The Heavy-Light Form Factor

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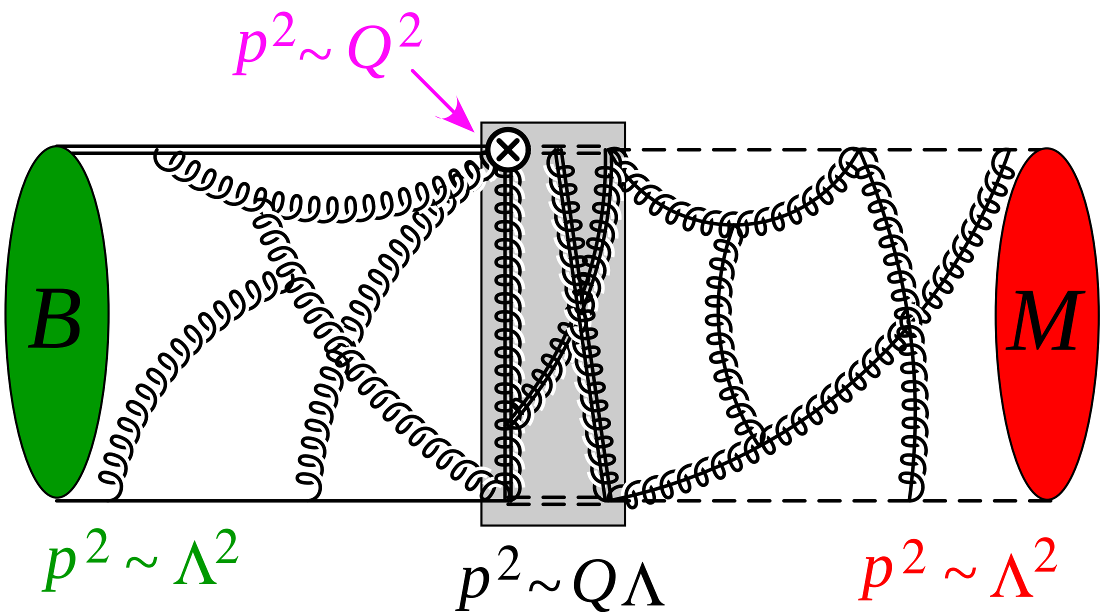
What will I do?

Heavy-light form factor as $F = f^{\rm F}(Q) + f^{\rm NF}(Q)$ where

$$f^F(Q) = N_0 \int_0^1 dz \int_0^1 dx \int_0^\infty dr_+ \, T(z, Q, \mu_0) \times J(z, x, r_+, Q, \mu_0, \mu) \phi_M(x, \mu) \phi_B(r_+, \mu)$$

$$f^{\rm NF}(Q) = C_k(Q, \mu)\, \zeta_k(Q, \mu)$$



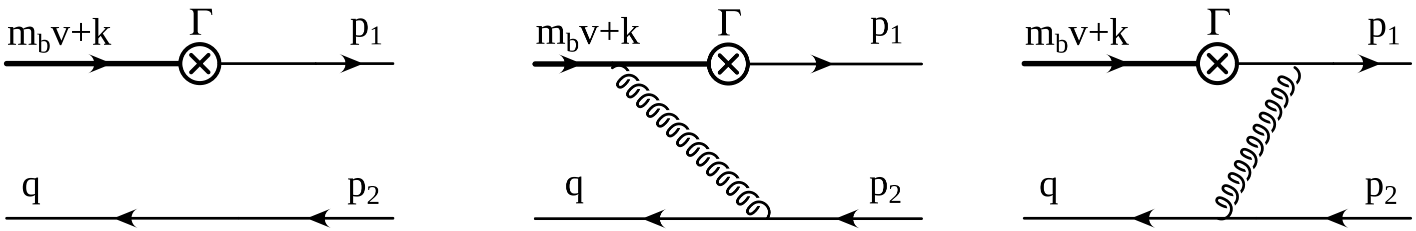
SCET Summary

- Collinear, soft and usoft degrees of freedom
- Integrating out off-shell fluctuations $\Rightarrow$ Wilson lines W and S
- Wilson lines ensure gauge invariance
- Simple power counting from scaling of fields in interaction vertices
- Leading and Subleading Lagrangians known
- Decoupling of usoft gluons by field redefinition with Wilson line Y

