

**Workshop on recent developments in string effective actions and D-instantons**

Max-Planck-Institute für Physik, Munich

# Branes in Flux Backgrounds and Dualities

Giovanni Villadoro

Harvard University

*based on:*

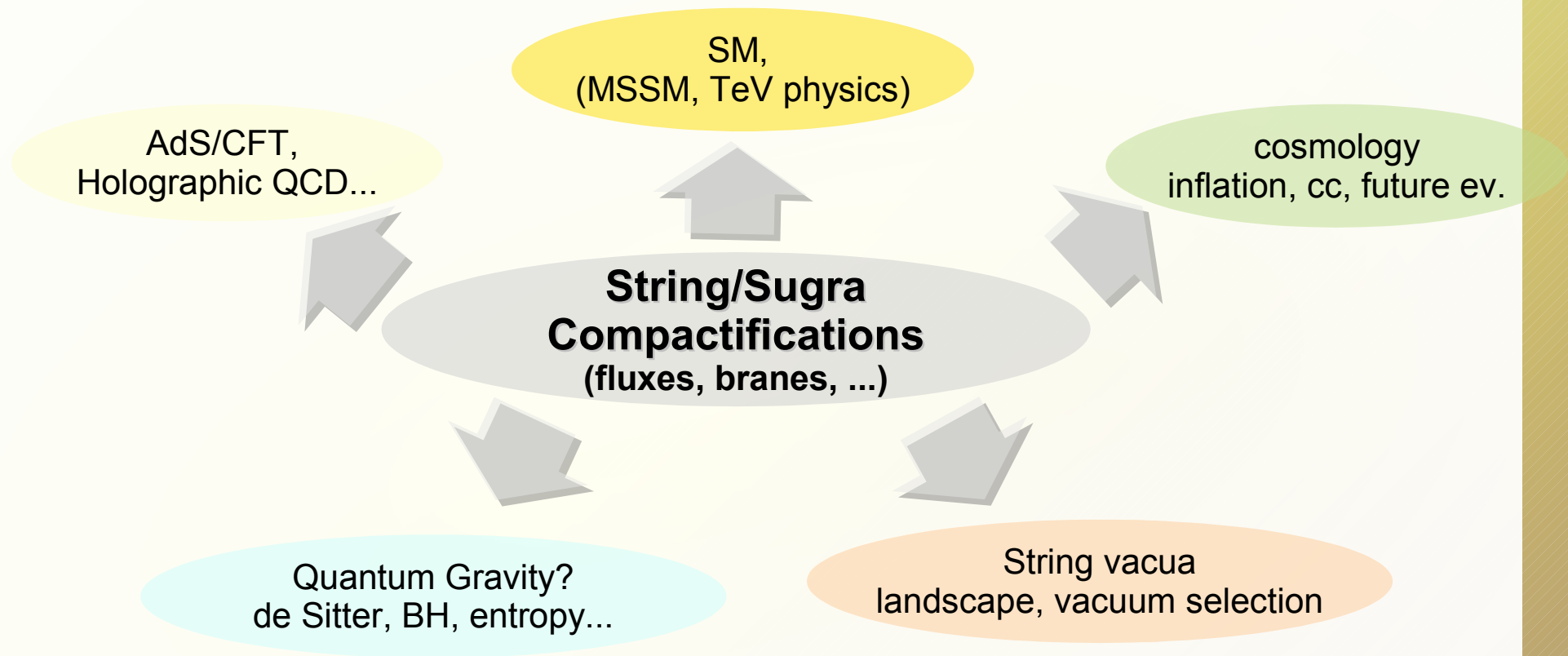
arXiv:0706.3049 -- PLB 652, 118

arXiv:0710.2551 -- JHEP

**Nov. 14-16, 2007**

in collaboration with **F. Zwirner**

# String/Supergravity Compactifications



**Important to understand the rules....**

# Dualities in String Constructions

what can and what cannot be done?

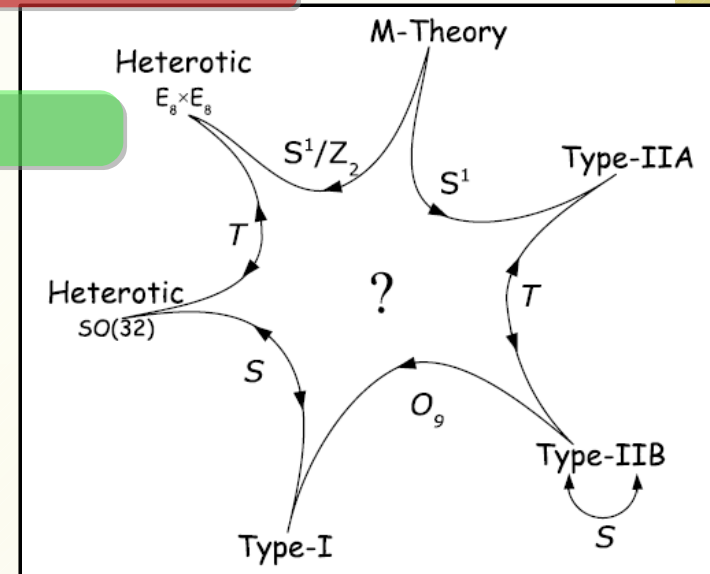
## Importance of dualities

### Fluxes:

- RR
- NSNS
- geometrical
- (non geometrical)

### Sources:

- D-branes
- NS5-branes
- KK-monopoles
- ...



