Gauge thresholds and Kähler metrics for toroidal intersecting D-brane models

Maximilian Schmidt-Sommerfeld

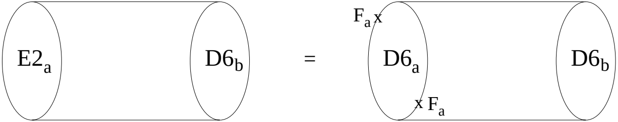
Workshop on recent developments in string effective action and D-instantons
M_{Pl}-Institut fuer Physik Muenchen

14.-16. November 2007

- "Thresholds for intersecting D-branes revisited"
  (Akerblom, Blumenhagen, Lüst, MSS), Phys. Lett. B652, 2007, 53-59, arXiv:0705.2150 [hep-th]
- "Instantons and Holomorphic Couplings in Intersecting D-brane Models"
  (Akerblom, Blumenhagen, Lüst, MSS), JHEP 08, 2007, 044, arXiv:0705.2366 [hep-th]
- "Gauge Thresholds and Kähler Metrics for Rigid Intersecting D-brane Models"
  (Blumenhagen, MSS), arXiv:0711.0866 [hep-th]

Why thresholds ?

- Relation between thresholds and instanton amplitude:



(Abel, Goodsell; Akerblom, Blumenhagen, Lüst, Plauschinn, MSS)

- Holomorphy of W_{np} and f_{1-loop} ?
- Explicit computation for intersecting brane models on $T^6/\mathbb{Z}_2 \times \mathbb{Z}'_2$

(Lüst, Stieberger; Akerblom, Blumenhagen, Lüst, MSS; Billo, Frau, Pesando, Di Vecchia, Lerda, Marotta; Blumenhagen, MSS)

- Use Kaplunovsky-Louis formula to extract holomorphic parts and Kähler metrics

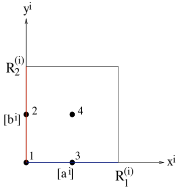
Setup

Type IIA orientifold on $T^2 \times T^2 \times T^2/\mathbb{Z}_2 \times \mathbb{Z}'_2$ (Blumenhagen, Cvetič, Marchesano, Shiu)

Rigid cycles

$\Rightarrow$ Asymptotically free gauge theories

$\Rightarrow$ Instantons with "correct" zero mode structure



- Hodge numbers $(h_{11}, h_{21}) = (3{,}51)$
- Branes charged under twisted sectors
- Branes wrap 1 cycle $n^i[a^i] + m^i[b^i]$ on the i'th torus

Full cycle including twisted sector charges:

$$\Pi = \frac{1}{4}\Pi^B + \frac{1}{2}\sum_i \sum_{k,l} \epsilon^i_{kl}\left(n^i[e^i_{kl}] \otimes [a^i] + m^i[e^i_{kl}] \otimes [b^i]\right)$$

Gauge threshold corrections

- Background field method: (Bachas, Fabre; Antoniadis, Bachas, Dudas)
  Start with partition functions, replace

$$\vartheta_{\alpha\beta}(0)/\eta^3 \rightarrow B\ \vartheta_{\alpha\beta}(-\epsilon_a)/\vartheta_1(-\epsilon_a)$$

Gauge threshold corrections

- Background field method: (Bachas, Fabre; Antoniadis, Bachas, Dudas)
  Start with partition functions, replace

$$\vartheta_{\alpha\beta}(0)/\eta^3 \rightarrow B\ \vartheta_{\alpha\beta}(-\epsilon_a)/\vartheta_1(-\epsilon_a)$$

and extract term quadratic in B:

$$\mathcal{A}_{ab}^{g(1)} \sim \int_0^\infty dl \sum_i \sigma_{ab}^i \widetilde{L}_{ab}^{(i)}$$

Gauge threshold corrections

- Background field method: (Bachas, Fabre; Antoniadis, Bachas, Dudas)
  Start with partition functions, replace

$$\vartheta_{\alpha\beta}(0)/\eta^3 \rightarrow B\ \vartheta_{\alpha\beta}(-\epsilon_a)/\vartheta_1(-\epsilon_a)$$

and extract term quadratic in B:

$$\mathcal{A}^{g(1)}_{ab} \sim \int_0^\infty dl \sum\nolimits_i \sigma^i_{ab} \widetilde{L}^{(i)}_{ab}$$

$$\mathcal{A}^{g(2)}_{ab} \sim \int_0^\infty dl \Big[\prod_{i=1}^{3} I^i_{ab} \sum_{i=1}^{3} \frac{\vartheta_1'}{\vartheta_1}(\theta^i_{ab}, 2il) + \sum_{i\neq j\neq k} I^i_{ab}\sigma^i_{ab} \left(\frac{\vartheta_1'}{\vartheta_1}(\theta^i_{ab}, 2il) + \frac{\vartheta_4'}{\vartheta_4}(\theta^j_{ab}, 2il) + \frac{\vartheta_4'}{\vartheta_4}(\theta^k_{ab}, 2il) \right) \Big]$$

