Instantons from Open Strings and D-branes

Alberto Lerda

U.P.O. *"A. Avogadro"* & I.N.F.N. - Alessandria

Munich, November 14, 2007

Foreword

- This talk is mainly based on:
  - R. Argurio, M. Bertolini, G. Ferretti, A. L. and C. Petersson, JHEP **0706** (2007) 067 , arXiv:0704.0262 [hep-th]
  - M. Billo, M. Frau, I. Pesando, P. Di Vecchia, A. L. and R. Marotta, JHEP **0710** (2007) 091 , arXiv:0708.3806 [hep-th]
  - M. Billo, M. Frau, I. Pesando, P. Di Vecchia, A. L. and R. Marotta, arXiv:0709.0245 [hep-th]
- For earlier works see:
  - M. Billo, M. Frau, F. Fucito and A. L., JHEP **0611**, 012 (2006), [hep-th/0606013]
  - M. Billo, M. Frau, I. Pesando, F. Fucito, A. L. and A. Liccardo, JHEP **0302**, 045 (2003), [hep-th/0211250]
  - M. B. Green and M. Gutperle, JHEP **0002** (2000) 014 [hep-th/0002011] + ...
- For recent related works, see other talks of this conference

Plan of the talk

1. Introduction and motivation

Plan of the talk

1. Introduction and motivation

2. Part I : Branes and instantons in flat space and CY singularities
   - $\mathcal{N} = 1$ SQCD

Plan of the talk

1. Introduction and motivation
2. Part I : Branes and instantons in flat space and CY singularities
   - $\mathcal{N} = 1$ SQCD
3. Part II : Wrapped magnetized D-branes and instantons
   - $\mathcal{N} = 2$ SQCD

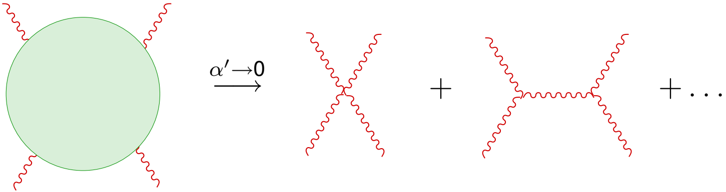
Plan of the talk

1. Introduction and motivation
2. Part I : Branes and instantons in flat space and CY singularities
   - $\mathcal{N} = 1$ SQCD
3. Part II : Wrapped magnetized D-branes and instantons
   - $\mathcal{N} = 2$ SQCD
4. Conclusions

Introduction

String theory is a very powerful tool to analyze field theories, and in particular gauge theories.

Behind this, there is a rather simple and well-known fact: in the field theory limit $\alpha' \to 0$, a single string scattering amplitude reproduces a sum of different Feynman diagrams



String theory S-matrix elements $\Longrightarrow$ vertices and effective actions in field theory

- String theory is useful not only to retrieve perturbative results !

- Using D-branes, also some non-perturbative properties of field theories can be analyzed in string theory, in particular INSTANTONS!

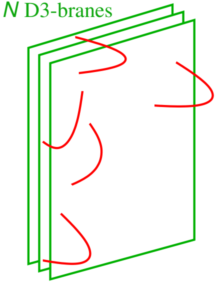
- String theory is useful not only to retrieve perturbative results !
- Using D-branes, also some non-perturbative properties of field theories can be analyzed in string theory, in particular INSTANTONS!
- Instantons are responsible for a particularly tractable class of non perturbative effects in (supersymmetric) gauge theories.
- In string theory instantons arise as (possibly wrapped) Euclidean branes.
- In addition to the usual field theory effects, stringy instantons can generate additional terms in the effective superpotentials or prepotentials, thus providing a rationale for their presence that would otherwise appear rather ad hoc.

Simplest example: branes in flat space

D-branes in flat space

The effective action of a SYM theory can be given a simple and calculable string theory realization using D-branes in flat space:

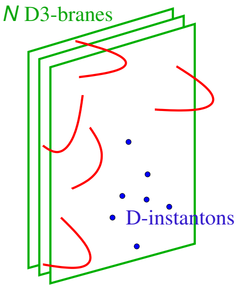
- The gauge degrees of freedom are realized by open strings attached to *N* D3 branes.



D-branes in flat space

The effective action of a SYM theory can be given a simple and calculable string theory realization using D-branes in flat space:

- The gauge degrees of freedom are realized by open strings attached to N D3 branes.

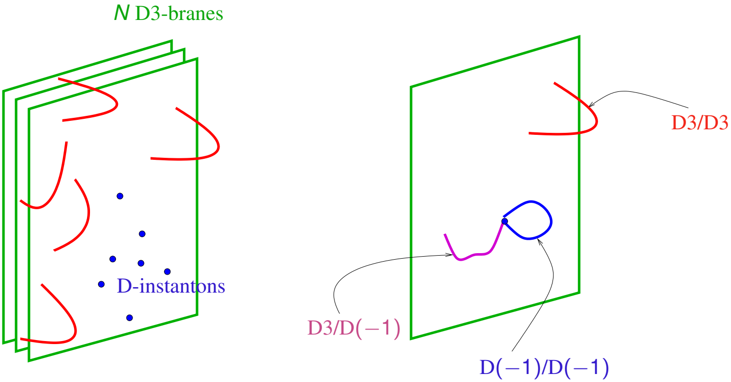


- The instanton sector of charge k is realized by adding k D-instantons.

[Witten 1995, Douglas 1995, Dorey 1999, ...]