Issues in $B \to L\ell\bar{\nu}$

- Two different types of contributions

"Soft" contribution   "Hard" contribution



$p_1 \sim p_\pi,\, p_2 \sim \Lambda_{\rm QCD}$ $p_1 \sim x p_\pi,\, p_2 \sim (1-x) p_\pi$

- Can hard contribution can be written as?

$$F = T \otimes \phi_B \otimes \phi_\pi$$

  - Given in terms of light cone wave functions
  - Can not have $1/x^2$ singularities in T

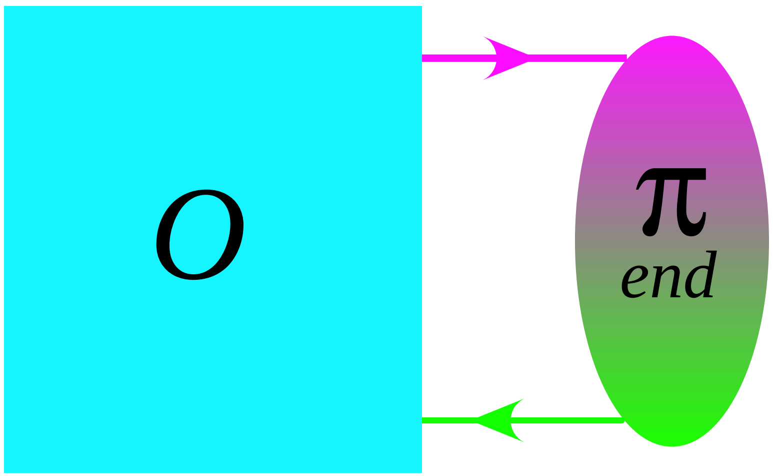
- Form-factor relations for "soft" contributions

Charles *et al.* ('99), CB, Fleming, Pirjol, Stewart ('00)

- Relative size of two terms unknown

Definition of the states

- Can pick any interpolating field
- Pick $|B_v\rangle \sim \bar{h}_v \gamma_5 q_s |0\rangle$, $|\pi_n\rangle \sim \bar{\xi}_n \bar{\not{n}} \gamma^5 \xi_n |0\rangle$
- Both valence quarks collinear
- Asymmetric configuration (endpoint of wave function)?



- Interaction $\bar{q} A_c \xi$ power suppressed

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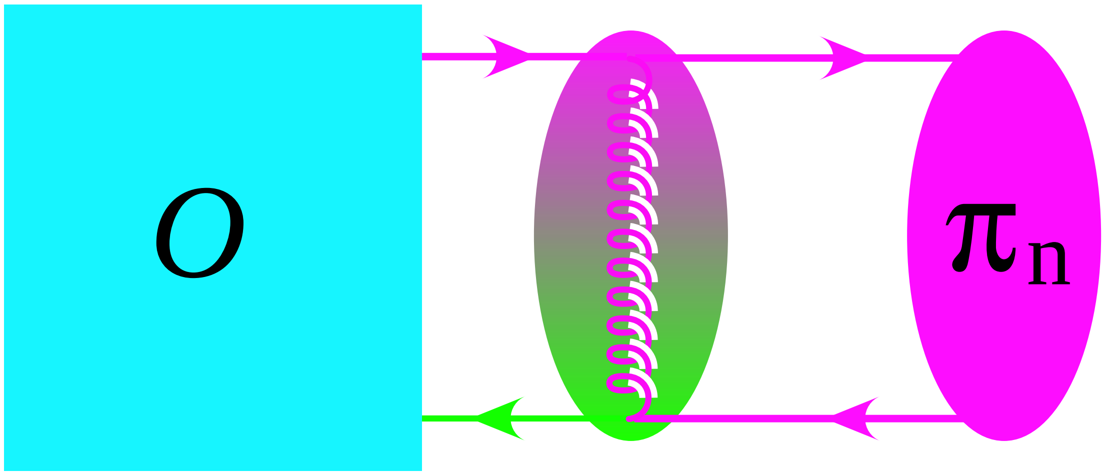