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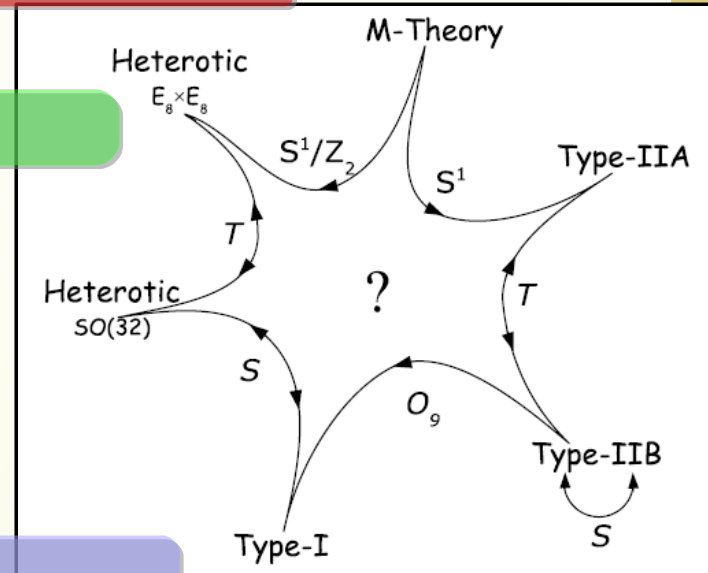
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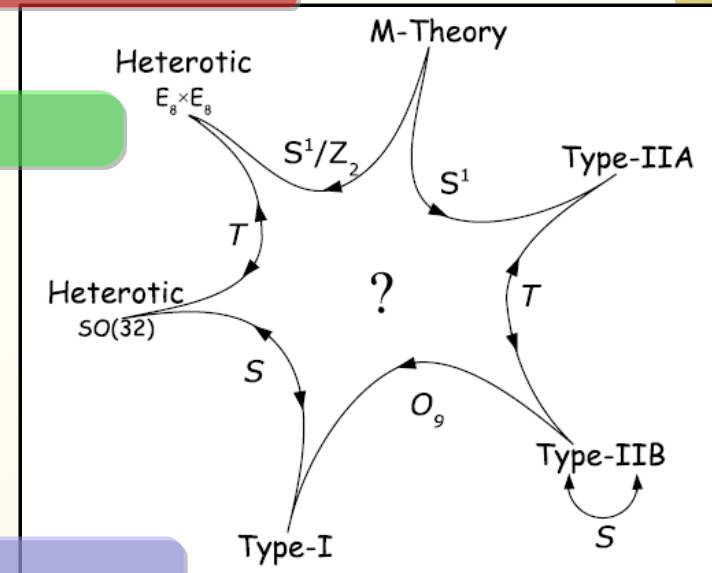
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*answer: satisfy e.o.m., BI, etc...*

*add  $\alpha'$  and  $g_s$  corrections (pert. and non pert.), etc...*

*very tough!!!*

*rather:  
start from a simpler solution  
and perturb it*

+

*global constraints*

**Classic example:**

RR-tadpole cancellation condition

Other constraints?

# Fluxes and Twisted Tori

## Fluxes:

● RR  $G^{(p)} = dC^{(p-1)} + \dots$

● NSNS  $H = dB + \dots$

● T-dual to geometrical fluxes:  $\omega$  (Scherk-Schwarz parameter)

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## Consistency Constraint on geometrical fluxes:

$$\omega \omega = \omega_{[a b}{}^c \omega_{c d]}{}^e = 0$$

It corresponds to the closure of the external derivative,  
(compatibility with compactness of space):

$$d \rightarrow d + \omega$$

$$dd=0 \rightarrow (d+\omega)(d+\omega) = d\omega + \omega \omega = 0$$

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## How strong the constraint $\omega \omega = 0$ is?

● Seems a strong topological constraint;

● *Hint:* DKPZ vacua from 4d N=4 gauged sugra seems to correspond to non trivial  $\omega \omega$  compactifications: [Derendinger, Kounnas, Petropoulos, Zwirner 2004](#)

● are they in the swampland?

