Instantons in N=2 D=4 string compactifications

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Introduction and Motivation

- Aim: to understand better nonperturbative aspects of string theory compactifications.
- Results: exact formulae for a series of instanton corrections, to ALL ORDER in the string coupling constant g_s.
- Models: Type II strings on Calabi-Yau threefolds, no fluxes (yet …).
- Conifold limit: stringy instantons in field theory.

Type II on CY

- Four-dimensional action is N=2 D=4 SUGRA coupled to vector- and hypermultiplets.
- The four-dimensional dilaton is part of a hypermultiplet, so the hypermultiplet moduli space receives quantum corrections.

Nonperturbative strings

- Instantons: wrapped Euclidean Dp-branes over p+1 cycles (Becker,Becker, Strominger, ‘95).
- IIA: D2 over three-cycles

  NS5 over CY

  IIB: D(-1),D1,D3,D5,NS5 over points, two-cycles, four-cycles, CY.

Type IIA on CY

Vector multiplets: special Kahler, holomorphic prepotential F(t):

$$F = F_{cl} + F_{pert} + F_{ws}$$

$$= \frac{1}{3!} d_{abc} t^a t^b t^c + \frac{i}{2(2\pi)^3} \zeta(3) \chi_E - \frac{i}{(2\pi)^3} \sum_{\{k_a\}} n_{k_a} Li_3(e^{2\pi i k_a t^a})$$

Worldsheet instantons: F1 over two-cycles.

Kahler moduli $t^a ; a = 1, \ldots, h^{1,1}$

Hypermultiplets

Hypermultiplets: quaternion-Kahler, also determined by a single function $\chi(\phi)$

$$g_{AB}(\phi) = D_A \partial_B \chi(\phi)$$

called hyperkahler potential (de Wit, Kleijn, S.V., 1999; de Wit, Rocek, S.V., 2000).

Complex coordinates:

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} \chi \qquad D_i \partial_j \chi = 0$$

Quantum corrections

- All quantum corrections encoded in a single function:

$$\chi = \chi_{cl} + \chi_{pert} + \chi_{inst}$$

- This function is invariant under symmetries of string theory, e.g. Sl(2,Z) of IIB.

C-map

Using T-duality (c-map), vectors and hypers are exchanged, and

$$\chi^{IIB} \simeq 4\tau_2^2 \, \mathrm{Im}[F_0(t) + \overline{t}^{\,\overline{a}} F_a(t)] + \frac{\chi_E}{24\pi}$$

Includes classical, perturbative (α' and $\tau_2 = g_s^{-1}$), and worldsheet instantons. (M.Rocek,C.Vafa,S.V.,'05; D.Robles-Llana, F.Saueressig,S.V.,'06; B. de Wit and F.Saueressig, '06)

SL(2,Z)

- This result is not SL(2,Z) invariant, as required by IIB. This is because D1 instantons are left out, and also D(-1) :

$$\tau = \tau_1 + i\tau_2 \to \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} F1 \\ D1 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F1 \\ D1 \end{pmatrix}$$

D(-1) and D1 instantons

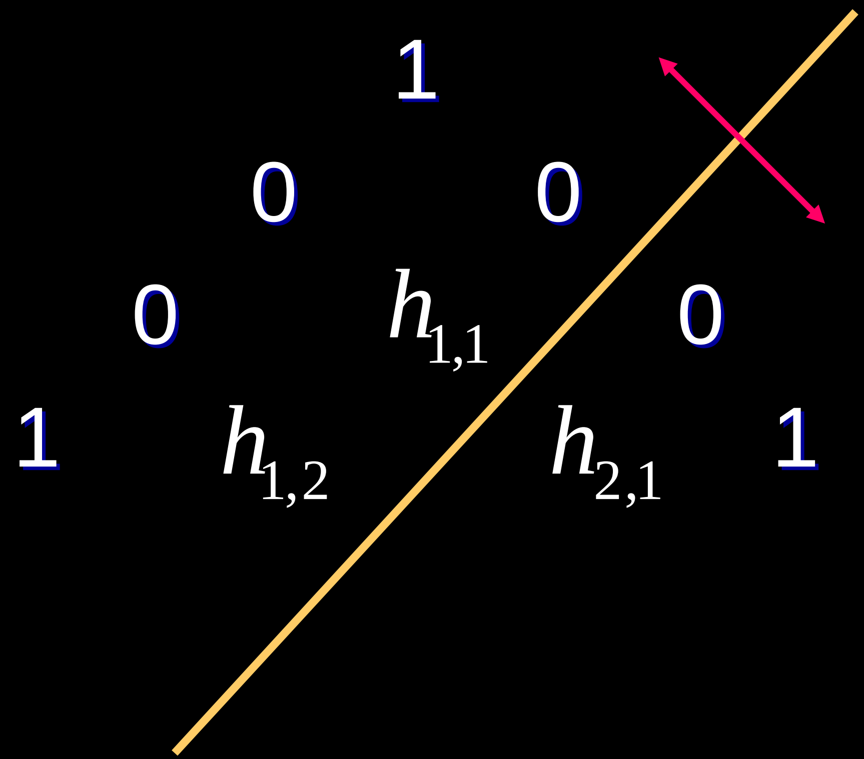
Applying the method of images:

$$\chi_{D(-1)} \simeq \frac{\tau_2}{2\pi^2} \sum_{m\neq 0, n>0} \frac{\chi_E}{2} \left|\frac{m}{n}\right| e^{2\pi i m n \tau_1} K_1(2\pi |mn| \tau_2)$$

$$\chi_{D1} \simeq -\frac{\tau_2}{2\pi^2} \sum_{\{k_a, m, n\}} n_{k_a} \left|\frac{k_a t^a + n}{m}\right| e^{2\pi i m(c-\tau_1(b+n))} K_1(2\pi |m(n+t)| \tau_2)$$

Sum over multi-instantons, RR scalars $c = k_a c^a$ and b arise from Sl(2,Z) completion.

Mirror symmetry



Exchanges two and three-cycles, IIA and IIB

Mirror symmetry

D1 instantons mapped to membrane instantons (over A-cycles)

$$b_1 = 0 \qquad S^2 \leftrightarrow S^3$$

D(-1) instanton mapped to membrane instanton corresponding to

$$h_{3,0} = 1$$

consistent with Strominger,Yau, Zaslow,'96

Mirror map

Relate IIA and IIB variables:

$$\phi_{IIA} = \phi_{IIB}$$

$$A^a = -(c^a - \tau_1 b^a)$$

$$A^1 = \tau_1$$

$$A^\Lambda = \{A^1, A^a\} = \int_{\gamma_3^\Lambda} C_3$$

Checks: classically ok, produces correct instanton actions, conifold limit ok (below).

(Marino,Minasian,Moore,Strominger,'99)

Membrane instantons

D2 instanton (half) contribution to the hypermultiplet moduli space :

$$\chi_{D2}^{IIA} \simeq -\frac{\tau_2}{2\pi^2} \sum_{k_\Lambda, m} n_{k_\Lambda} \left| \frac{k_\Lambda t^\Lambda}{m} \right| e^{-2\pi i m k_\Lambda A^\Lambda} K_1(2\pi\tau_2 \,|\, m k_\Lambda t^\Lambda \,|)$$

$$n_{(n,k_a=0)} = \frac{\chi_E}{2} \qquad k_\Lambda = (n, k_a)$$

Exact result to all orders in string coupling !

Conifold limit

In IIA, we can take the volume of a three-cycle and the string coupling constant to zero,

$$t^* \to 0 \qquad\qquad g_s \to 0$$

with fixed ratio such that instantons survive:

$$\frac{|t^*|}{g_s} = fixed$$

Conifold limit

In this limit, gravity decouples and the resulting moduli space is four-dimensional hyperkahler (Gibbons-Hawking):

$$ds^2 = V^{-1}(da - \vec{A}\cdot d\vec{y})^2 + V d\vec{y}^2$$

Coordinates $\vec{y} = (b, t, \bar{t})$ with a and b the RR scalars and $t = t^* / g_s$ the vanishing three-cycle

Resolving singularities

Metric determined by single function:

$$V = -\frac{1}{4\pi}\ln(t\bar{t}) + \frac{1}{2\pi}\sum_{m\neq 0} K_0(2\pi|mt|)e^{2\pi imb}$$

Singularity at $t=0$ is resolved by instantons (after Poisson resummation). This result proves the conjecture by Ooguri and Vafa,'96 !

Field theory ?

Is this the Higgs branch of some N=2 D=4 field theory ? Typically in SYM,

$$S_{inst} = \frac{8\pi}{g_{YM}^2}|k| + i\theta k$$

No field dependence in instanton action (however, constrained instantons …)

Exotic stringy instanton interpretation ?

Cosmic strings ? (Greene, Morrison, Vafa, Yau,'96)

Conclusions

- IIB: determined all D(-1),F1,D1 instanton corrections to hypermultiplet moduli space
- IIA: determined half of the D2 instanton corrections (A-cycles)
- Field theory limit agrees with Ooguri-Vafa
- Left to do: D2 over B-cycles (e.m. duality)

  NS5-branes