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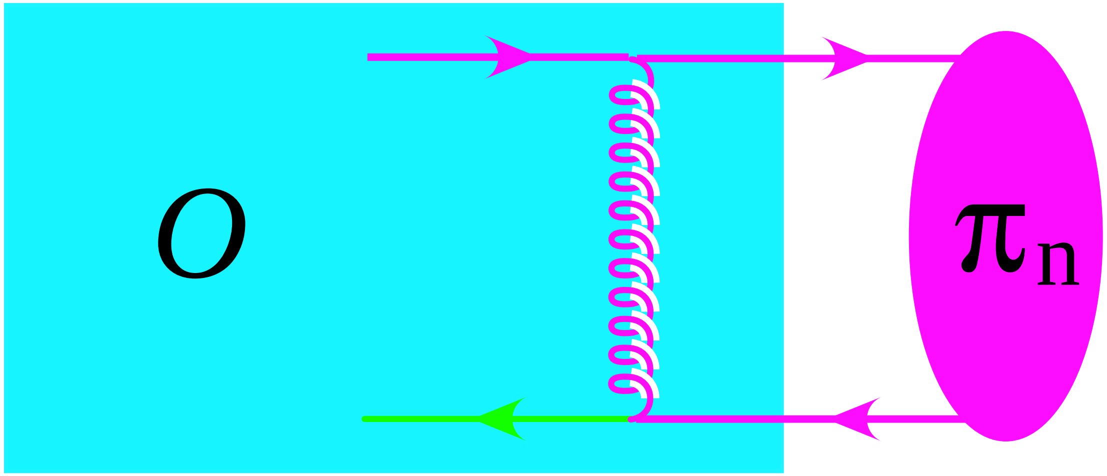

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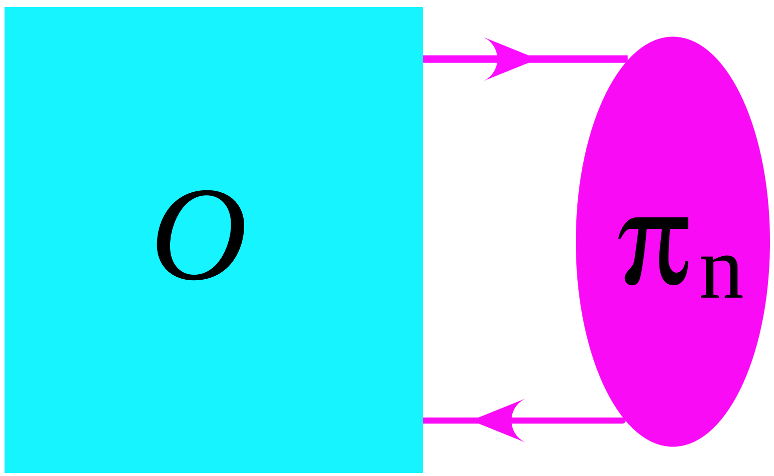

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Power corrections to SCET

- SCET is expansion in small parameter λ
- Subleading terms appear in Lagrangians and external currents
- For Lagrangians distinguish between $\mathcal{L}_{\xi\xi}$ and $\mathcal{L}_{\xi q}$
- For this analysis need $\mathcal{L}_{\xi\xi}^{(0)}$, $\mathcal{L}_{\xi\xi}^{(1)}$, $\mathcal{L}_{cg}^{(0)}$, $\mathcal{L}_{cg}^{(1)}$, $\mathcal{L}_{\xi q}^{(1)}$, $\mathcal{L}_{\xi q}^{(2)}$
- Lagrangians don't receive perturbative corrections, match at tree level

Power corrections to Currents

- At tree level find $J_i^{(0)} = \bar{\xi} W \Gamma_i h$

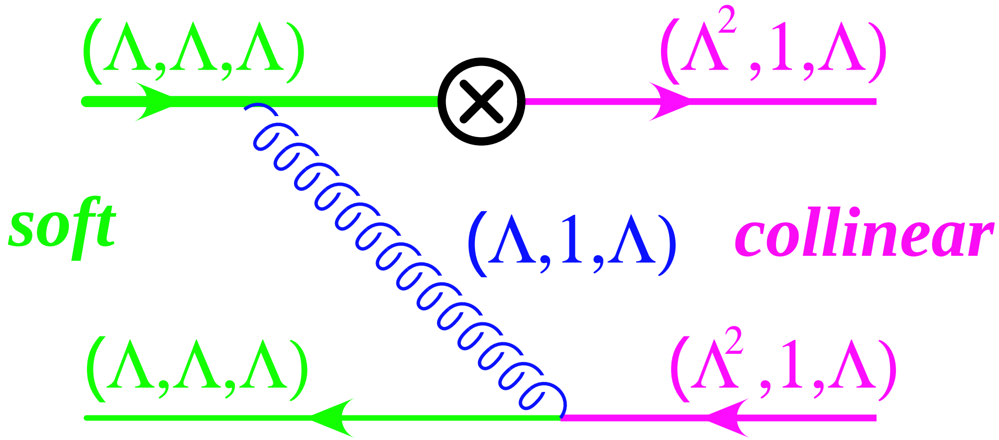
$$J_i^{(1a)} = \bar{\xi}\, \frac{\bar{\not n}}{2}\, (i\not D_c^{\perp})^{\dagger} W \frac{1}{\bar{\mathcal{P}}^{\dagger}}\, \Upsilon_i^{\alpha}\, h \qquad J_i^{(1b)} = \bar{\xi}\, \Theta_i^{\alpha}\, (i\not D_c^{\perp}) W \frac{1}{m_b}\, \frac{\not n}{2}\, h$$