- Bianchi identities produce global constraints in compactifications
  - RR sector (Dp-branes source RR-fields)

$$D G = Q_{\text{RR}}$$

$$(d + \omega + H) G = \sum Q(\pi_p)$$

- Integrability condition:

$$\omega G + H \wedge G = \sum Q(\pi_p)$$

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- NSNS sector (F1/NS5-branes source NSNS-fields)

$$D H = Q_H$$

$$(d + \omega) H = \sum Q(v_5)$$

- $\Rightarrow \omega H = \sum Q(v_5)$

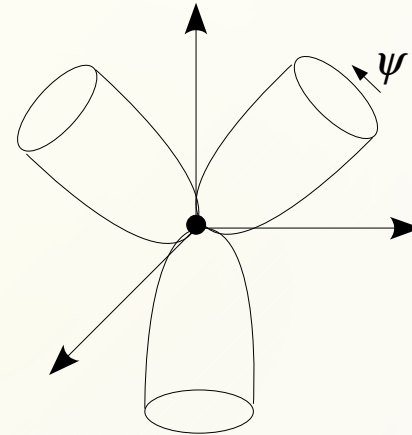
# KK-monopoles source geometrical fluxes

What about the metric?

KK-monopoles are magnetic sources for the metric

$$ds_{TN}^2 = \left(1 + \frac{m}{r}\right) (dr^2 + r^2 d\Omega^2) + \left(1 + \frac{m}{r}\right)^{-1} (d\psi + V^\psi)^2$$

$$V^\psi = m(1 - \cos\theta) d\phi, \quad dV^\psi = m \sin\theta d\theta \wedge d\phi = \omega^\psi$$



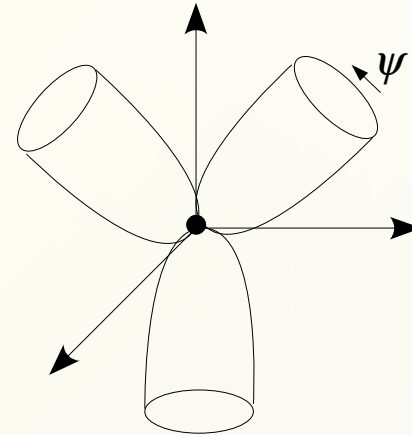
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$d\omega^\psi = [\kappa_5]^\psi \Rightarrow$  KK5 source geometrical fluxes !

Kachru, Schulz, Tripathy, Trivedi 2002

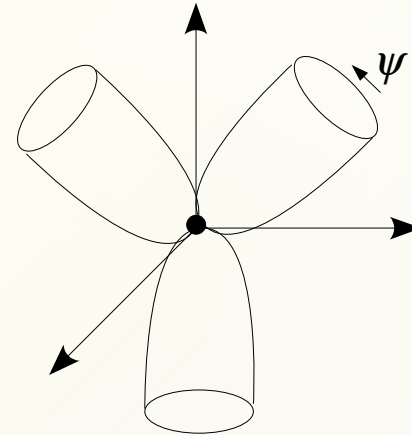
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This is manifest via T-duality:

$$dH_\psi = [v_5]_\psi \xleftarrow{T_\psi} d\omega^\psi = [\kappa_5]^\psi$$

Bianchi, Fucito, Rossi, Martellini '94; Ooguri, Vafa '95  
Bergshoeff, Janssen, Ortin '97

# Beyond Twisted Tori - I

- KK-monopoles on twisted torus

$$d\omega + \omega \omega = [\kappa_5]$$

- after integration...

$$\omega \omega = Q_{\text{KK}}$$

- This means that  $dd \neq 0 \rightarrow$

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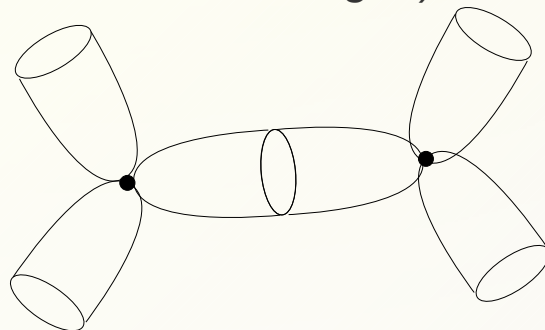
$$\omega\omega = Q_{\text{KK}}$$

- This means that  $dd \neq 0 \rightarrow$

- Is geometrically inconsistent?

