\eta' K^{*-}$
ηK^-	$2.2^{+2.7+1.9}_{-2.0-0.8}$	$13.8^{+4.8+19.8}_{-4.2-6.7}$	ηK^{*-}

Pattern explained by non-singlet penguin interference (Lipkin)



absent for π^0
constructive for η', destructive η

- Factorisation is relevant, but no precise statement
No difficulty with large $\eta' K$ rate

Conclusion

- Much theoretical progress has been made on exclusive B decays to light mesons

  ↪ Nice challenge for theorists due to interplay of soft and collinear physics

  ↪ Motivations for factorisation with EFT see following talks

- Phenomenological successes as well as a few ununderstood data (→ failures of theory ??)

  In general difficulty of theoretical uncertainties and small scales + input parameter uncertainties