- Including perturbative corrections

$$\begin{aligned} J_i^{(0)}(\omega) &= [\bar{\xi} W]_{\omega} \Gamma_i h \\ J_i^{(1a)}(\omega) &= [\bar{\xi}\, i\overleftarrow{D}^{\perp}_{c\,\alpha} W]_{\omega} \frac{1}{\bar{\mathcal{P}}^{\dagger}}\, \Upsilon_i^{\alpha}\, h\,, \\ J_i^{(1b)}(\omega_1, \omega_2) &= \frac{1}{m_b} [\bar{\xi} W]_{\omega_1} \Theta_i^{\alpha} \Big[\frac{1}{\bar{\mathcal{P}}} W^{\dagger} i g B^{\perp}_{c\,\alpha} W\Big]_{\omega_2} h \end{aligned}$$

- Running of Wilson coefficients gives Sudakov logs
- By RPI $C_i^{(0)}(\omega) = C_i^{(1a)}(\omega)$
- Same Sudakov logs in $C_i^{(0)}(\omega)$ and $C_i^{(1a)}(\omega)$ (more later)

Kinematics Revisited



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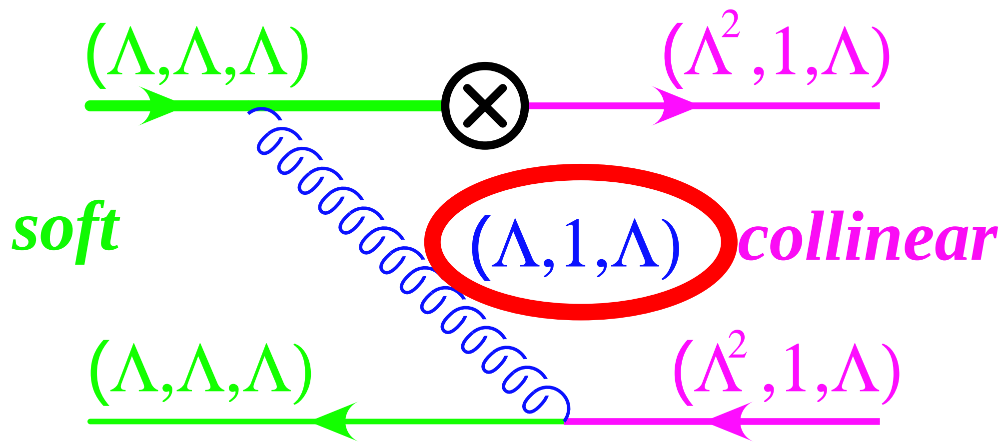

- Gluon is off-shell, needs to be integrated out
- Matches onto four-quark operators
- What do we know about the resulting matrix element?
- Can we write it in terms of ϕ_π and ϕ_B?
- Yes, if $O_{4q} \sim [\bar{\xi}W]_{\omega_1}[W^\dagger\xi]_{\omega_2}[\bar{h}S]_{\kappa_1}[S^\dagger h]_{\kappa_2}$