Stringy description of gauge instantons

	0	1	2	3	4	5	6	7	8	9
D3	−	−	−	−	*	*	*	*	*	*
D(−1)	*	*	*	*	*	*	*	*	*	*



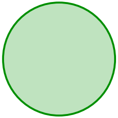
Moduli vertices and instanton parameters

Open strings with at least one end on a D(–1) carry no momentum: they are moduli, rather than dynamical fields.

	ADHM	Meaning	Vertex	Chan-Paton
D(–1)/D(–1) (NS)	a'_μ	*centers*	$\psi^\mu\, \mathrm{e}^{-\varphi}$	adj. U(k)
	χ_a	*aux.*	$\psi^a\, \mathrm{e}^{-\varphi(z)}$	⋮
	D_c	*Lagrange mult.*	$\bar\eta^c_{\mu\nu}\psi^\nu\psi^\mu$	⋮
D(–1)/D(–1) (R)	$M^{\alpha A}$	*partners*	$S_\alpha S_A\, \mathrm{e}^{-\frac{1}{2}\varphi}$	⋮
	$\lambda_{\dot\alpha A}$	*Lagrange mult.*	$S^{\dot\alpha} S^A\, \mathrm{e}^{-\frac{1}{2}\varphi}$	⋮
D(–1)/D3 (NS)	$w_{\dot\alpha}$	*sizes*	$\Delta S^{\dot\alpha}\, \mathrm{e}^{-\varphi}$	$k \times N$
	$\bar w_{\dot\alpha}$	*sizes*	$\overline{\Delta} S^{\dot\alpha}\, \mathrm{e}^{-\varphi}$	$N \times k$
D(–1)/D3 (R)	μ^A	*partners*	$\Delta S_A\, \mathrm{e}^{-\frac{1}{2}\varphi}$	$k \times N$
	$\bar\mu^A$	⋮	$\overline{\Delta} S_A\, \mathrm{e}^{-\frac{1}{2}\varphi}$	$N \times k$

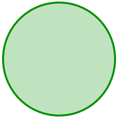
Disk amplitudes and effective actions

D3 disks

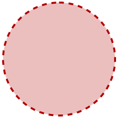


Disk amplitudes and effective actions

D3 disks

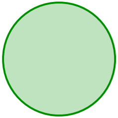


D(–1) disks

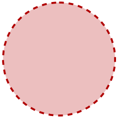


Disk amplitudes and effective actions

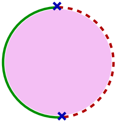
D3 disks



D(–1) disks

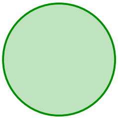


Mixed disks

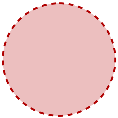


Disk amplitudes and effective actions

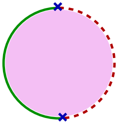
D3 disks

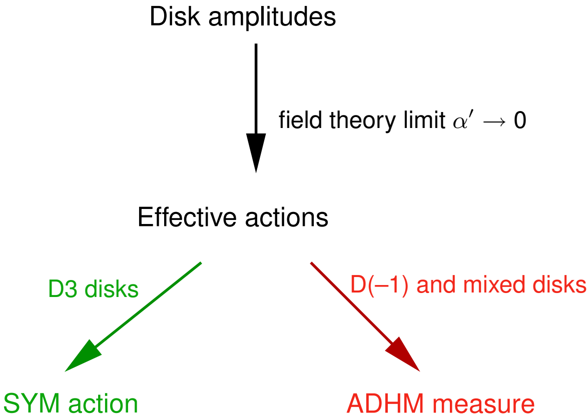


D(–1) disks



Mixed disks





Collecting all diagrams D(–1) and mixed disk diagrams with insertion of all moduli vertices, we can extract the instanton moduli action

$$S_1 = \mathrm{tr}\Big\{ -[a_\mu,\chi^a]^2 - \frac{i}{4}M^{\alpha A}[\chi_{AB},M^B_\alpha] + \chi^a\bar{w}_{\dot\alpha}w^{\dot\alpha}\chi_a + \frac{i}{2}\bar\mu^A\mu^B\chi_{AB}$$
$$- iD^c\left(\bar{w}^{\dot\alpha}(\tau^c)_{\dot\alpha}^{\ \dot\beta}w_{\dot\beta} + i\bar\eta^c_{\mu\nu}[a^\mu,a^\nu]\right)$$
$$+ i\lambda^{\dot\alpha}_A\left(\bar\mu^A w_{\dot\alpha} + \bar{w}_{\dot\alpha}\mu^A + \sigma^\mu_{\beta\dot\alpha}[M^{\beta A},a_\mu]\right)\Big\}$$

where $\chi_{AB} = \chi_a(\Sigma^a)_{AB}$.

- S_1 is just a gauge theory action dimensionally reduced to $d = 0$ in the ADHM limit.
- The last two lines in S_1 correspond to the bosonic and fermionic ADHM constraints.

