Holomorphic Couplings and D-Instantons

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Workshop on Recent Developments in
String Effective Actions and D-instantons

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- N.A., R. Blumenhagen, D. Lüst, M. Schmidt-Sommerfeld: "Instantons and Holomorphic Couplings in Intersecting D-brane Models" (JHEP 0708: 044, 2007)
- N.A., R. Blumenhagen, D. Lüst, M. Schmidt-Sommerfeld: "Thresholds for Intersecting D-branes Revisited" (Phys. Lett. B 652: 53-59, 2007)

Overview

- Main point: Holomorphy of instanton induced (superpotential-)couplings

  Showing this uses the Kaplunovsky-Louis formula

- Kaplunovsky-Louis formula also important for showing connection between gauge threshold corrections and one-loop corrections to gauge kinetic function

- We begin with the latter in the IIA orientifold setting

Gauge Threshold Corrections and One-Loop Corrections to the Gauge Kinetic Function

Review: Tree Level Gauge Kinetic Function

The Setting: $\mathcal{N} = 1$ Type IIA intersecting D6-brane model on $M_4 \times K$, where K is some orbifold (or Calabi-Yau)

- Want tree level gauge kinetic function f_a on brane D6_a
- Introduce symplectic basis of three cycles (A_I, B^I), $I = 1, \ldots, h_{2,1}$, $A_I \circ B^J = \delta_I{}^J$, on K, and ...
- Expand the cycle $\Pi_a \subset K$ around which brane D6_a wraps as

$$\Pi_a = M_a^I A_I + N_{a,I} B^I$$

Review: Tree Level Gauge Kinetic Function

Then define

- Complexified complex structure moduli:

$$U_I^c = \frac{1}{2\pi\ell_s^3}\left[e^{-\phi_4}\int_{A_I}\mathrm{Re}(\widehat{\Omega}_3) - i\int_{A_I} C_3\right]$$

By dimensionally reducing the DBI and CS action for the brane $D6_a$ one can then deduce that the tree level gauge kinetic function is

$$f_a = \sum_{I=0}^{h_{2,1}} M_a^I U_I^c$$

Constraints on Corrections to f_a

Holomorphy and shift symmetry constrain the form of corrections to the gauge kinetic function (“non-renormalization theorem”):

$$f_a = \sum_I M_a^I U_I^c + f_a^{\text{1-loop}}(e^{-T_i^c}) + f_a^{\text{np}}(e^{-U_I^c}, e^{-T_i^c}) \,,$$

where

$$T_i^c = \frac{1}{\ell_s^2}\left(\int_{C_i} J_2 - i\int_{C_i} B_2\right) \,.$$

Using this, we deduce $f_a^{\text{1-loop}}$ from gauge threshold corrections!

Gauge Threshold Corrections

- Running of the coupling for field theory on D6_a:

$$\frac{8\pi^2}{g_a^2(\mu)} = \frac{8\pi^2}{g_{a,\text{string}}^2} + \frac{b_a}{2}\log\frac{M_s^2}{\mu^2} + \frac{\Delta_a}{2}$$

- $\Delta_a = \Delta_a^{\text{n.h.}} + \text{Re}(\Delta_a^{\text{hol.}})$: gauge threshold corrections
- Kaplunovsky-Louis formula:

$$\frac{8\pi^2}{g_a^2(\mu^2)} = 8\pi^2\text{Re}(f_a^{\text{tree+1-loop}}) + \text{running} + \text{non-hol. Kähler stuff}$$

Comparing gives

$$f_a = f_a^{\text{tree}} + f_a^{\text{1-loop}}(e^{-T_i^c}) = \sum_I M_a^I U_I^c + \Delta_a^{\text{hol.}}$$

One Loop Corrections to Gauge Kinetic Function

For $\mathcal{N}=1$ sector in $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$ ($h_{2,1}=3\neq 51$) model ($\sum_I \theta^I_{ab}=0$):

$$\Delta_a^{\mathcal{N}=1}=-\frac{b_a}{16\pi^2}\log\left[\frac{\Gamma(\theta^1_{ab})\Gamma(\theta^2_{ab})\Gamma(1+\theta^3_{ab})}{\Gamma(1-\theta^1_{ab})\Gamma(1-\theta^2_{ab})\Gamma(-\theta^3_{ab})}\right],$$

with $\theta^{1,2}_{ab}>0$, $\theta^3_{ab}<0$ (Lüst, Stieberger; Akerblom, Blumenhagen, Lüst, Schmidt-Sommerfeld)

- This has no holomorphic part:

$$(\Delta_a^{\mathcal{N}=1})^{\text{hol.}}=0\Rightarrow(f_a^{\mathcal{N}=1})^{\text{1-loop}}=0$$

- Similarly, for $\mathcal{N}=2$ sector:

$$(\Delta_a^{\mathcal{N}=2})^{\text{hol.}}=\log\eta(i\,T^c_I)\Rightarrow(f_a^{\mathcal{N}=2})^{\text{1-loop}}=\log\eta(iT^c_I)$$

Related work: Billo, Frau, Pesando, Di Vecchia, Lerda, Marotta; Blumenhagen, Schmidt-Sommerfeld

Holomorphy of Instanton Contributions to the Superpotential

Holomorphy of Instanton Contributions?