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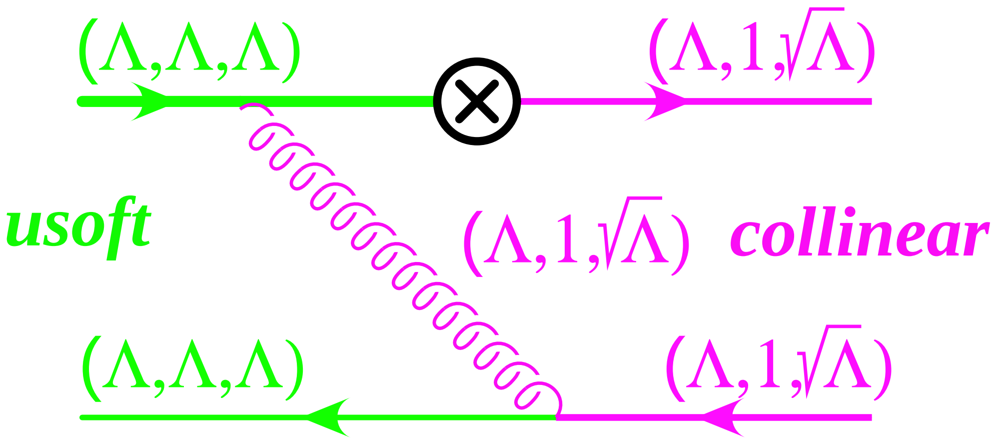
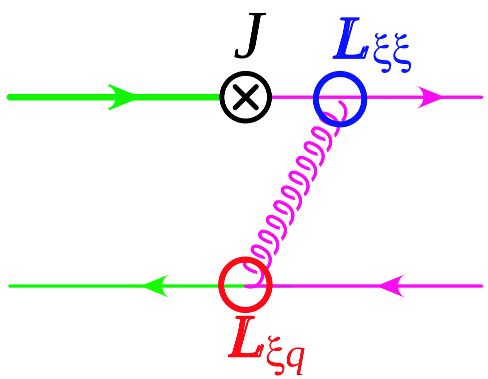


- Gluon is off-shell, needs to be integrated out
- Matches onto four-quark operators
- What do we know about the resulting matrix element?
- Analyze the factorization on collinear and (u)soft
- Study in intermediate SCET_I , where we have better understanding of collinear and usoft

Two step matching procedure

1. Matching QCD $\rightarrow$ SCET$_{\rm I}$
   - Integrate out fluctuations with $p^2 \geq m_b^2$
2. Factorization in SCET$_{\rm I}$
   - Factorize usoft from collinear DOF
3. Matching SCET$_{\rm I}$ $\rightarrow$ SCET$_{\rm II}$
   - Integrate out fluctuations with $p^2 \geq m_b \Lambda_{\rm QCD}$
4. Matrix elements in SCET$_{\rm II}$
   - Take matrix elements with physical external states
   - Identify non-perturbative parameters

Matching QCD $\to$ SCET$_{\rm I}$



- All particles propagating DOF
  $\Rightarrow$ match currents and Lagrangians
- $\mathcal{L}_{\xi q}$ starts at $\mathcal{O}(\lambda)$
  $\Rightarrow$ power suppression needed

Full theory reproduced by the following T-products

$$T_1 = T\big[J^{(1a)}, i\mathcal{L}_{\xi q}^{(1)}\big], \qquad T_2 = T\big[J^{(1b)}, i\mathcal{L}_{\xi q}^{(1)}\big],$$

$$T_3 = T\big[J^{(0)}, i\mathcal{L}_{\xi q}^{(2b)}\big], \qquad T_4 = T\big[J^{(0)}, i\mathcal{L}_{\xi q}^{(2a)}\big],$$

$$T_5 = T\big[J^{(0)}, i\mathcal{L}_{\xi\xi}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\big], \qquad T_6 = T\big[J^{(0)}, i\mathcal{L}_{cg}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\big].$$

- Contributions start at λ^2

Factorization in SCET$_{\text{I}}$

- Factorization between collinear and usoft by field redefinition

$$\xi \to Y \xi^{(0)}\,, \qquad A_c^\mu \to Y A_c^{\mu(0)} Y^\dagger$$
$$\mathcal{L}_{\xi\xi}^{(0)}(\xi, A_c, A_{us}) \to \mathcal{L}_{\xi\xi}^{(0)}(\xi^{(0)}, A_c^{(0)}, 0)$$

- No coupling in leading order Lagrangian
- No non-factorizable pieces from matrix element
- What about factorization of the operator?

