$B \to \pi$ and $B \to \rho$ from QCD Sum Rules on the Light-Cone

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Setting the Stage for $B \to \pi \ldots$

(... come back to other decays later)

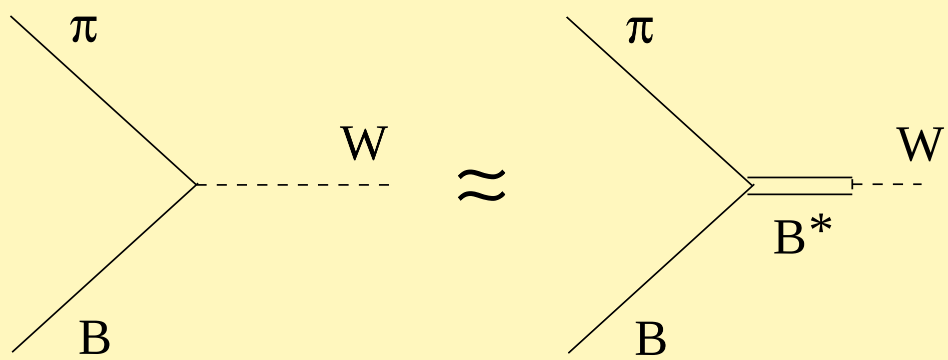
Definition of form factors:

$$\langle \pi | \bar{u} \gamma_\mu b | B \rangle = f_+(q^2)(p_{B\mu} + p_{\pi\mu}) + f_-(q^2) q_\mu \quad [q_\mu = p_{B\mu} - p_{\pi\mu}]$$

$B \to \pi e \nu$: f_- suppressed by m_e^2/m_B^2, $0 \le q^2 \le (m_B - m_\pi)^2$.

Naïve expectation: f_+ dominated by B^*-pole ($m_{B^*} = 5.32\,\mathrm{GeV}$):

$$f_+(q^2) \propto \frac{1}{m_{B^*}^2 - q^2}$$



Correct expression: $f_+(q^2) = \dfrac{c}{m_{B^*}^2 - q^2} + \displaystyle\int_{(m_B+m_\pi)^2}^{\infty} dt\, \frac{\rho(t)}{t - q^2}$

Some Ideas How to Calculate

- Quark models: started 85 with relativistic harmonic oscillator + pole-dominance (Bauer/Stech/Wirbel)

- Lattice: favourite channel, *but* restriction to small π energies. . .

- QCD sum rules à la SVZ: trouble with nonperturbative terms for $m_b \to \infty$

- pQCD methods à la Brodsky-Lepage (to be cont'd)

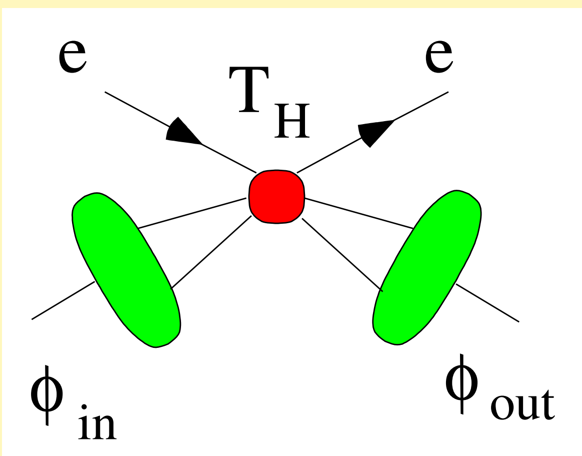
- QCD sum rules on the light-cone: hybrid of SVZ and pQCD, good for large π energies
  (to be cont'd)

Hard & Soft pQCD I

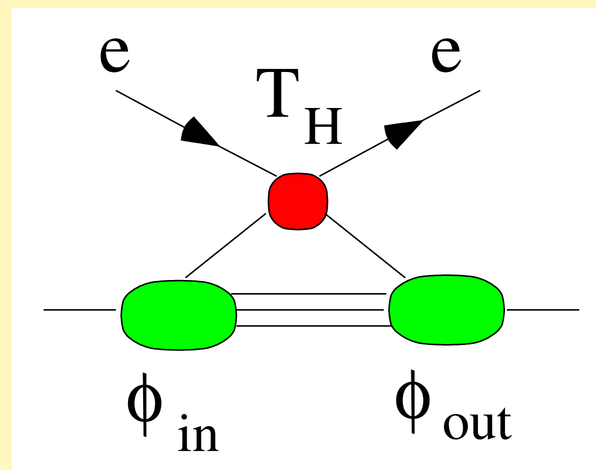
At high enough momentum transfer Q^2, exclusive amplitude M dominated by states with "valence" quark content; factorization applies:

$$M = \prod_j \phi_{\mathrm{out},j}(n_j) \otimes T_H(n_j, n_i) \otimes \prod_i \phi_{\mathrm{in},i}(n_i)$$