- Divergences for $l \rightarrow \infty$ cancel using tadpole constraint

Holomorphic gauge kinetic function

- KL-formula (Kaplunovsky, Louis)

$$\frac{8\pi^2}{g_a^2(\mu^2)} \quad = \quad 8\pi^2\,\Re(f_a) + \Delta_0 + \sum_r \Delta_r = \frac{8\pi^2}{g_{a,tree}^2} + \sum_b \mathcal{A}_{ab}^g$$

Holomorphic gauge kinetic function

- KL-formula (Kaplunovsky, Louis)

$$\frac{8\pi^2}{g_a^2(\mu^2)} = 8\pi^2\,\Re(f_a) + \Delta_0 + \sum_r \Delta_r = \frac{8\pi^2}{g_{a,tree}^2} + \sum_b \mathcal{A}_{ab}^g$$

Holomorphic gauge kinetic function

- KL-formula (Kaplunovsky, Louis)

$$\frac{8\pi^2}{g_a^2(\mu^2)} = 8\pi^2\,\Re(f_a) + \Delta_0 + \sum_r \Delta_r = \frac{8\pi^2}{g_{a,tree}^2} + \sum_b \mathcal{A}_{ab}^g$$

$$\Delta_0 = T(G_a)\left(-\frac{3}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right] - \frac{1}{2}\mathcal{K} + \ln\left[\frac{1}{g_a^2(\mu^2)}\right]\right)$$

Holomorphic gauge kinetic function

- KL-formula (Kaplunovsky, Louis)

$$\frac{8\pi^2}{g_a^2(\mu^2)} = 8\pi^2\,\Re(f_a) + \Delta_0 + \sum_r \Delta_r = \frac{8\pi^2}{g_{a,tree}^2} + \sum_b \mathcal{A}^g_{ab}$$

$$\Delta_0 = T(G_a)\left(-\frac{3}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right] - \frac{1}{2}\mathcal{K} + \ln\left[\frac{1}{g_a^2(\mu^2)}\right]\right)$$

$$\Delta_r = T_a(r)\left(\frac{n_r}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{n_r}{2}\mathcal{K} - \ln\left[\det K_r(\mu^2)\right]\right)$$

Holomorphic gauge kinetic function

- KL-formula (Kaplunovsky, Louis)

$$\frac{8\pi^2}{g_a^2(\mu^2)} = 8\pi^2\,\Re(f_a) + \Delta_0 + \sum_r \Delta_r = \frac{8\pi^2}{g_{a,tree}^2} + \sum_b \mathcal{A}_{ab}^g$$

$$\Delta_0 = T(G_a)\left(-\frac{3}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right] - \frac{1}{2}\mathcal{K} + \ln\left[\frac{1}{g_a^2(\mu^2)}\right]\right)$$

$$\Delta_r = T_a(r)\left(\frac{n_r}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{n_r}{2}\mathcal{K} - \ln\left[\det K_r(\mu^2)\right]\right)$$

- Compare Δ_0 with

$$\mathcal{A}'^{g(1)}_{aa} \propto 3\ln\left[\frac{M_s^2}{\mu^2}\right] - \sum_i \ln\left[(V_a^i)^2\right] - \sum_i 4\ln\left[\eta(iR_1^{(i)}R_2^{(i)})\right]$$

Holomorphic gauge kinetic function

- KL-formula (Kaplunovsky, Louis)

$$\frac{8\pi^2}{g_a^2(\mu^2)} = 8\pi^2\,\Re(f_a) + \Delta_0 + \sum_r \Delta_r = \frac{8\pi^2}{g_{a,tree}^2} + \sum_b \mathcal{A}_{ab}^g$$

$$\Delta_0 = T(G_a)\left(-\frac{3}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right] - \frac{1}{2}\mathcal{K} + \ln\left[\frac{1}{g_a^2(\mu^2)}\right]\right)$$

$$\Delta_r = T_a(r)\left(\frac{n_r}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right] + \frac{n_r}{2}\mathcal{K} - \ln\left[\det K_r(\mu^2)\right]\right)$$

- Compare Δ_0 with

$$\mathcal{A}'^{g(1)}_{aa} \propto 3\ln\left[\frac{M_s^2}{\mu^2}\right] - \sum_i \ln\left[(V_a^i)^2\right] - \sum_i 4\ln\left[\eta(iR_1^{(i)}R_2^{(i)})\right]$$

- Conclude: One-loop correction to gauge kinetic function

$$\delta_a f_a^{1-loop} = \frac{N_a}{4\pi^2}\sum_{i=1}^{3}\ln\left[\eta(iT_i^c)\right],$$