- **NO!**

*the total charge of KK5 is also non zero,  
open twisted cycles from the twist close on the KK5 “charge”.  
(As flux line for RR charges)*



# Beyond Twisted Tori - II

● Can be cross-checked via M-theory uplift:

● start from IIA with D6 on twisted tori:

$$d G^{(2)} + \omega G^{(2)} = [\pi_6]$$

● uplift to M-theory:

$$d \omega + \omega \omega = [\kappa_6]$$

● shrink another circle to IIA (9-11 flip):  $d \omega + \omega \omega = [\kappa_5]$

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● Equivalence among KK-monopoles and geometry

● An example: string-string duality chain

	heterotic	$\xrightarrow{S}$	type-I	$\xrightarrow{T_4}$	type-IIB	$\xrightarrow{S}$	type-IIB	$\xrightarrow{T_1}$	type-IIA
on	$T^4$		$T^4$		$T^4/Z'_2$		$T^4/Z''_2$		$T^4/Z_2$
with	$U(1)^{16}$		D9/O9		D5/O5		NS5/NSO5		KK5/KKO5

$\sim K_3$

## • The DKPZ example:

N=4 gauged sugras  $\Leftrightarrow$  type-IIA  $AdS_4$  flux vacua

$f_{ab}^c$ , gauge structure constants  $\Leftrightarrow$   $G, H, \omega$  fluxes

$f_{ab}^c f_{cd}^e = 0$ , Jacobi identities  $\Leftrightarrow$   $\omega G + H \wedge G = 0$ ,  $\omega \omega = 0 \dots$ , BI

## • Non 1-to-1 correspondence between BI and JI

DKPZ consistent gauged SUGRA vacua with  $\omega \omega \neq 0$

## • *We now understand the type-IIA uplift of DKPZ vacua:*

**equivalent to compactifications with non zero KK5 charge!**

# Beyond Twisted Tori - IV

- Further comments
- KK5-moduli  $\Leftrightarrow$  CY/twisted fields
  - trade  $\omega$  with  $Q_{\text{KK}}$  to remove twisted fields...
  - Equivalent to stabilization of twisted sector via Scherk-Schwarz fluxes.

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- KK5 can be added in usual N=1 compactifications without breaking SUSY as D6-branes; new class of compactifications (matching with EFT gauged SUGRAs)
- In terms of the T-dual invariant external derivative the NSNS BI reads
  - $D = d + \omega + H + \dots$
  - with Bianchi id. :  $DD = Q_{\text{NS}} = Q_{\text{H}} + Q_{\text{KK}} + \dots$
- Analogous sources for non-geometrical fluxes ???

# General Constraints on Branes and Fluxes

- General topological constraints from Bianchi identities (RR, NSNS tadpoles)

$$DD=0 \quad D = d + \omega + H \quad \Rightarrow \quad \omega \omega = 0, \omega H = 0$$

$$DG = Q_{RR} \quad Q_{RR} = \sum [\pi] e^{\mathcal{F}} \quad \Rightarrow \quad \omega G + H \wedge G = [\pi] e^{\mathcal{F}}$$

$$0 = DDG = D([\pi] e^{\mathcal{F}}) = 0$$

$$\Rightarrow (d + \omega)[\pi] = 0 \quad \Rightarrow \quad ((d + \omega)\mathcal{F} + H)[\pi] = 0$$

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- $\Rightarrow \mathcal{F} = (d+\omega)A - B$  gauge invariant combination of the DBI

- Integrability  $\Rightarrow H \wedge [\pi] = 0$

$$\Leftrightarrow \int_{\gamma_3 \subset \pi_p} H = 0$$

Equivalent to Freed-Witten anomaly condition (with no  $W_3$ )  
(and twisted K-theory)

- Importance for consistency of EFT
- RR-tadpole/RR-BI:
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- gauge invariance of  $W$ :

Camara, Font, Ibanez '05; GV, Zwirner '06

$$W \supset \int H \wedge C^{(3)}, \quad L = \left( \partial_\mu C^{(3)} + [\pi_6] A_\mu \right)^2 \quad \text{from} \quad \int_{\pi_6} C^{(5)} \wedge F$$

$$\Rightarrow \delta_\lambda C^{(3)} = [\pi_6] \lambda; \quad \delta_\lambda W = \int H \wedge [\pi_6] \lambda = \lambda \int_{\gamma_3 \subset \pi_6} H = 0,$$

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- Instantons:

Kashani-Poor, Tomasiello '05

$$N=2: \quad \text{ED2} \rightarrow \exp \left( \int_{\pi_2} \text{Re} \Omega + i C^{(3)} \right)$$

$$\left( \partial_\mu C^{(3)} + H C_\mu^{(1)} \right) \Rightarrow \delta_\lambda C^{(3)} = H \lambda,$$

$$\delta_\lambda \exp \left( \int_{\pi_2} \text{Re} \Omega + i C^{(3)} \right) \propto \lambda \int_{\pi_2} H = 0$$