Instanton calculus for charged matter couplings:[1]

- In order to compute (b/c you're interested in the Yukawas Y)

$$\langle \Phi_{a_1,b_1} \cdots \Phi_{a_M,b_M} \rangle_{\text{E2}} = \frac{e^{\frac{\mathcal{K}}{2}} Y_{\Phi_{a_1,b_1},\ldots,\Phi_{a_M,b_M}}}{\sqrt{K_{a_1,b_1} \cdots K_{a_M,b_M}}} ,$$

- Compute:

$$\langle \Phi_{a_1,b_1} \cdots \Phi_{a_M,b_M} \rangle_{\text{E2}} \sim \int d\{\text{E2}\} \sum_{\text{conf.}} \exp(Z_0') \, \langle \widehat{\Phi}_{a_1,b_1}[\vec{x}_1] \rangle_{\lambda_{a_1},\bar{\lambda}_{b_1}} \cdots \langle \widehat{\Phi}_{a_L,b_L}[\vec{x}_L] \rangle_{\lambda_{a_L},\bar{\lambda}_{b_L}}$$

Problem: How to extract Yukawas? Recall that they must be holomorphic, as they are part of the superpotential!

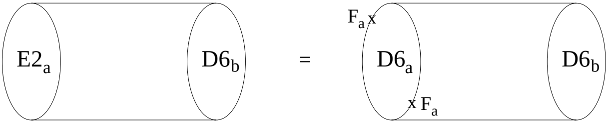
[1] Blumenhagen, Cvetič, Weigand; Ibáñez, Uranga

Holomorphy of Instanton Contributions?!

- Calculating a correlator in the E2 background involves the quantity ("one-loop Pfaffian")

$$Z'_0 = \sum_b {Z'}^A(\mathrm{E2}, \mathrm{D6}_b) + {Z'}^M(\mathrm{E2}, \mathrm{O6})$$

- It can be shown that[1] (similarly for the Möbius diagrams)



i.e. an equality between an instantonic one-loop amplitude and a threshold correction. We have seen that such stringy threshold corrections are (in general) non-holomorphic!

[1] Abel, Goodsell; N.A., Blumenhagen, Lüst, Plauschinn, Schmidt-Sommerfeld

Resolving the puzzle

- As we have seen earlier, the Kaplunovsky-Louis formula connects the threshold corrections to gauge kinetic function, Kähler potential, and matter Kähler metrics

- Applied to the gauge thresholds in the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold, we relate the instantonic one-loop diagram Z_0 to the Kähler data

$$Z_0 \sim \mathrm{Re}(f^{(1)}) + \log \frac{M_{\mathrm{Pl}}^2}{\mu^2} + \mathcal{K}_{\mathrm{tree}} + \log \frac{V_3}{g_s} + \det K_{\mathrm{tree}}^{ab}$$

Resolving the puzzle

- The CFT correlators in the instanton calculus formula also depend on the non-holomorphic Kähler potential

$$\langle \widehat{\Phi}_{a,b}[\vec{x}] \rangle_{\lambda_a,\overline{\lambda}_b} = \frac{e^{\frac{\mathcal{K}}{2}} Y_{\Phi_{a_1,b_1},\ldots,\Phi_{a_M,b_M}}}{\sqrt{K_{\lambda_a,a} K_{a,x_1} \cdots K_{x_n,b} K_{b,\overline{\lambda}_b}}}$$

- Putting together the Kaplunovsky-Louis formula and these correlators, one finds cancellation of non-holomorphic terms between instantonic one-loop diagrams and correlators, yielding the manifestly holomorphic couplings

$$Y_{\Phi_{a_1,b_1},\ldots,\Phi_{a_M,b_M}} \sim \sum_{\text{conf.}} \exp(-f_a^{(0,1)})\, Y_{\lambda_{a_1} \widehat{\Phi}_{a_1,b_1}[\vec{x}_1] \overline{\lambda}_{b_1}} \cdots Y_{\lambda_{a_1} \widehat{\Phi}_{a_L,b_L}[\vec{x}_L] \overline{\lambda}_{b_L}}$$

Summary

- In the type IIA case can use gauge threshold corrections to obtain one-loop corrections to gauge kinetic function (too)

  (Of course, this also applies to the dual IIB case)

- Application of Kaplunovsky-Louis formula to (charged matter coupling) instanton calculus shows that instanton contributions to superpotential are holomorphic

- Outlook: Non-holomorphic pieces of gauge threshold corrections can be used to constrain matter Kähler metrics.[1] For this, and further applications, see Max Schmidt-Sommerfeld's talk

[1] N.A., Blumenhagen, Lüst, Schmidt-Sommerfeld; Billo, Frau, Pesando, Di Vecchia, Lerda, Marotta; Blumenhagen, Schmidt-Sommerfeld