The interactions of the charged moduli and the scalar fields X^a of the gauge theory are described by

$$
\begin{aligned}
S_2 &= \mathrm{tr}\Big\{\bar{w}_{\dot\alpha} X^a X_a w^{\dot\alpha} + \frac{i}{2}(\Sigma^a)_{AB}\bar\mu^A X_a\,\mu^B\Big\} \\
&= \mathrm{tr}\Big\{\frac{1}{8}\,\epsilon^{ABCD}\,\bar{w}_{\dot\alpha} X_{AB} X_{CD}\, w^{\dot\alpha} + \frac{i}{2}\,\bar\mu^A X_{AB}\,\mu^B\Big\} \\
&= \mathrm{tr}\Big\{\frac{1}{2}\,\bar{w}_{\dot\alpha}\,\{\Phi^i,\Phi_i^\dagger\}\,w^{\dot\alpha} + \frac{i}{2}\,\bar\mu^i\Phi_i^\dagger\mu^4 - \frac{i}{2}\,\bar\mu^4\Phi_i^\dagger\mu^i - \frac{i}{2}\,\epsilon_{ijk}\bar\mu^i\Phi^j\mu^k\Big\}
\end{aligned}
$$

where $X_{AB} = X_a\,(\Sigma^a)_{AB}$ in the **6** of $\mathrm{SU}(4)$, or in $\mathrm{SU}(3)$ notation (which will be the most convenient)

$$
\Phi^i \equiv \frac{1}{2}\epsilon^{ijk}X_{jk}\,, \qquad \Phi_i^\dagger \equiv X_{i4}
$$

An instanton contribution is given by the integral over ALL MODULI of the total action $S_{mod} \equiv S_1 + S_2$:

$$Z \propto \int d\{a, \chi, M, \lambda, D, w, \bar{w}, \mu, \bar{\mu}\}\, e^{-S_{mod}}$$

- Some zero modes are special:

$$x^\mu = \mathrm{tr} a^\mu \quad \text{and} \quad \theta^{\alpha A} = \mathrm{tr} M^{\alpha A}$$

  are the Goldstone modes of the broken (super)translations and S_{mod} does not depend on them.

- The other neutral (anti-chiral) fermionic zero-modes

$$\lambda_{\dot{\alpha} A}$$

  are the Lagrangian multipliers for the fermionic ADHM constraints.

In the case of reduced SUSY, some of the $\theta^{\alpha A}$ will not be present.

- $\mathcal{N} = 2$

$$Z = \int dx^4 \, d\theta^4 \, \mathcal{F} \quad \text{where} \quad \mathcal{F} \propto \int d\{\text{remaining moduli}\} \, e^{-S_{mod}}$$

- $\mathcal{N} = 1$

$$Z = \int dx^4 \, d\theta^2 \, W \quad \text{where} \quad W \propto \int d\{\text{remaining moduli}\} \, e^{-S_{mod}}$$

In the case of reduced SUSY, some of the $\theta^{\alpha A}$ will not be present.

- $\mathcal{N} = 2$

$$Z = \int dx^4\, d\theta^4\, \mathcal{F} \quad \text{where} \quad \mathcal{F} \propto \int d\{\text{remaining moduli}\}\, e^{-S_{mod}}$$

- $\mathcal{N} = 1$

$$Z = \int dx^4\, d\theta^2\, W \quad \text{where} \quad W \propto \int d\{\text{remaining moduli}\}\, e^{-S_{mod}}$$

- The integral over the anti-chiral zero modes $\lambda_{\dot{\alpha}A}$ enforces the fermionic ADHM constraints.

- In general, one must investigate under what conditions these instanton contributions to the prepotential $\mathcal{F}$ or to the superpotential W are non vanishing.

$\mathcal{N} = 1$ example

$\mathcal{N} = 1$ SYM theories from fractional branes

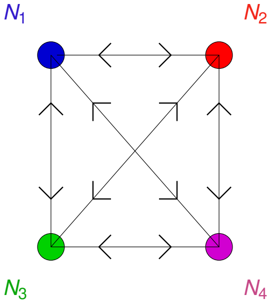
- They can be realized by the massless d.o.f. of open strings attached to fractional D3-branes in the orbifold background

$$\mathbb{R}^{1,3} \times \mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$$

where the orbifold group acts as

$$g_1 : \{z^1, z^2, z^3\} \rightarrow \{z^1, -z^2, -z^3\}, \; g_2 : \{z^1, z^2, z^3\} \rightarrow \{-z^1, z^2, -z^3\}$$

- In this orbifold there 4 types of fractional D3 branes, giving rise to the following quiver gauge theory



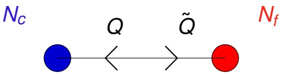
- This quiver theory has the advantage of being non-chiral

- It is automatically free of gauge anomalies.

- It has a large freedom in fractional D3 branes content: (N_1, N_2, N_3, N_4).
  If we take $N_1 = N_c$, $N_2 = N_f$, $N_3 = N_4 = 0$, the theory living on the N_c D3 branes is

  $\mathcal{N} = 1$ SQCD with gauge group $\mathrm{SU}(N_c)$ and N_f flavors

  and is represented by the following (simplified) quiver



- Now we add instantons !

Exotic Instantons

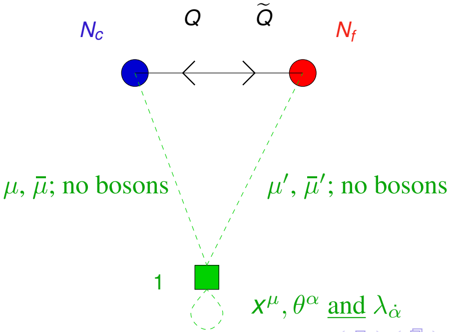
- Let us choose the fractional D3 brane content to be

$$\text{D3} \quad : \quad (N_1, N_2, N_3, N_4) \equiv (N_c, N_f, 0, 0)$$

and the fractional D-instanton content to be

$$\text{D}(-1) \quad : \quad (k_1, k_2, k_3, k_4) \equiv (0, 0, 1, 0)$$

- The instanton is associated to a node that is not occupied in the gauge theory !