$\phi(u)$, $0 \le u \le 1$: probability amplitude for collinear quarks with momentum up and $(1-u)p$, resp., to form hadron with momentum p ($p^2 \ll Q^2$)



Purely hard process: dominant in "classical" applications of pQCD, e.g. EM π FF



"Soft" or "Feynman" mechanism: strongly asymmetric kinematical configuration of partons

Hard & Soft pQCD II

- processes involving only light mesons: dominated by hard contributions (gluon exchange)
- heavy mesons: soft and hard processes parametrically of same order, although soft processes damped by Sudakov logs (Chernyak/Zhitnitsky 1990)
- effectiveness of Sudakov-damping at $Q^2 = m_b^2$ questionable
- theory of light-meson distribution amplitudes ϕ mature, but for heavy mesons largely *terra incognita*
- calculation of $B \to \pi$ by hard-gluon exchange spoiled by soft divergences (Szczepaniak/Henley/Brodsky 1990)
- alternative approach: factorization in SCET (Bauer/Pirjol/Stewart, hep-ph/0211069) (come back to that later)

$\rightsquigarrow$ need method to capture both hard-gluon-exchange and soft Feynman-mechanism! Avoid ϕ_B!

Enter the stage

QCD Sum Rules on the Light-Cone

$$i\int d^4y e^{iqy}\langle\pi(p)|T[\bar{u}\gamma_\mu b](y)[m_b\bar{b}i\gamma_5 d](0)|0\rangle \stackrel{\text{LCE}}{=} \sum_n T_H^{(n)}\otimes\phi_\pi^{(n)}$$

- $\phi_\pi^{(n)}$: π distribution amplitudes (DAs)
- n: twist
- $T_H^{(n)}$: perturbative amplitudes

$$= 2p_\mu\left(f_+(q^2)\frac{m_B^2 f_B}{m_B^2-p_B^2} + \text{higher poles and cuts}\right) + \ldots$$

$\rightsquigarrow$ no ϕ_B as B described by Euclidean current + plus analytical continuation

$\rightsquigarrow$ LC-expansion starts at $O(1)$, not $O(\alpha_s)$ $\to$ soft terms included

Features of LCSRs

- LCE effectively in $1/m_b \to$ need to include higher-twist terms
- $\sum T_H^{(n)} \otimes \phi_\pi^{(n)}$ implies factorization – valid at higher twist?
  - calculate $O(\alpha_s)$, known for
    T2 (π (Khodjamirian et al. 97, Ball et al. 97), ρ (Ball/Braun 98))
    T3 (π (Ball/Zwicky 2001))
    $\to$ factorization OK
- use standard SR techniques: Borel-transformation, continuum model

Formal Definition of Twist-2 DAs

$p^2 = 0, z^2 = 0$, $p \propto (1,0,0,1)$, $z \propto (1,0,0,-1)$

Projections on light-cone coordinates:

$$a_+ = a \cdot z, \quad a_- = (a \cdot p)/(z \cdot p), \quad a_{\perp\mu} = a_\mu - a_+ p_\mu/(p \cdot z) - a_- z_\mu$$

$$\langle 0|\bar{u}(z)\,[z,-z]\,\gamma_+\gamma_5 d(-z)|\pi^-(p)\rangle = i f_\pi (p \cdot z) \int_0^1 du\, e^{i\xi pz}\, \phi_\pi(u)$$

$$\langle 0|\bar{u}(z)\,[z,-z]\,\gamma_+ d(-z)|\rho^-(p,\lambda)\rangle = f_\rho m_\rho (e^{(\lambda)} \cdot z) \int_0^1 du\, e^{i\xi pz} \phi_\parallel(u)$$

$$\langle 0|\bar{u}(z)\,[z,-z]\,\sigma_{\cdot\perp} d(-z)|\rho^-(p,\lambda)\rangle = i f_\rho^T e_\perp^{(\lambda)} (p \cdot z) \int_0^1 du\, e^{i\xi pz} \phi_\perp(u)$$

$$\xi = 2u - 1$$

N.B.: certain similarity to definition of DIS parton distribution functions. However:

PD functions	$\leftrightarrow$	probability
DAs	$\leftrightarrow$	amplitude

Partial Wave Expansion of Twist 2 DAs

Exploit conformal symmetry of massless QCD, expand in conformal spin.

$$\phi(u,\mu^2) = 6u(1-u)\left(1+\sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)\right)$$

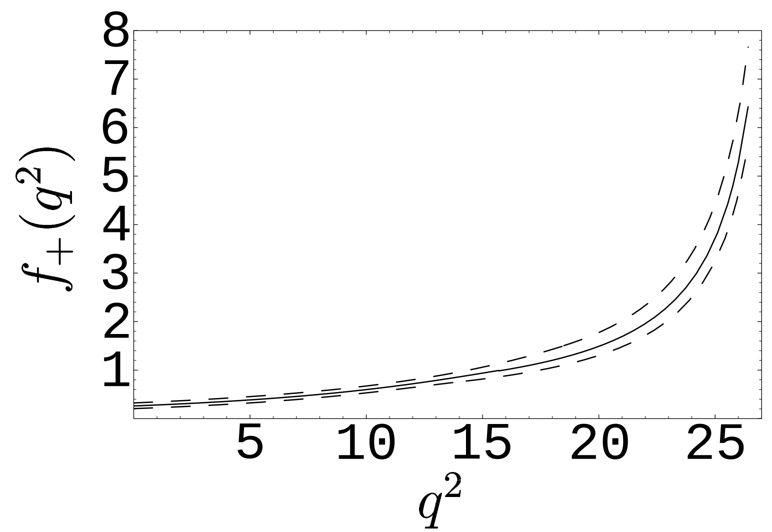
- $6u(1-u)$: asymptotic DA, valid for $\mu \to \infty$, conformal spin $= 2$.
- $a_n(\mu)$: Gegenbauer moments; nonperturbative parameters, renormalize multiplicatively in LO QCD by virtue of conformal symmetry; $a_n(\mu) \to 0$ for $\mu \to \infty$
- $C_n^{3/2}$: Gegenbauer polynomials, orthogonal over asymptotic DA; conformal spin $2+n$

a_n determined from experiment (e.g. $e^+e^- \to e^+e^-\pi^0$ (CLEO)).

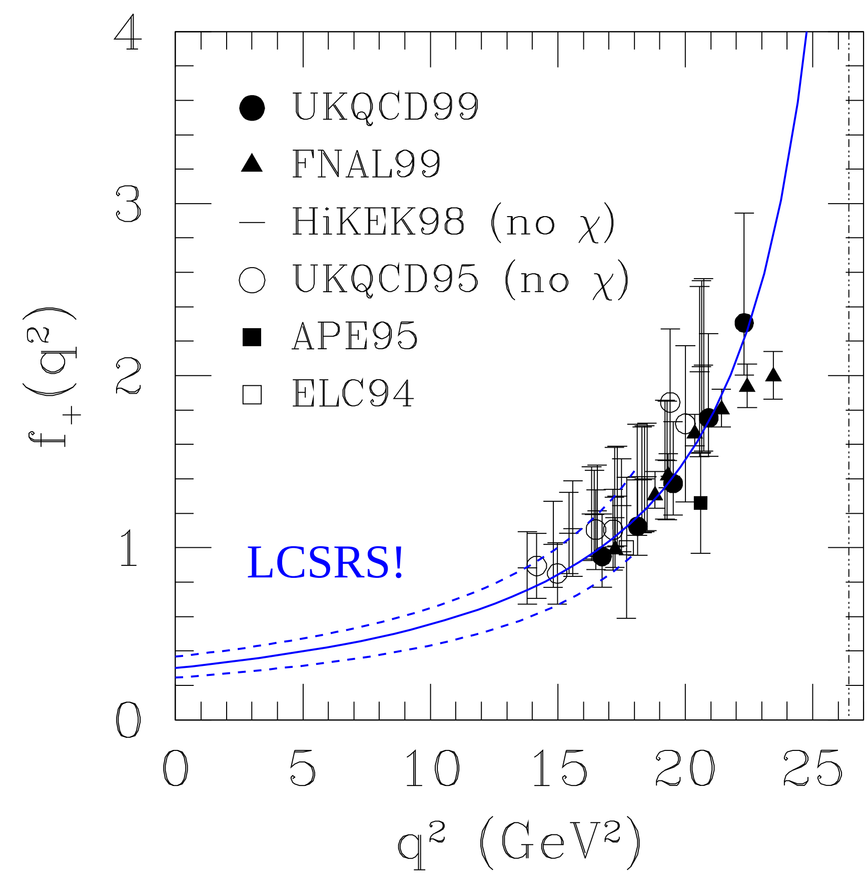
And now, Ladies and Gentlemen:

Results!

Ball/Zwicky 2001: $f_+^{B\to\pi}(0) = 0.26 \pm 0.06 \pm 0.05$.
(uncertainty in input parameters + guesstimate of systematic error)



Use LCSRs for $q^2 \leq 15\,\text{GeV}^2$, pole-dominance above.



The Quest for the q^2-Dependence

- poles + cuts: $f_+(q^2) = \dfrac{c}{m_{B^*}^2 - q^2} + \displaystyle\int_{(m_B+m_\pi)^2}^{\infty} dt\, \frac{\rho(t)}{t-q^2}$

- LCSR: $$F(q^2) = \frac{F(0)}{1 - a_F\,\frac{q^2}{m_B^2} + b_F\left(\frac{q^2}{m_B^2}\right)^2}$$ works extremely well

- region of applicability of LCSRs: $E_\pi \gg \Lambda_{\rm QCD}$, OK for $q^2 < 15\,\text{GeV}^2$
  - consistent with LEET relations (Orsay group)

And what about uncertainties?

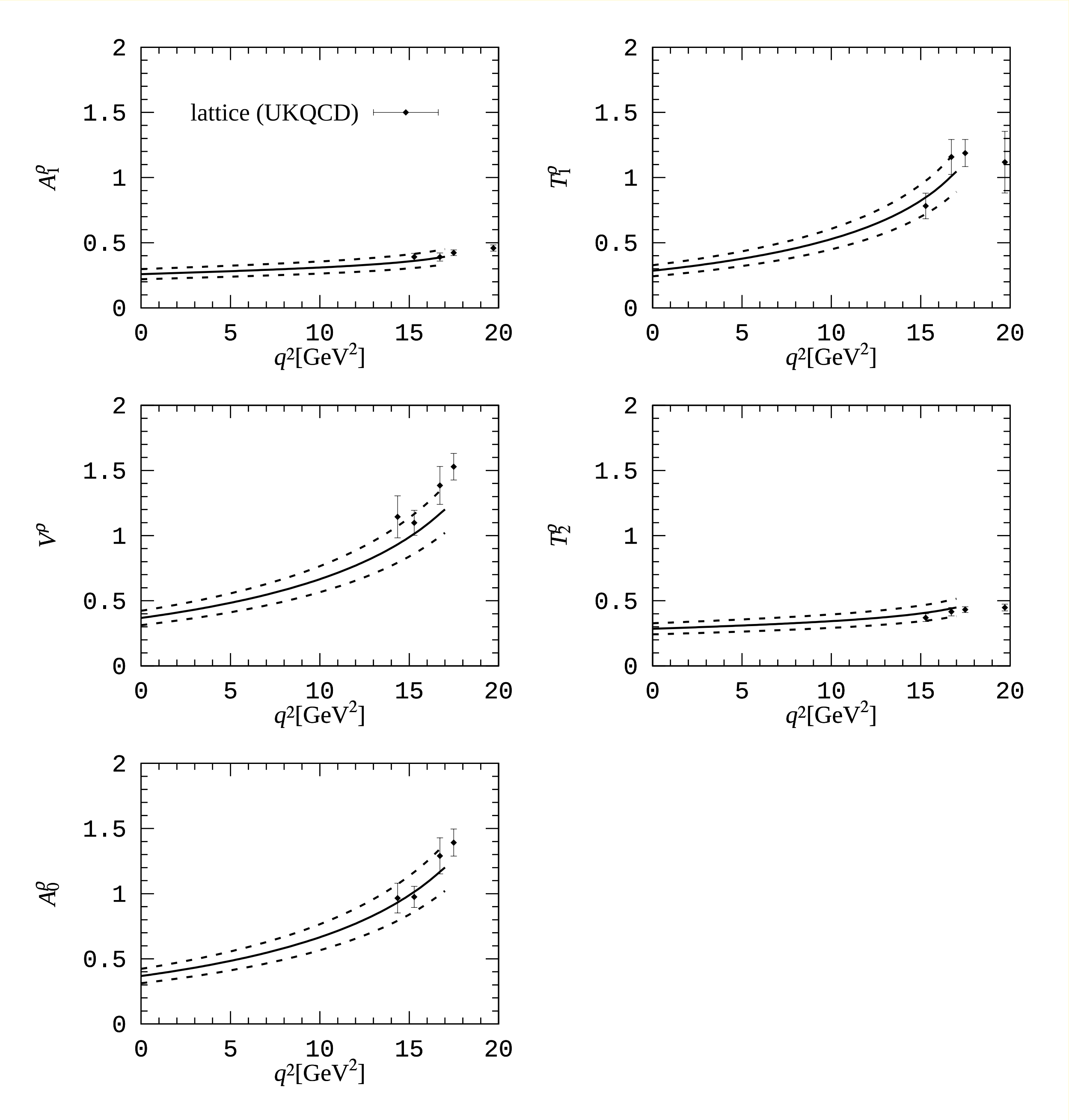
- systematic uncertainties from various approximations involved
  - Borel-transformation/ dep. on Borel-parameter
  - continuum model/subtraction