Holomorphic gauge kinetic function

GT from sector yielding chiral bifundamentals

$$\mathcal{A}'^{g(2)}_{ab} \sim \Upsilon_{ab} \left(\ln \left[\frac{M_s^2}{\mu^2} \right] + \ln \left[\prod_{k=1}^{3} \left(\frac{\Gamma(1-|\theta^k_{ab}|)}{\Gamma(|\theta^k_{ab}|)} \right)^{\mathrm{sgn}(\theta^k_{ab})} \right] \right)$$

Holomorphic gauge kinetic function

GT from sector yielding chiral bifundamentals

$$\mathcal{A}'^{g(2)}_{ab} \sim \Upsilon_{ab} \left(\ln \left[\frac{M_s^2}{\mu^2} \right] + \ln \left[\prod_{k=1}^{3} \left(\frac{\Gamma(1 - |\theta^k_{ab}|)}{\Gamma(|\theta^k_{ab}|)} \right)^{\mathrm{sgn}(\theta^k_{ab})} \right] \right)$$

are cancelled in the KL-formula if the Kähler metric is

$$K^{(1)}_{ab} = (SU_1 U_2 U_3)^{-\frac{1}{4}} (T_1 T_2 T_3)^{-\frac{1}{2}} \prod_{i=1}^{3} \left(\frac{\Gamma(1 - |\theta^i_{ab}|)}{\Gamma(|\theta^i_{ab}|)} \right)^{-\mathrm{sgn}(\theta^i_{ab})/2}$$

Universal gauge threshold corrections

Consider an additional factor in the Kähler metric:

$$K_{ab}^{(2)} = \prod_{i=1}^{3} U_i^{-\left(\xi\ \mathrm{sgn}(\Upsilon_{ab})\ \theta^i_{ab}\right)} \times T_i^{-\left(\zeta\ \mathrm{sgn}(\Upsilon_{ab})\ \theta^i_{ab}\right)}$$

Universal gauge threshold corrections

Consider an additional factor in the Kähler metric:

$$K_{ab}^{(2)} = \prod_{i=1}^{3} U_i^{-\left(\xi \ \mathrm{sgn}(\Upsilon_{ab}) \ \theta^i_{ab}\right)} \times T_i^{-\left(\zeta \ \mathrm{sgn}(\Upsilon_{ab}) \ \theta^i_{ab}\right)}$$

this leads to terms on the FTS of the KL-formula

$$n_a^1 n_a^2 n_a^3 \left[\sum\nolimits_b N_b \widetilde{m}_b^1 \widetilde{m}_b^2 \widetilde{m}_b^3 \sum\nolimits_{l=1}^3 \theta_b^l \left(\xi \ln U_l + \zeta \ln T_l\right)\right]$$

$$\sum_{i;k,l} n_a^i \left(\epsilon^i_{a,kl} \pm \epsilon^i_{a,R(k)R(l)}\right) \sum_b N_b \widetilde{m}_b^i \epsilon^i_{b,kl} \sum_j \theta_b^j (\xi \ln U_j + \zeta \ln T_j)$$

Universal gauge threshold corrections

Consider an additional factor in the Kähler metric:

$$K^{(2)}_{ab} = \prod_{i=1}^{3} U_i^{-\left(\xi\ \mathrm{sgn}(\Upsilon_{ab})\ \theta^i_{ab}\right)} \times T_i^{-\left(\zeta\ \mathrm{sgn}(\Upsilon_{ab})\ \theta^i_{ab}\right)}$$

this leads to terms on the FTS of the KL-formula

$$n^1_a n^2_a n^3_a \left[\sum\nolimits_b N_b \widetilde{m}^1_b \widetilde{m}^2_b \widetilde{m}^3_b \sum\nolimits_{I=1}^{3} \theta^I_b \left(\xi \ln U_I + \zeta \ln T_I\right)\right]$$

$$\sum_{i;k,l} n^i_a \left(\epsilon^i_{a,kl} \pm \epsilon^i_{a,R(k)R(l)}\right) \sum_b N_b \widetilde{m}^i_b \epsilon^i_{b,kl} \sum_j \theta^j_b (\xi \ln U_j + \zeta \ln T_j)$$

Compare with the tree level gauge kinetic function

$$f_{a,tree} \sim S^c n^1_a n^2_a n^3_a + \sum_{i=1}^{3} \sum_{k,l=1}^{4} n^i_a \left(\epsilon^i_{a,kl} \pm \epsilon^i_{a,R(k)R(l)}\right) W^c_{ikl}$$

Universal gauge threshold corrections

Conclude: Dilaton and cmplx structure moduli shifted at 1-loop

$$S \to S - \frac{1}{32\pi^2}\sum_b N_b \widetilde{m}_b^1 \widetilde{m}_b^2 \widetilde{m}_b^3 \left(\sum_I \theta_b^I(\xi \ln U_I + \zeta \ln T_I)\right)$$