The distinctive features of these exotic D-instantons are:

- there are no bosonic moduli in the charged sectors ($\rightarrow$ no w's nor $\bar{w}$'s)

- there are no ADHM-like constraints

- Thus

$$S_1 = 0 \qquad , \qquad S_2 = \frac{i}{2}\,\bar{\mu}\,Q_f\,\mu'^f - \frac{i}{2}\,\bar{\mu}'_f\,\tilde{Q}^f\,\mu$$

  (notice that the Q dependence is holomorphic).

The distinctive features of these exotic D-instantons are:

- there are no bosonic moduli in the charged sectors ($\rightarrow$ no w's nor $\bar{w}$'s)
- there are no ADHM-like constraints
- Thus

$$S_1 = 0 \qquad , \qquad S_2 = \frac{i}{2}\,\bar{\mu}\,Q_f\,\mu'^f - \frac{i}{2}\,\bar{\mu}'_f\,\tilde{Q}^f\,\mu$$

  (notice that the Q dependence is holomorphic).
- But

$$W \propto \int d\{\lambda, D, \mu, \bar{\mu}, \mu', \bar{\mu}'\}\, e^{-S_2} \equiv 0$$

  because of the λ integration !!
- These exotic configurations do NOT contribute to the superpotential, unless the λ's are removed! see however C. Petersson's talk

Orientifold Projection Ω

- In addition to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, let us introduce an O3-plane generated by

$$\Omega = \omega\, I_6$$

  where ω is the world-sheet parity, and I_6 is the parity in the transverse space.

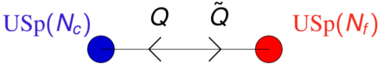
- For the D3 branes, we choose an anti-symmetric representation for Ω on the open string Chan-Paton factors

$$\gamma_-(\Omega) = \mathrm{diag}\,(\epsilon_1 \ldots \epsilon_4) \quad \text{with} \quad \epsilon_i^T = -\epsilon_i \quad , \quad \epsilon_i^2 = 1$$

- The gauge theory fields must satisfy

$$A_\mu = -\gamma_-(\Omega) A_\mu^T \gamma_-(\Omega)^{-1}$$

  implying that the gauge groups are now $USp(N)$. For example,



with $\tilde{Q} = \epsilon_2\, Q^T\, \epsilon_1$

Now let us add D-instantons

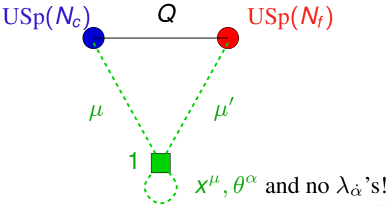
- For consistency, on the D-instantons we must choose a symmetric (orthogonal) representation for Ω

$$\gamma_+(\Omega) = 1\!\!1$$

- This implies

$$\begin{aligned} a_\mu &= (a_\mu)^T \quad , \quad M^\alpha = (M^\alpha)^T \quad , \\ \lambda_{\dot\alpha} &= -(\lambda_{\dot\alpha})^T \quad , \quad \bar\mu \propto \mu^T \end{aligned}$$

- Thus, for any single fractional instanton, the λ zero-modes are projected out!
- For a single exotic instanton in this orientifold, the quiver diagram is



Exotic contributions to superpotential

- Now we have
$$S_1 = 0 \qquad , \qquad S_2 = -i\,\mu^T\,\epsilon_1\,Q_f\,\mu'^f$$
and
$$W \propto \int d\mu d\mu'\ e^{-S_1 - S_2} = \int d\mu d\mu'\ e^{i\mu^T \epsilon_1 \Phi \mu'} \propto \delta_{N_f,N_c}\,\mathrm{det}(Q)$$

- Introducing the $\mathrm{USp}(N_c)$ "meson" matrix $M = Q\epsilon_1 Q$ in the antisymmetric of $\mathrm{USp}(N_f)$, we can write
$$W \propto \mathrm{pf}(M)$$

Exotic contributions to superpotential

- Now we have

$$S_1 = 0 \qquad , \qquad S_2 = -i\,\mu^T\,\epsilon_1 Q_f\,\mu'^f$$

and

$$W \propto \int d\mu d\mu'\ e^{-S_1 - S_2} = \int d\mu d\mu'\ e^{i\mu^T\epsilon_1\Phi\mu'} \propto \delta_{N_f,N_c}\,\mathrm{det}(Q)$$

- Introducing the $\mathrm{USp}(N_c)$ “meson” matrix $M = Q\epsilon_1 Q$ in the antisymmetric of $\mathrm{USp}(N_f)$, we can write

$$W \propto \mathrm{pf}(M)$$

- The condition $N_f = N_c$ is the same as for the existence of contributions from ordinary instantons [Intriligator and Pouliot, 1995]

- In this case, the ADS-like term can be computed from the $(1, 0, 0, 0)$ fractional instanton (just as in the $SU(N)$ case) and one finds

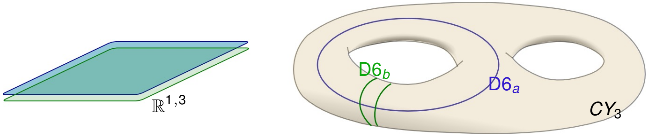
$$W \propto \frac{\Lambda^{2N_c+3}}{\mathrm{pf}(M)}$$

Wrapped branes scenarios

Wrapped D-branes

One can also use systems of wrapped D-branes in $\mathbf{R}^{1,3} \times CY_3$ to engineer 4-d SYM theories with chiral matter and interesting phenomenology:

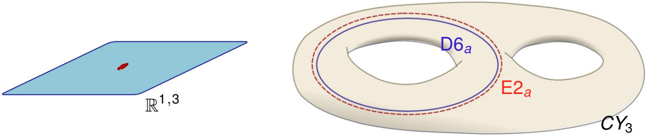
- Type IIB : magnetized D9 branes
- Type IIA : intersecting D6 (easier to visualize)



- families from multiple intersections, tuning different coupling constants
- low energy described by SUGRA with vector and matter multiplets
- can be derived directly from string amplitudes

Instantons and Euclidean branes (I): gauge instantons

- In type IIA Euclidean D2 branes that wrap the same 3-cycle as the D6 branes are point-like in $\mathbf{R}^{1,3}$ and correspond to ordinary gauge instantons of the SYM theory on the D6 branes
- This is analogous to the D3/D(-1) system in flat space:
  - the ADHM construction is obtained from strings attached to the instantonic branes [Witten 1995, Douglas 1995, Dorey 1999, ...]

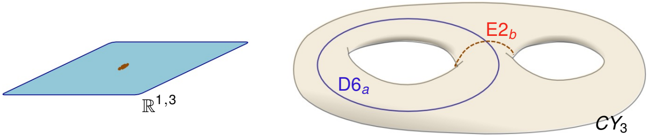


- In type IIB we should consider instead $D9_a$/$E5_a$ branes wrapped on the entire CY space.

Instantons and Euclidean branes (II): exotic instantons

- Type IIA E2 branes wrapped differently from the color D6 branes are still point-like in $\mathbf{R}^{1,3}$ but they do not correspond to ordinary gauge instanton configurations.
- Their "field theory" interpretation is not yet clear but still they may give important non-perturbative contributions to the effective action, e.g. Majorana masses for neutrinos, moduli stabilizing terms, . . .

[Blumenhagen et al, 2006; Ibanez and Uranga, 2006;...]



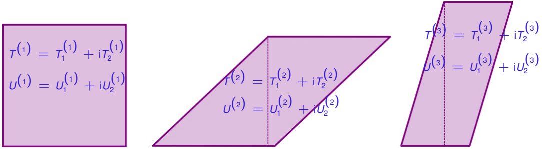
- In type IIB we consider instead systems with $D9_a$ and $E5_b$ branes wrapped on the entire CY space but with different magnetizations (*i.e.* $a \neq b$).

$\mathcal{N} = 2$ example

The background geometry

Internal space:

$$\frac{\mathcal{T}_2^{(1)} \times \mathcal{T}_2^{(2)}}{\mathbb{Z}_2} \times \mathcal{T}_2^{(3)}$$



- Orbifold action:

$$(Z^{(1)}, Z^{(2)}, Z^{(3)}) \to (-Z^{(1)}, -Z^{(2)}, Z^{(3)})$$

- Compactification moduli in the supergravity basis: [Lüst et al, 2004; ...]

$$\mathrm{Im}(s) \equiv s_2 = \frac{1}{4\pi}\,\mathrm{e}^{-\phi_{10}}\,T_2^{(1)}T_2^{(2)}T_2^{(3)}$$

$$\mathrm{Im}(t^{(i)}) \equiv t_2^{(i)} = \mathrm{e}^{-\phi_{10}}\,T_2^{(i)}\,, \quad u^{(i)} = u_1^{(i)} + \mathrm{i}\,u_2^{(i)} = U^{(i)}$$

- Bulk Kähler potential:

$$K = -\log(s_2) - \sum_i \log(t_2^{(i)}) - \sum_i \log(u_2^{(i)})$$

$\mathcal{N} = 2$ SQCD from magnetized branes

The gauge sector

- Place a stack of N_a fractional D9 branes in the representation R_0 of the orbifold group ("color branes" 9_a)
- The massless spectrum of the $9_a/9_a$ strings gives rise, in $\mathbf{R}^{1,3}$, to $\mathcal{N} = 2$ vector multiplets for the gauge group $\mathrm{SU}(N_a)$

$\mathcal{N} = 2$ SQCD from magnetized branes

The gauge sector

- Place a stack of N_a fractional D9 branes in the representation R_0 of the orbifold group ("color branes" 9_a)

- The massless spectrum of the $9_a/9_a$ strings gives rise, in $\mathbf{R}^{1,3}$, to $\mathcal{N} = 2$ vector multiplets for the gauge group $\mathrm{SU}(N_a)$

- The gauge action is

$$S_{gauge} = \int d^4x \, \mathrm{Tr}\Big\{ \frac{1}{2g_a^2} F_{\mu\nu}^2 + 2\, K_\Phi \, D_\mu \bar{\Phi} D^\mu \Phi + \cdots \Big\}$$

where

$$\frac{1}{g_a^2} = \frac{1}{4\pi} \, \mathrm{e}^{-\phi_{10}} \, T_2^{(1)} T_2^{(2)} T_2^{(3)} = s_2 \quad , \quad K_\Phi = \frac{1}{t_2^{(3)} u_2^{(3)}}$$

$\mathcal{N} = 2$ SQCD from magnetized branes

The matter sector

- Add D9-branes in the representation R_1 of the orbifold group ("flavor branes" 9_b) with quantized magnetic fluxes along the first two torii

$$f_b^{(i)} = \frac{m_b^{(i)}}{n_b^{(i)}} \qquad m_b^{(i)}, n_b^{(i)} \in \mathbb{Z} \quad \text{for } i = 1, 2$$

- $\mathcal{N} = 2$ susy requires $\nu_b^{(1)} = \nu_b^{(2)}$, where

$$f_b^{(i)} / T_2^{(i)} = \tan \pi \nu_b^{(i)} \quad \text{with} \quad 0 \leq \nu_b^{(i)} < 1$$