$\rightsquigarrow$ estimate at 20% (?)

- uncertainties from input-parameters
  - π, ρ, K, K^* DAs: exp. or SRs or lattice?
  - m_b

- adds up to a total of what?

Ball/Braun 98



$$\langle V(p)|(V-A)_\mu|B(p_B)\rangle = -i\epsilon^*_\mu(m_B+m_V)A_1^V(q^2)$$
$$+i(p_B+p)_\mu(\epsilon^* p_B)\,\frac{A_2^V(q^2)}{m_B+m_V} + \; iq_\mu(\epsilon^* p_B)\,\frac{2m_V}{q^2}$$
$$\times\left(A_3^V(q^2)-A_0^V(q^2)\right)+\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma\,\frac{2V^V(q^2)}{m_B+m_V}\,.$$

LCSRs, Factorization and SCET I

Bauer/Pirjol/Stewart, hep-ph/0211069: to all orders in α_s and leading order in $1/Q$:

$$f_+^{B\to\pi}\left(E_\pi = \frac{m_B}{2}\right) = N_0 \int dz dz dk_+ T(z,\mu_0) J(z,x,k_+;\mu_0,\mu)\phi_\pi(x,\mu)\phi_B(k_+,\mu) + C(Q,\mu)\zeta(Q,\mu)$$

($\mu = O(m_b)$, $\mu_0 = O(\sqrt{\Lambda_{\rm QCD} m_b})$, $Q = E_\pi$?).

Feature: separation into factorizable and nonfactorizable parts scale- and scheme-dependent

Open questions:

- demonstration of independence of μ and μ_0
- evolution of ζ and ϕ_B with μ
- for phenomenological purposes need parametrization of DAs and ζ

LCSRs, Factorization and SCET II

(Formal) HQL of LCSR for f_+ (only twist-2 term, Bagan/Ball/Braun, hep-ph/9709243):

$$\frac{f^{\rm stat}}{f_\pi}[m_b^{3/2} f_+(0)] = -\omega_0^2 \phi'_\pi(1,\mu_0)\left[1+\frac{\alpha_s}{\pi}C_F\left(\frac{1+\pi^2}{4}+\ln\frac{m_b}{2\omega_0}-\frac{1}{2}\ln^2\frac{m_b}{2\omega_0}+\frac{1}{2}\ln\frac{2\omega_0}{\mu_0}\right)\right]$$

$$-\omega_0^2\frac{\alpha_s}{\pi}C_F\left[\left(1-\ln\frac{2\omega_0}{\mu_0}\right)\int_0^1 du\left(\frac{\phi_\pi(u)}{\bar u^2}+\frac{\phi'_\pi(1)}{\bar u}\right)-\ln\frac{2\omega_0}{\mu_0}\int_0^1 du\,\frac{\phi_\pi(u)}{\bar u}\right]$$