Factorization in SCET$_\mathrm{I}$

- "Factorizable" pieces

$$J^{(0)} \to [\bar\xi^{(0)} W^{(0)}]\Gamma[Y^\dagger h_v]$$

- "Non-factorizable" pieces

$$\mathcal{L}^{(1)}_{\xi\xi} \to \bar\xi^{(0)}[Y^\dagger i\not{D}^{us}_\perp Y]\frac{1}{\bar n\cdot D^{(0)}_c} i\not{D}^{c(0)}_\perp \frac{\not{\bar n}}{2}\xi$$

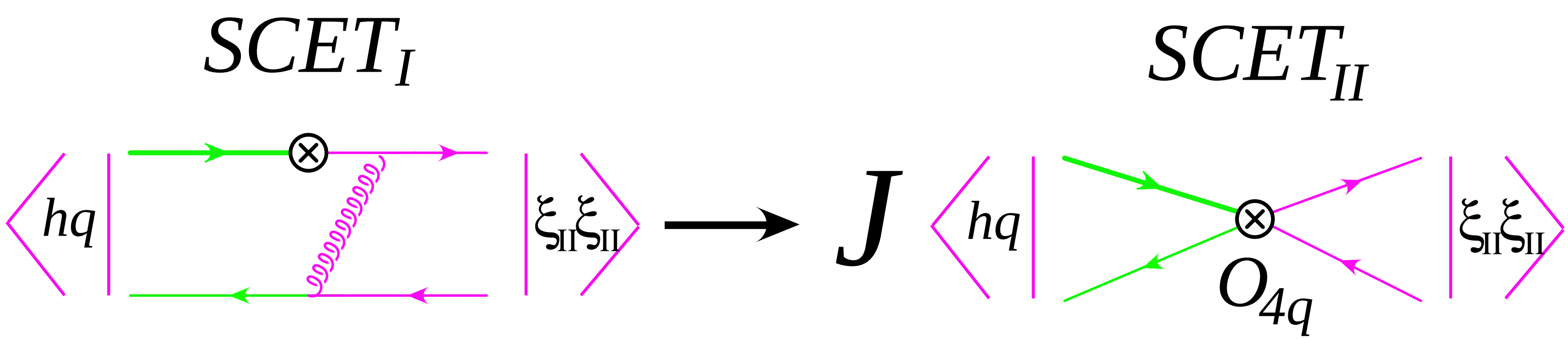
$$T_1^{\rm F} = T\big[J^{(1a)}, i\mathcal{L}^{(1)}_{\xi q}\big], \qquad T_2^{\rm F} = T\big[J^{(1b)}, i\mathcal{L}^{(1)}_{\xi q}\big],$$

$$T_3^{\rm F} = T\big[J^{(0)}, i\mathcal{L}^{(2b)}_{\xi q}\big], \qquad T_4^{\rm NF} = T\big[J^{(0)}, i\mathcal{L}^{(2a)}_{\xi q}\big],$$

$$T_5^{\rm NF} = T\big[J^{(0)}, i\mathcal{L}^{(1)}_{\xi\xi}, i\mathcal{L}^{(1)}_{\xi q}\big], \qquad T_6^{\rm NF} = T\big[J^{(0)}, i\mathcal{L}^{(1)}_{cg}, i\mathcal{L}^{(1)}_{\xi q}\big].$$

Matching $\mathrm{SCET_I} \rightarrow \mathrm{SCET_{II}}$

- Fluctuations in $\mathrm{SCET_I}$ collinear particles in $\mathrm{SCET_I}$ too large for physical pion
- Need to eliminate fluctuations with $p^2 \sim m_b \Lambda_{\mathrm{QCD}}$
- Match onto $\mathrm{SCET_{II}}$



- Fields ξ_{II} are contained in Hilbert space of $\mathrm{SCET_I}$
  $\xi_{II}(k) = \xi_I(\lambda^4, 1, \lambda^2)$

Matrix elements in SCET$_{\mathrm{II}}$

- Big question: What do we know about the matrix element $\langle\pi|O_{4q}|B\rangle$?
- If $O_{4q} \sim [\bar\xi W]_{\omega_1}[W^\dagger\xi]_{\omega_2}[\bar h S]_{\kappa_1}[S^\dagger h]_{\kappa_2}$ (Valid for T_i^F)

$$\langle\pi|O^F_{4q}|B\rangle \sim \int_0^1 dz \int_0^1 dx \int_0^\infty dr_+ \, T(z,Q,\mu_0) J(z,x,r_+,Q,\mu_0,\mu)\phi_M(x,\mu)\phi_B(r_+,\mu)$$