$\mathcal{N} = 2$ SQCD from magnetized branes

The matter sector

- Add D9-branes in the representation R_1 of the orbifold group ("flavor branes" 9_b) with quantized magnetic fluxes along the first two torii

$$f_b^{(i)} = \frac{m_b^{(i)}}{n_b^{(i)}} \qquad m_b^{(i)}, n_b^{(i)} \in \mathbb{Z} \quad \text{for } i = 1, 2$$

- $\mathcal{N} = 2$ susy requires $\nu_b^{(1)} = \nu_b^{(2)}$, where

$$f_b^{(i)} / T_2^{(i)} = \tan \pi \nu_b^{(i)} \quad \text{with} \quad 0 \leq \nu_b^{(i)} < 1$$

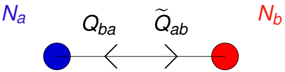
- The $9_b/9_a$ strings are twisted by the relative angles

$$\theta_{ba}^{(i)} \equiv \nu_b^{(i)} - \nu_a^{(i)}$$

  and their massless modes fill up $\mathcal{N} = 2$ hypermultiplets in the fundamental representation of $\mathrm{SU}(N_a)$.

$\mathcal{N} = 2$ SQCD from magnetized branes

The matter sector



- The degeneracy of these hypermultiplet is $N_b|I_{ab}|$ where I_{ab} is the # of Landau levels for the (a, b) "intersection"

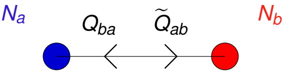
$$I_{ab} = \prod_{i=1}^{2} \left(m_a^{(i)} n_b^{(i)} - m_b^{(i)} n_a^{(i)} \right)$$

- Thus in $\mathcal{N} = 1$ language

$$\{Q_f\,,\ \widetilde{Q}_f^\dagger\,,\ \chi_f\,,\ \widetilde{\chi}_f^\dagger\} \quad \text{with} \quad f = 1, ..., N_f = N_b|I_{ab}|$$

$\mathcal{N} = 2$ SQCD from magnetized branes

The matter sector



- The matter action is

$$S_{matter} = \int d^4x \sum_{f=1}^{N_F} K_Q \Big\{ D_\mu Q^{\dagger\,f} D^\mu Q_f + D_\mu \tilde{Q}^f D^\mu \tilde{Q}_f^\dagger + \ldots \Big\}$$

together with a superpotential $W = \sum_f \tilde{Q}^f \, \Phi \, Q_f$ and possibly a mass term

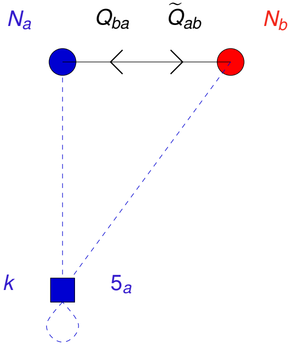
- The Kähler metric for Q and $\widetilde{Q}$ is

$$\mathrm{e}^{K/2} K_Q^{-1} = g_a \sqrt{K_\Phi} \quad \Rightarrow \quad K_Q = \frac{1}{\left(t_2^{(1)} t_2^{(2)} u_2^{(1)} u_2^{(2)}\right)^{1/2}}$$

Now let us add instantons!

- Add k E5 branes whose internal structure is the same as that of the color branes (5_a branes $\rightarrow$ ordinary gauge instantons)

- The quiver diagram becomes



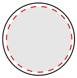
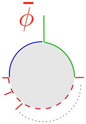
where the dotted lines represent the ADHM instanton moduli.

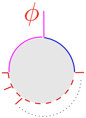
Moduli and instanton parameters

	ADHM	Meaning	Chan-Paton
$5_a/5_a$ (NS)	a'_μ	*centers*	adj. $\mathrm{U}(k)$
	χ	*aux.*	$\vdots$
	D	*Lagrange mult.*	$\vdots$
$5_a/5_a$ (R)	$M^{\alpha A}$	*partners*	$\vdots$
	$\lambda_{\dot\alpha A}$	*Lagrange mult.*	$\vdots$
$5_a/9_a$ (NS)	$w_{\dot\alpha}$	*sizes*	$k \times N_a$
	$\bar w_{\dot\alpha}$	*sizes*	$N_a \times k$
$5_a/9_a$ (R)	μ^A	*partners*	$k \times N_a$
	$\bar\mu^A$	$\vdots$	$N_a \times k$
$5_a/9_b$ (R)	μ'^A	*partners*	$k \times N_f$
	$\bar\mu'^A$	$\vdots$	$N_f \times k$

Instanton contributions to the effective action

We first compute the moduli action from all (mixed) disk diagrams

$S_{mod}(\phi, \bar{\phi}; \widehat{\mathcal{M}}_k) =$  $+$ 

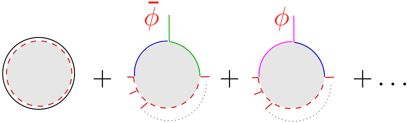
$+$ 

$+ \ldots$

Instanton contributions to the effective action

We first compute the moduli action from all (mixed) disk diagrams

$$S_{mod}(\phi, \bar{\phi}; \widehat{\mathcal{M}}_k) =$$



We then integrate over the moduli and get the instanton induced effective action

$$S^k_{eff} = \mathcal{C}_k \int d^4x\, d^4\theta\; d\widehat{\mathcal{M}}_k\, \mathrm{e}^{-\mathcal{S}_{mod}(\phi,\bar{\phi};\widehat{\mathcal{M}}_k)}$$

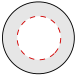
where

$$\mathcal{C}_k = \Lambda^{k(2N_a - N_F)}\, \mathrm{e}^{\mathcal{A}'_{5_a}} = \Lambda^{kb_1}\, \mathrm{e}^{\mathcal{A}'_{5_a}}$$

with

[Blumenhagen et al, 2006;...]

$$\mathcal{A}_{5_a} \equiv$$



(no moduli insertions)

Instanton contributions to the effective action

The integral over the instanton moduli can be evaluated using localization methods and the explicit form of the effective action S^k_{eff} can be obtained
[Nekrasov, 2002; ...; Fucito et al., 2004;...]