# New constraints via dualities

- Are there other constraints?
- Dualities suggest there are:
  - T-duality

$$H \wedge [\pi] = 0 \quad \leftarrow T \rightarrow \quad \omega[\pi] = 0, \quad \partial \pi = 0$$

$$\begin{aligned} (\partial_\mu C^{(4)} + \omega C_\mu^{(4)}) &\Rightarrow \delta_\lambda C^{(4)} = \omega \gamma_3 \lambda \\ \delta_\lambda \left[ \exp\left(\int_{\pi_3} J \wedge J + i C^{(4)}\right) \right] &\propto \int_{\pi_3} \omega \gamma_3 \propto \int_{\partial \pi_3} \gamma_3 = 0 \end{aligned}$$

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S-duality

need also NSNS sources

$\Rightarrow DD = Q_{\text{NS}} \neq 0$

$$DDG = DQ_{\text{RR}} = Q_{\text{NS}} G$$

$$[\nu_5] \wedge G = 0 \quad \Leftrightarrow \quad \int_{\gamma_{p-1} \subset \nu_5} G^{(p)} = 0, \quad (\text{in type-II B S-dual to FW})$$

$$[\kappa_5]^\xi G_\xi = 0 \quad \Leftrightarrow \quad \int_{\gamma_{p-1} \subset \kappa_5} G_\xi^{(p)} = 0$$

# Localized BI from M-theory

## ● S-duality has a geometrical interpretation in M-theory

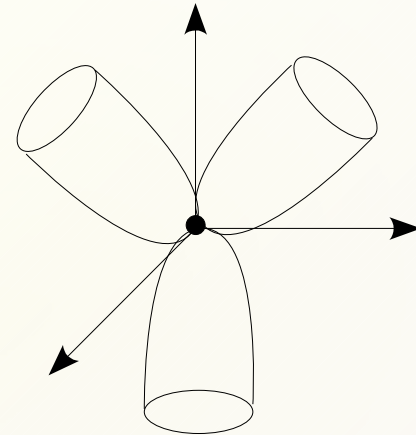
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$$dV^{11} = m \sin\theta d\theta \wedge d\phi = dC^{(1)} = G^{(2)}$$

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$$\xi^{(2)} = d\xi^{(1)} \propto G^{(2)} + \dots = *_4 \xi^{(2)}$$



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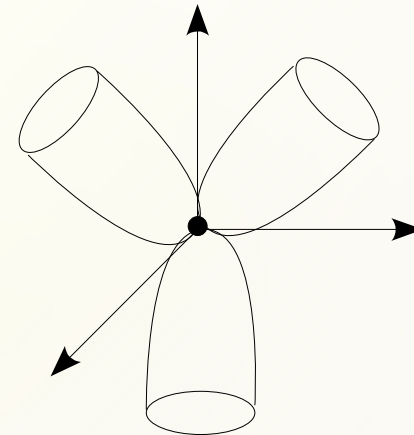
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- In this background a 4-form flux decomposes as:

$$F^{(4)} = G^{(4)} + \xi^{(1)} \wedge H + \xi^{(2)} \wedge \mathcal{F}^{(2)} + \dots$$

and from its BI follows

$$dF^{(4)} = \underbrace{dG^{(4)}}_{=0} - \underbrace{\xi^{(1)} \wedge dH}_{=0} + \xi^{(2)} \wedge \underbrace{(d\mathcal{F}^{(2)} + H)}_{=0} + \dots = 0,$$

RR BI

NSNSN BI

localized BI

# New constraints from M-theory

• S-duality  $\Leftrightarrow$  9-11 flip

- Start from the same background but shrink a circle inside the KK6-monopole (say  $\eta$ )  $\rightarrow$  KK6  $\rightarrow$  KK5

$$F^{(4)} = G^{(4)} + \xi^{(1)} \wedge G_{\xi}^{(4)} + \eta \wedge H + \xi^{(2)} \wedge \mathcal{K}^{(2)} + \dots,$$

The BI for  $F^{(4)}$  now gives

$$dF^{(4)} = dG^{(4)} - \eta \wedge dH - \xi^{(1)} \wedge dG_{\xi}^{(4)} + \xi^{(2)} \wedge (d\mathcal{K}_{\xi}^{(3)} + G_{\xi}^{(4)}) + \dots = 0,$$

=0

*localized BI for KK5-vectors*

• and from the integrability condition

$$[\kappa_5]_{\xi} \overline{G}_{\xi}^{(4)} = 0 \quad \Leftrightarrow \quad \int_{\gamma_3 \subset \kappa_5} G_{\xi}^{(4)} = 0.$$