Universal gauge threshold corrections

Conclude: Dilaton and cmplx structure moduli shifted at 1-loop

$$S \to S - \frac{1}{32\pi^2} \sum_b N_b \widetilde{m}_b^1 \widetilde{m}_b^2 \widetilde{m}_b^3 \left(\sum_I \theta_b^I (\xi \ln U_I + \zeta \ln T_I) \right)$$

$$\delta^{(1)} W_{ikl} = -\frac{1}{64\pi^2} \sum_b N_b \widetilde{m}_b^i \epsilon_{b,kl}^i \sum_j \theta_b^j (\xi \ln U_j + \zeta \ln T_j)$$

Universal gauge threshold corrections

Conclude: Dilaton and cmplx structure moduli shifted at 1-loop

$$S \to S - \frac{1}{32\pi^2} \sum_b N_b \widetilde{m}_b^1 \widetilde{m}_b^2 \widetilde{m}_b^3 \left(\sum_I \theta_b^I (\xi \ln U_I + \zeta \ln T_I) \right)$$

$$\delta^{(1)} W_{ikl} = -\frac{1}{64\pi^2} \sum_b N_b \widetilde{m}_b^i \epsilon_{b,kl}^i \sum_j \theta_b^j (\xi \ln U_j + \zeta \ln T_j)$$

There is a bit more ...

$$\mathcal{A}_{ab}^{g(2)} \supset \sum_i I_{ab}^i \, (\theta_a^i - \theta_b^i) \, \sigma_{ab}^i$$

Universal gauge threshold corrections

Conclude: Dilaton and cmplx structure moduli shifted at 1-loop

$$S \to S - \frac{1}{32\pi^2}\sum_b N_b \widetilde{m}_b^1 \widetilde{m}_b^2 \widetilde{m}_b^3 \left(\sum_I \theta_b^I (\xi \ln U_I + \zeta \ln T_I)\right)$$

$$\delta^{(1)} W_{ikl} = -\frac{1}{64\pi^2}\sum_b N_b \widetilde{m}_b^i \epsilon_{b,kl}^i \sum_j \theta_b^j (\xi \ln U_j + \zeta \ln T_j)$$

There is a bit more ...

$$\mathcal{A}_{ab}^{g(2)} \supset \sum_i I_{ab}^i \, (\theta_a^i - \theta_b^i) \, \sigma_{ab}^i$$

summing over all branes this can be cast into

$$\sum_{i;k,l} n_a^i (\epsilon_{a,kl}^i \pm \epsilon_{a,R(k)R(l)}^i) \sum_b N_b \widetilde{m}_b^i \theta_b^i \epsilon_{b,kl}^i$$

Universal gauge threshold corrections

Conclude: Dilaton and cmplx structure moduli shifted at 1-loop

$$S \to S - \frac{1}{32\pi^2} \sum_b N_b \widetilde{m}_b^1 \widetilde{m}_b^2 \widetilde{m}_b^3 \left(\sum_I \theta_b^I (\xi \ln U_I + \zeta \ln T_I) \right)$$

$$\delta^{(1)} W_{ikl} = -\frac{1}{64\pi^2} \sum_b N_b \widetilde{m}_b^i \epsilon_{b,kl}^i \sum_j \theta_b^j (\xi \ln U_j + \zeta \ln T_j)$$

There is a bit more ...

$$\mathcal{A}_{ab}^{g(2)} \supset \sum_i I_{ab}^i \, (\theta_a^i - \theta_b^i) \, \sigma_{ab}^i$$

summing over all branes this can be cast into

$$\sum_{i;k,l} n_a^i (\epsilon_{a,kl}^i \pm \epsilon_{a,R(k)R(l)}^i) \sum_b N_b \widetilde{m}_b^i \theta_b^i \epsilon_{b,kl}^i$$

... another contribution to the shift:

$$\delta^{(2)} W_{ikl} = -\frac{1}{32\pi^2} \sum_b N_b \widetilde{m}_b^i \epsilon_{b,kl}^i \theta_b^i \ln[2]$$

Summary

- Gauge threshold corrections for $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orbifold computed (Lüst, Stieberger; Akerblom, Blumenhagen, Lüst, MSS; Blumenhagen, MSS)
- One-loop corrections to the gauge kinetic function extracted
- Form of Kähler metric determined up to constants (Akerblom, Blumenhagen, Lüst, MSS; Billo, Frau, Pesando, Di Vecchia, Lerda, Marotta; Blumenhagen, MSS)
- "Universal" corrections found leading to redefinition of complex structure moduli
- Instantonic one-loop amplitudes for $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orbifold known