For our purposes it is convenient to write the result in a $\mathcal{N} = 1$ superspace notation

$$S^k_{eff} \quad = \quad \Lambda^{kb_1}\, \mathrm{e}^{\mathcal{A}'_{5_a}} \left\{ \int d^4x_0\, d^2\theta \left[\frac{1}{2g_a^2}\, \tau^k_{uv}(\phi)\, W^\alpha_u W_{\alpha v} \right] + \int d^4x_0\, d^2\theta\, d^2\bar{\theta} \left[\frac{1}{g_a^2}\, \bar{\phi}_u\, \sigma^k_u(\phi) \right] \right\}$$

where the instanton induced couplings

$$\tau^k_{uv}(\phi) \quad \text{and} \quad \sigma^k_u(\phi)$$

are holomorphic functions of the canonically normalized scalar ϕ of the gauge multiplet

Instanton contributions to the effective action

Changing to the SUGRA effective fields Φ, and using the homogeneity properties of the instanton induced couplings, we obtain

$$S^k_{eff} = \Lambda^{kb_1}\, \mathrm{e}^{\mathcal{A}'_{5_a}}\, (g_a\sqrt{K_\Phi})^{-kb_1} \left\{ \int d^4x_0\, d^2\theta \left[\frac{1}{2g_a^2}\, \tau^k_{uv}(\Phi)\, W^\alpha_u W_{\alpha v}\right] + \int d^4x_0\, d^2\theta\, d^2\bar{\theta} \left[K_\Phi \bar{\Phi}_u\, \sigma^k_u(\Phi)\right] \right\}$$

where

$$K_\Phi = (t_2^{(3)} u_2^{(3)})^{-1}$$

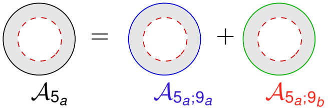
is the Kähler metric of the adjoint scalars. All non-holomorphic terms in the prefactor coming from

the annulus amplitude $\mathcal{A}'_{5_a}$ and the Kähler metric K_Φ

should cancel !!

The annulus amplitude

$\mathcal{A}_{5_a}$ is a sum of annulus amplitudes with a boundary on the 5_a branes



- Imposing the appropriate b.c.'s and GSO, these annulus amplitudes are given by

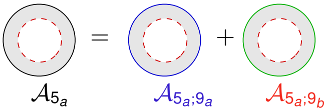
$$\int_0^\infty \frac{d\tau}{2\tau}\left[\mathrm{Tr}_{\mathrm{NS}}\left(P_{\mathrm{GSO}}\,P_{\mathrm{orb.}}\,q^{L_0}\right)-\mathrm{Tr}_{\mathrm{R}}\left(P_{\mathrm{GSO}}\,P_{\mathrm{orb.}}\,q^{L_0}\right)\right]$$

- Both in the colored amplitude $\mathcal{A}_{5_a;9_a}$ and the flavored amplitude $\mathcal{A}_{5_a;9_b}$, the contributions from all excited string states cancel

- Only the KK copies of the instanton zero-modes on the untwisted torus $\mathcal{T}_2^{(3)}$ give a contribution leading to a (non-holomorphic) dependence on the Kähler and complex structure moduli $t_2^{(3)}$, $u_2^{(3)}$

The annulus amplitude

$\mathcal{A}_{5_a}$ is a sum of annulus amplitudes with a boundary on the 5_a branes



- These annulus amplitudes are UV and IR divergent. The UV divergences (IR in the closed string channel) cancel if tadpole cancellation assumed. The IR divergence is regulated with a scale μ

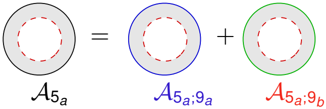
- The explicit result of the string calculation is

$$\mathcal{A}_{5_a} = -kb_1 \left(\frac{1}{2} \log(\alpha' \mu^2) + \log \left(\eta(U^{(3)}) \right)^2 + \frac{1}{2} \log \left(U_2^{(3)} T_2^{(3)} \right) \right)$$

[Billo et al. 2007; see also Lüst and Stieberger, 2003]

The annulus amplitude

$\mathcal{A}_{5_a}$ is a sum of annulus amplitudes with a boundary on the 5_a branes



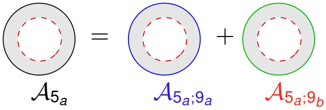
- There is a relation between instantonic annulus amplitudes and threshold corrections to the gauge coupling constant
  [Abel and Goodsell, 2006; Akerblom et al, 2006]

- Indeed in SUSY theories the mixed annulus amplitudes compute the running coupling constant by expanding around the instanton background
  [Billo et al, 2007]

$$\mathcal{A}_{5_a} = -\frac{8\pi^2 k}{g_a^2(\mu)}\Big|_{1-\mathrm{loop}}$$

The annulus amplitude

$\mathcal{A}_{5_a}$ is a sum of annulus amplitudes with a boundary on the 5_a branes