# General constraints and the duality web

Using further T and S transformations the duality chain closes

$$\begin{array}{ccccccc}
 \int_{\gamma_3 \subset \kappa_6} F_\xi^{(4)} = 0 & \xrightarrow{S^1_{(\xi)}} & \int_{\gamma_3 \subset \pi_6} H = 0 & \xleftrightarrow{T_\eta} & \int_{\gamma_3 \subset \pi_5} H = 0 & & \\
 \text{M} & & \text{IIA} & & \text{IIB} & & \\
 & & & & & & \left. \vphantom{\int_{\gamma_3 \subset \pi_5}} \right) \text{S} \\
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 \end{array}$$

- Analogously starting from M5-branes

$$\begin{array}{ccccc}
 \int_{\gamma_3 \subset \mu_5} F_{11}^{(4)} = 0 & \xrightarrow{S^1} & \int_{\gamma_3 \subset \pi_4} H = 0 & \xleftrightarrow{T_x} & \int_{\gamma_3 \subset \pi_5} H = 0 \\
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 \end{array}$$

# New constraints and EFTs

## • New constraints and EFT

### • Example #1: Type-IIA with NS5-instantons and $G^{(4)}$ -flux

$$dH^{(7)} + \frac{1}{2} G^{(4)} \wedge G^{(4)} = 0, \quad \Rightarrow \quad H_{\mu}^{(7)} = \partial_{\mu} B^{(6)} + \frac{1}{2} G^{(4)} \wedge C_{\mu}^{(3)}, \quad \Rightarrow \quad \delta_{\lambda} B^{(6)} = \frac{1}{2} G^{(4)} \wedge \gamma_2,$$

$$\delta_{\lambda} \left[ \exp \left( \int_{\nu_5} B^{(6)} \right) \right] \propto \int_{\gamma_4 \subset \nu_5} G^{(4)} = 0,$$

### • Example #2: Type-IIB with E3 instantons and $G^{(3)}$ / $H$ fluxes

$$dG^{(5)} + H \wedge G^{(3)} = 0, \quad \Rightarrow \quad G_{\mu}^{(5)} = \partial_{\mu} C^{(4)} + \frac{1}{2} G^{(3)} \wedge B_{\mu} - \frac{1}{2} H \wedge C_{\mu}^{(2)},$$

$$\Rightarrow \quad \delta_{\lambda} C^{(4)} = G^{(3)} \wedge \gamma_1 \lambda_1 - H \wedge \gamma_1 \lambda_2,$$

$$\delta_{\lambda} \left[ \exp \left( i \int_{\pi_3} C^{(4)} \right) \right] \propto \int_{\gamma_3 \subset \pi_3} G^{(3)} \lambda_1 - H \lambda_2 = 0,$$

- Generic case: non-free branes

- MMS-instantons:

Maldacena, Moore, Seiberg '01

A  $Dp$ -brane may support  $N$  units of  $H$  flux if  $N$   $D(p-2)$ -branes end on it; analogously  $N$   $Dp$ -branes on a  $H$ -flux background may be unstable to decay into a  $ED(p+2)$ -brane instanton.

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- The stability condition corresponds to

$$[\pi_p] \neq H \wedge [\pi'_{p+2}]$$

- The FW+MMS conditions correspond to a twisted Kohnomology:

$$\begin{array}{ll}
 \text{FW:} & (d+H)[\pi_p]=0 \quad \Leftrightarrow \quad \partial_H \pi_p=0 \\
 \text{MMS:} & [\pi_p] \neq (d+H)[\pi'_{p+2}] \quad \Leftrightarrow \quad \pi_p \neq \partial_H \pi'_{p+2}
 \end{array}$$

# MMS from M-theory

## • MMS from M-theory and new decay transitions

- Consider the  $F^{(4)}$  BI with M5 sources:

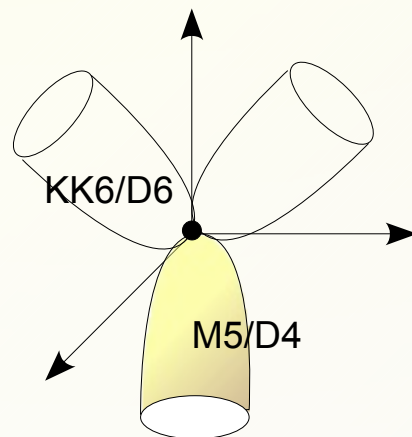
$$dF^{(4)} = [\mu_5]$$

- decomposing the sources

$$[\mu_5] = [\pi_4] - \xi^{(1)} \wedge [\nu_5] + \xi^{(2)} \wedge [\pi_4^\perp] \theta \phi + \dots,$$

we get

$$dG^{(4)} = [\pi_4], \quad dH = [\nu_5], \quad d\mathcal{F} + H = [\pi_4^\perp] \theta \phi.$$



$$\bar{H} = N_{D4}^\perp$$

**M-theory gives the geometrical interpretation for MMS decays:**  
*D4 wrap a trivial cycle and decay*

# New decays from M-theories

- Again 9-11 flip in M-theory gives the S-dual of MMS:

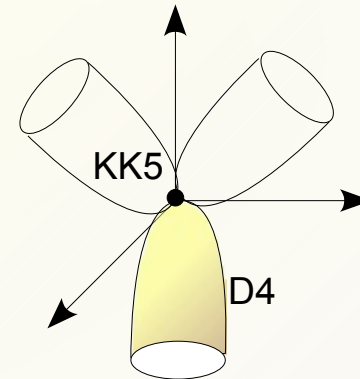
$$dF^{(4)} = [\mu_5]$$

- now  $\eta$  is the 11<sup>th</sup> dimension

$$[\mu_5] = [\pi_4] - \eta \wedge [\nu_5] - \xi^{(1)} \wedge [\pi_4]_\xi + \xi_{(2)} \wedge [\pi_4^\perp]_{\theta\phi} + \dots,$$

and we get:

$$d\mathcal{K}^{(2)} + G_\xi^{(4)} = [\pi_4^\perp]_{\theta\phi}.$$



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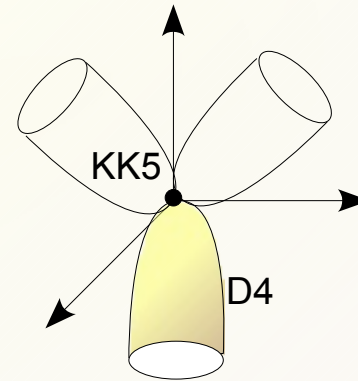
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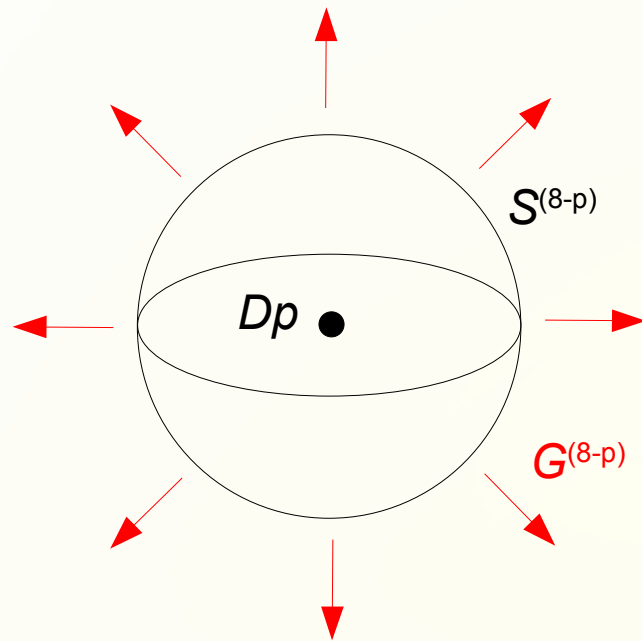
S and T duality web for MMS-like conditions

$dF^{(4)} = [\mu_5]$	$\xrightarrow{S_\xi^1}$	$\int_{\gamma_3 \subset \pi_6} H = N_{D4}^\perp$	$\xleftrightarrow{T_\eta}$	$\int_{\gamma_3 \subset \pi_5} H = N_{D3}^\perp$	) S
M		IIA		IIB	
$dF^{(4)} = [\mu_5]$	$\xrightarrow{S_\eta^1}$	$\int_{\gamma_3 \subset \kappa_5} G_\xi^{(4)} = N_{D4}^\perp$	$\xleftrightarrow{T_\xi}$	$\int_{\gamma_3 \subset \nu_5} G^{(3)} = N_{D3}^\perp$	

# The general proof - I

● The general proof:

● A  $Dp$ -brane magnetic source for RR  $G^{(8-p)}$  flux



● Consider BI for  $G^{(10-p)}$  RR forms:

$$dG^{(10-p)} + H \wedge G^{(8-p)} = \sum_q Q_{(p-2)}(\pi_q),$$

● and integrate it over  $S^{(8-p)} \times \gamma_3$ ,  
where  $\gamma_3 \subset \pi_6$

we get

$$\int_{S^{(8-p)} \times \gamma_3} dG^{(10-p)} + H \wedge G^{(8-p)} = N_{Dp} \int_{\gamma_3} H = \sum \int_{S^{(8-p)} \times \gamma_3} [\pi_{p-2}] = N_{(\pi_{p-2}) \cap (S^{(8-p)} \times \gamma_3)} = N_{D(p-2)}^\perp$$

$\gamma_3$	$\times$	$\mathbb{R}^{p-2}$	$\times$	$\mathbb{R}$	$\times$	$S^{(8-p)}$
$Dp$		$Dp$				$G^{(8-p)}$
$H$		$D(p-2)$		$D(p-2)$		