Object	Origin	perturbative?	relevant scale
$T(\ldots)$	QCD $\to$ SCET$_{\mathrm{I}}$ matching	$\alpha_s(Q)$	
$J(\ldots)$	SCET$_{\mathrm{I}}$ $\to$ SCET$_{\mathrm{II}}$ matching	$\alpha_s(\sqrt{Q\Lambda_{\mathrm{QCD}}})$	$\mu_0 \sim \sqrt{Q\Lambda_{\mathrm{QCD}}}$
$\phi_M(\ldots)$	light meson LCDA	No	$\mu \sim \Lambda_{\mathrm{QCD}}$
$\phi_B(\ldots)$	B-meson LCDA	No	$\mu \sim \Lambda_{\mathrm{QCD}}$

$$f_M\phi_M(x,\mu) = \frac{i}{\bar n\cdot p}\langle M_{II}(p)|\bar\xi W\,\Gamma\,\delta\left((1-2z)\bar n\cdot p + \bar{\mathcal{P}}_+\right)W^\dagger\xi|0\rangle$$

No $1/x^2$ or $1/r_+^2$ singularities in T and J if defined this way

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$$\langle\pi|O_{4q}^{NF}|B\rangle \sim T \otimes J \otimes \Psi_M \otimes \Psi_B$$

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$J(\ldots)$	SCET$_{\mathrm{I}}$ $\to$ SCET$_{\mathrm{II}}$ matching	$\alpha_s(\sqrt{Q\Lambda_{\mathrm{QCD}}})$	$\mu_0 \sim \sqrt{Q\Lambda_{\mathrm{QCD}}}$
$\Psi_M(\ldots)$	light meson matrix element	No	$\mu \sim \Lambda_{\mathrm{QCD}}$
$\Psi_B(\ldots)$	B-meson matrix element	No	$\mu \sim \Lambda_{\mathrm{QCD}}$

The non-perturbative objects $\Psi_M(\ldots)$ and $\Psi_B(\ldots)$ are *NOT* both 1-d LCDA's!

Matrix elements in SCET_{II}

- Big question: What do we know about the matrix element $\langle\pi|O_{4q}|B\rangle$?
- Matrix elements of "non-factorizable" T-products give rise to new non-perturbative functions
- Why not write $\langle\pi|O_{4q}^{NF}|B\rangle \sim C(Q,\mu)\zeta(\sqrt{Q\Lambda_{\text{QCD}}},\mu)$

Object	Origin	perturbative?	relevant scales
$C(\ldots)$	QCD $\rightarrow$ SCET_{I} matching	$\alpha_s(Q)$	
$\zeta(\ldots)$	all remaining effects	No	$\mu \sim \Lambda_{\text{QCD}}, \sqrt{Q\Lambda_{\text{QCD}}}$

- Are there less functions $\zeta(\ldots)$ than there are form factors?
- Yes! FF relations hold for $\zeta(\ldots)$ if $J^{(0)}$ in T-product
- Only two (three) functions required for all FF's

One last point

- All T-products except T_1 and T_2 contain leading order current J_0
- The resulting contributions all obey FF relations
- Can move all these contributions into definition of $\zeta(\ldots)$
- Leads to definition of $\zeta(\ldots)$