- Introducing the Planck mass

$$M_P^2 = \frac{1}{\alpha'} \, \mathrm{e}^{-\phi_{10}} \, s_2$$

and using the effective field theory variables, we get

$$\begin{aligned} \mathcal{A}_{5_a} &= \frac{kb_1}{2} \log \frac{M_P^2}{\mu^2} + \mathcal{A'}_{5_a} \\ &= \frac{kb_1}{2} \log \frac{M_P^2}{\mu^2} + kb_1 \left(- \log \left(\eta(u^{(3)})\right)^2 + \frac{1}{2} \log(g_a^2) + \frac{1}{2} \log K_\Phi \right) \end{aligned}$$

The holomorphic life of wrapped D-brane instantons

Thus

$$\mathrm{e}^{\mathcal{A}'_{5_a}} = \Big(\eta(u^{(3)})\Big)^{-2kb_1} \left(g_a \sqrt{K_\Phi}\right)^{kb_1}$$

and

$$S^k_{eff} = \Lambda^{kb_1}\, \mathrm{e}^{\mathcal{A}'_{5_a}} \left(g_a\sqrt{K_\Phi}\right)^{-kb_1} \left\{ \int d^4x_0\, d^2\theta \left[\frac{1}{2g_a^2}\,\tau^k_{uv}(\Phi)\, W^\alpha_u W_{\alpha v}\right] + \int d^4x_0\, d^2\theta\, d^2\bar{\theta} \left[K_\Phi \bar{\Phi}_u\, \sigma^k_u(\Phi)\right] \right\}$$

The holomorphic life of wrapped D-brane instantons

Thus

$$e^{\mathcal{A}'_{5_a}} = \left(\eta(u^{(3)})\right)^{-2kb_1} \left(g_a \sqrt{K_\Phi}\right)^{kb_1}$$

and

$$S^k_{eff} = \left(\Lambda\, \eta(u^{(3)})^{-2}\right)^{kb_1} \left\{ \int d^4x_0\, d^2\theta \left[\frac{1}{2g_a^2}\, \tau^k_{uv}(\Phi)\, W^\alpha_u W_{\alpha v}\right] + \int d^4x_0\, d^2\theta\, d^2\bar{\theta} \left[K_\Phi\, \bar{\Phi}_u\, \sigma^k_u(\Phi)\right] \right\}$$

- The instanton induced effective action has the correct holomorphic structure as required by SUSY !
- There is a holomorphic rescaling of the Wilsonian scale Λ into

$$\Lambda' = \Lambda\, \eta(u^{(3)})^{-2}$$

Further considerations

Using the $\mathcal{N} = 2$ bulk Kähler potential

$$\widetilde{K} \equiv -\log(s_2) - \log(t_2^{(3)}) - \log(u_2^{(3)}) = K - 2\log K_Q$$

[see Bachas et al, 1997;...; Berg et al, 2005]

the instantonic annulus amplitude can be written as

$$\mathcal{A} = k\left[f + \frac{b_1}{2}\left(\log\frac{M_P^2}{\mu^2} + \widetilde{K}\right)\right]$$

with f a holomorphic function $\sim \log[\eta(u^{(3)})]$

- This is a particular case of the general $\mathcal{N} = 2$ "Kaplunovsky-Louis" formula

$$\mathcal{A} = k\left[f + \frac{b_1}{2}\log\frac{M_P^2}{\mu^2} - T(G)\log\left(\frac{1}{g^2}\right) + T(G)\log(K_\Phi) - \sum_r N_r\,T(r)\,\widetilde{K}\right]$$

[see de Wit et al. 1995]

where

$$T(r)\,\delta_{AB} = \mathrm{Tr}_r\left(T_A T_B\right) \quad , \quad T(G) = T(\mathrm{adj}) \quad , \quad N_r = \#\text{ of hypers in rep. } r$$

Conclusions

- D3/D(–1) systems at orbifold singularities provide very efficient string set-ups to perform explicit instanton calculations in gauge theories

- The (ordinary or exotic) instanton corrections to the superpotential or the prepotential can be computed from mixed **disk diagrams**

Conclusions

- D3/D(–1) systems at orbifold singularities provide very efficient string set-ups to perform explicit instanton calculations in gauge theories
- The (ordinary or exotic) instanton corrections to the superpotential or the prepotential can be computed from mixed **disk diagrams**
- Also systems of wrapped magnetized D9/E5 branes are very useful to study instanton effects in toroidal orbifold/orientifold compactifications of type IIB string theory
- In this case the mixed **annulus diagrams** are crucial to provide the correct holomorphic structure of the effective action required by SUSY

Conclusions

- D3/D(–1) systems at orbifold singularities provide very efficient string set-ups to perform explicit instanton calculations in gauge theories

- The (ordinary or exotic) instanton corrections to the superpotential or the prepotential can be computed from mixed **disk diagrams**

- Also systems of wrapped magnetized D9/E5 branes are very useful to study instanton effects in toroidal orbifold/orientifold compactifications of type IIB string theory

- In this case the mixed **annulus diagrams** are crucial to provide the correct holomorphic structure of the effective action required by SUSY

- The analysis of $\mathcal{N} = 2$ SQCD can be generalized to $\mathcal{N} = 1$ SQCD or other $\mathcal{N} = 1$ theories. In this case the annulus amplitudes contain interesting information on the Kähler metric for twisted chiral matter

[Akerblom et al. 2007, Billo et al, 2007, Blumenhagen et al 2007]