# The general proof - II

• Analogously the S-dual constraints:

- for example, an NS5-brane sources one unit of H flux over the  $S^3$  surrounding the brane
- consider again the BI for RR  $G^{(8-p)}$  forms:

$$dG^{(8-p)} + H \wedge G^{(6-p)} = \sum_q Q_p(\pi_q)$$

- and integrate over  $S^3 \times \gamma_{(6-p)}$ , where  $\gamma_{(6-p)} \subset v_5$

we get

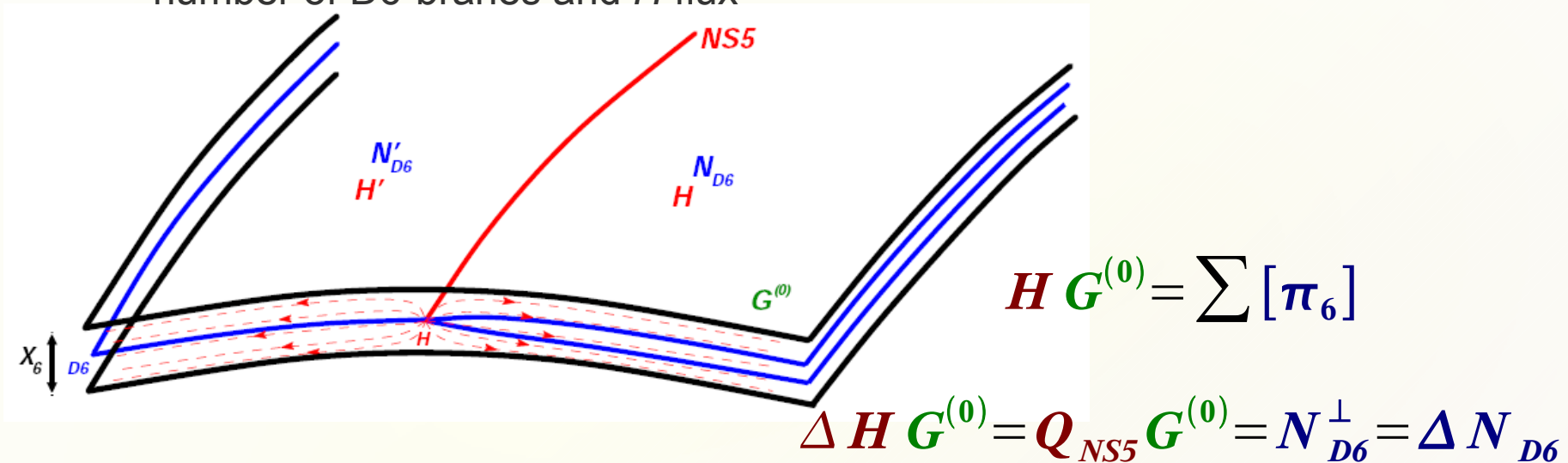
$$\int_{S^3 \times \gamma_{6-p}} dG^{(8-p)} + H \wedge G^{(6-p)} = N_{NS5} \int_{\gamma_{6-p}} G^{(6-p)} = \sum_q \int_{S^3 \times \gamma_{6-p}} [\pi_p] = N_{Dp}^\perp$$

$$N_{NS5} \int_{\gamma_{6-p}} G^{(6-p)} = N_{Dp}^\perp$$

- Analogously for with SS fluxes, KK-monopoles, etc...

# Interpolation between vacua

- MMS-like conditions fundamental to ensure BI in interpolating solutions
  - Example: NS5 interpolating between two massive IIA vacua with different number of D6-branes and  $H$  flux



- Analogously in type-IIB: e.g. NS5/D5 interpolate between IIB vacua with different  $G^{(3)}$  and  $H$  flux and number of D3-branes

$$H \wedge G^{(3)} = \sum [\pi_3] \quad \text{Kachru, Pearson, Verlinde '01}$$

$$\Delta H \wedge G^{(3)} = Q_{NS5} \wedge G^{(3)} = N_{D3}^\perp = \Delta N_{D3}$$

# Conclusions & Outlook

## ● KK-monopole compactifications:

- going beyond twisted tori
  - (uplift of some gauged SUGRAs)
- interpretation as geometry and duality with flux/brane setups

## ● Localized BI

- generalization of FW and MMS constraints
  - embedding in S and T duality chains and M-theory uplift
  - geometrical interpretation
- new source of vacuum instability and interpolation of vacua
- constraints for EFT (instanton contributions, anomalies...)

## ● Outlook

- Compactifications with KK6: stability, EFT, phenomenology....
- Study of vacuum stability and interpolating solutions
- (generalized)-MMS configurations and effects on EFT (instantons...)
- ...