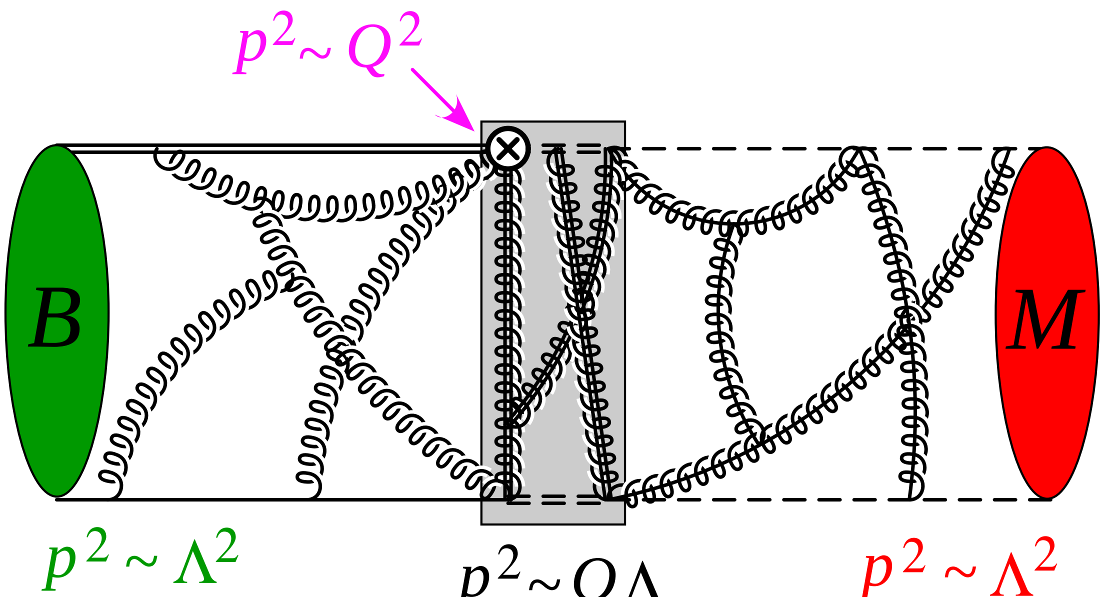
$$C(Q,\mu)\zeta(\sqrt{Q\Lambda_{\rm QCD}},\mu) = \sum_{i=3\ldots6} \langle\pi|T_i|B\rangle$$

- Furthermore T_1 and T_2 give factorizable terms with only $\phi_B^+(r_+)$
- No need for $\phi_B^-(r_+)$ in convolution

Final result

Near $q^2 = 0$ is $F(Q) = f^F(Q) + f^{\rm NF}(Q)$

$$
\begin{aligned}
f^F(Q) &= N_0 \int_0^1 dz \int_0^1 dx \int_0^\infty dr_+ \, T(z, Q, \mu_0) \\
&\quad \times J(z, x, r_+, Q, \mu_0, \mu) \phi_M(x, \mu) \phi_B^+(r_+, \mu) \\
f^{\rm NF}(Q) &= C_k\Big(Q, \mu\Big) \, \zeta_k\Big(\sqrt{Q\Lambda_{\rm QCD}}, \mu\Big)
\end{aligned}
$$



Relative size of two terms

- Factorizable and non-factorizable same order in power counting
- Relative size of the two terms is decided by powers of α_s (logarithms)
- Is there Sudakov suppression of one of the terms?
- Logarithms of NF piece determined by running of $C^{(0)}$
- Running known and contains Sudakov logarithms

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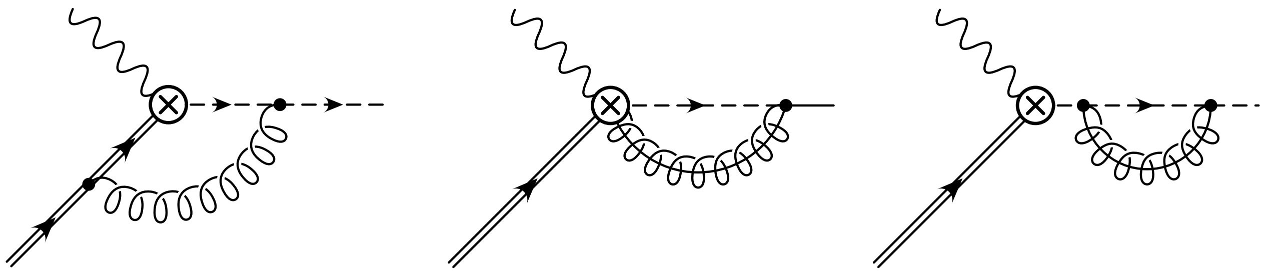
$$\mu \frac{d}{d\mu} C(\mu) = \gamma(\mu) C(\mu) \,, \quad \gamma(\mu) = \alpha_s(\mu) \left(A + B \log \mu \right)$$



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- Running known and contains Sudakov logarithms
- Logarithms of factorizable piece by running of $C^{(1a,b)}$
- Since $C^{(1a)} = C^{(0)}$ only unknown is $C^{(1b)}$
- But definitely Sudakov logarithms in factorizable piece
- No reason to be of different size

What have we learnt?

There are four main points we can show with our analysis

- Cleanly separated factorizable and non-factorizable contributions
- Factorizable term contains no $1/x^2$ singularities
- Non-factorizable term obeys the form-factor relations
- Factorizable and non-factorizable same order in power counting
- No reason for different size in logarithm