- no endpoint-singularities
- $\omega_0 \approx 1\,$GeV hadronic parameter characterizing B
- hard-collinear terms; $\ln m_b/(2\omega_0) \approx 1$: not *really* large
- explicit cancellation of μ_0-dependence b.v.o. evolution equation of ϕ_π;
  choose $\mu_0^2 = m_B^2 - m_b^2 = 2m_b\bar\Lambda \longleftrightarrow \mu_0 \approx 2.4\,$GeV

LCSRs, Factorization and SCET III

However that's not all: contributions from twist-3 DAs ϕ_p and ϕ_σ: ($\mu_\pi = m_\pi^2/(m_u+m_d)$)

$$\frac{f^{\rm stat}}{f_\pi}\,[m_b^{3/2} f_+(0)]_{\rm twist-3} \;=\; \mu_\pi\,\omega_0\,\phi_p(1,\mu_0)\left[1+\frac{\alpha_s}{\pi}C_F\left(-1+\frac{\pi^2}{4}-\frac{1}{2}\ln^2\frac{2\omega_0}{m_b}+\ln\frac{2\omega_0}{\mu_0}\right.\right.$$

$$\left.\left.-\ln\frac{2\omega_0}{m_b}\ln\frac{2\omega_0}{\mu_0}\right)\right]+\mu_\pi\,\omega_0\,C_F\frac{\alpha_s}{\pi}\left(2\ln\frac{2\omega_0}{\mu_0}-\frac{3}{2}\right)\int_0^1 du\frac{\phi_p(u)-\phi_p(1)}{\bar u}$$

$$-\mu_\pi\,\omega_0\,\frac{\phi'_\sigma(1,\mu_0)}{6}\left[1+C_F\frac{\alpha_s}{\pi}\left(\frac{\pi^2-6}{4}-2\ln\frac{2\omega_0}{m_b}-\frac{1}{2}\ln^2\frac{2\omega_0}{m_b}+2\ln\frac{2\omega_0}{\mu_0}+\ln\frac{2\omega_0}{m_b}\ln\frac{2\omega_0}{\mu_0}\right.\right.$$

$$-\mu_\pi\omega_0 C_F\frac{\alpha_s}{4\pi}\int_0^1 du\,\frac{\phi_\sigma(u)+\bar u\phi'_\sigma(1)}{\bar u^2}.$$

Problems:

- mixed logs $\to$ next slide
- $\phi_{p,\sigma}$ mix with twist-3 quark-quark-gluon DA under renormalization

However: (tree-level) contribution of 3-particle twist-3 quark-quark-gluon DA suppressed by $1/m_b$.

LCSRs, Factorization and SCET III

Simplify by using EOM for ϕ's: $\phi_p(1) + \phi'_\sigma(1)/6 = 0$ (valid exactly):

$$\frac{f^{\rm stat}}{f_\pi}\,[m_b^{3/2} f_+(0)]_{\rm twist-3} = \mu_\pi\, 2\omega_0\, \phi_p(1,\mu_0)\,\Big[1+ \frac{\alpha_s}{\pi} C_F \left(\frac{\pi^2}{4} - \frac{5}{4} - \frac{1}{2}\ln^2\frac{m_b}{2\omega_0} + \ln\frac{m_b}{2\omega_0} + \frac{3}{2}\ln\frac{2\omega_0}{\mu_0}\right)\Big] + \text{hard terms}$$

- mixed logs cancel
- same hard logs as for twist-2 contribution
- soft log cancels μ_0 dependence of $\phi_p(1,\mu_0)$ (to leading order in conformal expansion)

What happens to these contributions in SCET???

Summary

- npQCD devilishly complicated, no edging away from QCD-infested measurements
- QCD SRs on the light-cone appear well suited to describe heavy-to-light transitions $B \to \pi, \rho, K, K^*$ for small to moderate momentum transfer
- not much room for theoretical improvement, except for parameters describing distribution amplitudes (lattice?)
- test predictions for shape to gain confidence in absolute values; challenge/confirm sympathy with lattice