Non-perturbative superpotentials across lines of marginal stability

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Outline

Motivation

Some background
- Instanton effects
- Some useful geometries

Non-gauge D-brane instantons and marginal stability
- 2-instanton processes
- Non-perturbative lifting of zero modes

Gauge D-brane instantons and marginal stability
- Spacetime vs. instanton analysis
- Seiberg duality

Gauge instantons ⇔ non-gauge instantons

D3-branes at singularities (w/ L. Ibanez)

F/M-theory picture and topology change

Conclusions

Motivation

Non-perturbative superpotentials from euclidean D-brane instantons

Instantons contributing to superpotential: fully wrapped BPS D-branes with two fermion zero modes

(other fermion modes lifted by interaction)

Superpotential seems to depends on the precise list of such BPS D-branes

BPS condition: implies a holomorphic and a D-term condition

Latter depends non-trivially on closed string moduli:

$\Rightarrow$ lines of marginal stability (Kahler moduli in IIB, complex in IIA)

Real codimension-1 regions in moduli space where spectrum of BPS D-branes changes abruptly

(D-brane instanton becomes unstable and splits into sub-objects)

Fate of the non-perturbative superpotential in the crossing?

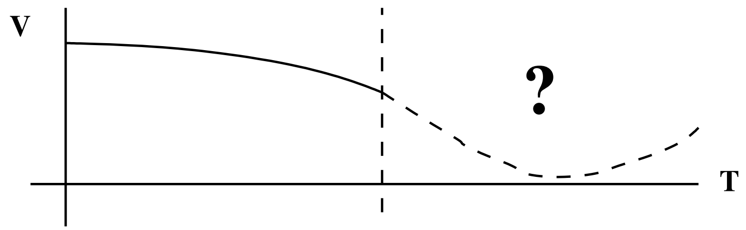
(Holomorphy implies continuity; microscopic explanation?)

Motivation (cont.)

A phenomenological motivation

Consider the approach to stabilize Kahler moduli in IIB compactifications via a non-perturbative superpotential, e.g. from gaugino condensates [Kachru,Kallosh,Linde,Trivedi]

- Sit at a point in Kahler moduli space, where sets of susy D7-branes generate a superpotential
- Minimize it
- The original D7-brane are NON-susy at the new minimum!
- Non-zero D-term, D7's recombine into new susy system
- Does the new system generate the same superpotential?
- If not, "minimum" is really out of regime of validity of superpotential used to find it!



Results

Non-perturbative superpotential is continuous across lines of marginal stability

Microscopically, thanks to contributions from multi-instanton processes

At points where a D-brane instanton becomes unstable against and its contribution disappears, there exists a multi-instanton process (involving the decay products) which reconstructs precisely the same contribution

Extra zero modes of an instanton can be soaked up by others in multi-instanton processes

Non-perturbative lifting of fermion zero modes

- Superpotential contributions from U(1) instantons
- Evades usual criteria (e.g. of arithmetic genus, etc)

Many examples involving non-gauge or gauge D-brane instantons

Instanton effects

Different kinds of instantons to keep in mind

Gauge instantons

Instanton D-brane wraps same cycle as 4d gauge D-brane

Ex. ADS "fractional" instantons

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}}$$

[..., many authors]

Non-gauge instantons

General D-brane instantons

In perturbative models, need O(1) Chan-Paton group

Beasley-Witten instantons

BPS D-branes with more than 2 decoupled fermion zero modes

Generate multi-fermion F-term, sketchily

$$\int d^4x\, d^2\theta\, w_{\bar{i}_1\bar{j}_1\ldots\bar{i}_n\bar{j}_n}(\Phi)\, \bar{D}\bar{\Phi}^{\bar{i}_1}\, \bar{D}\bar{\Phi}^{\bar{j}_1} \ldots \bar{D}\bar{\Phi}^{\bar{i}_n}\, \bar{D}\bar{\Phi}^{\bar{j}_n}$$

Useful geometries

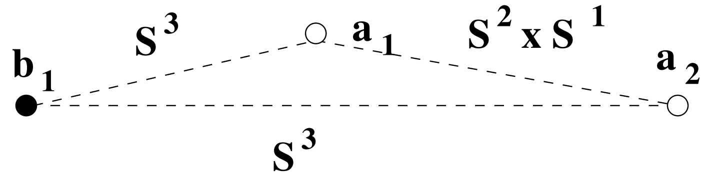
[Ooguri,Vafa]

Consider non-compact geometries with compact 3-cycles on which we have D2-brane instantons

Double C* fibrations over the complex plane, 3-cycles are double circle fibrations over segments between degenerations

$$xy = \prod_{k=1}^{P} (z - a_k)$$

$$x'y' = \prod_{k'=1}^{P'} (z - b_k)$$

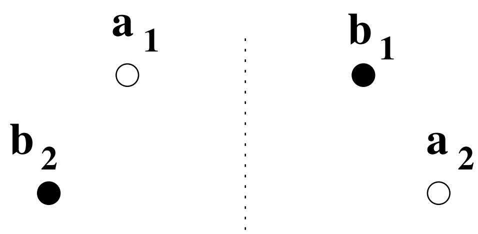


BPS D2' wrap horizontal segments calibrated wrt $\Omega = dz\, \frac{dx}{x}\, \frac{dx'}{x'}$

Useful orientifold, mod by $\Omega R(-)^F$ with

$$z \longrightarrow -\bar{z} \quad ; \quad (x, y) \leftrightarrow (\bar{x}', \bar{y}')$$

O6 in the vertical direction, exchanging a and b degenerations



Useful geometries (cont.)

Useful T-dual realization as D- branes suspended among NS5-branes

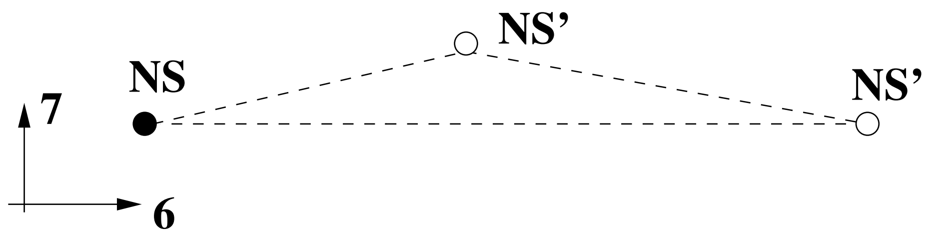
[Hanany, Witten]

T-duality along the two circle directions on the fibers

NS along 012345

NS' along 012389

D0 along segment in 67



Picture allows to read out spectrum and interactions of zero modes

e.g. "hypermultiplets" at touching of horizontal segments

cubic "superpotentials" for adjoint (non-rigidity) zero modes

possible quartic "superpotentials" for bifundamentals

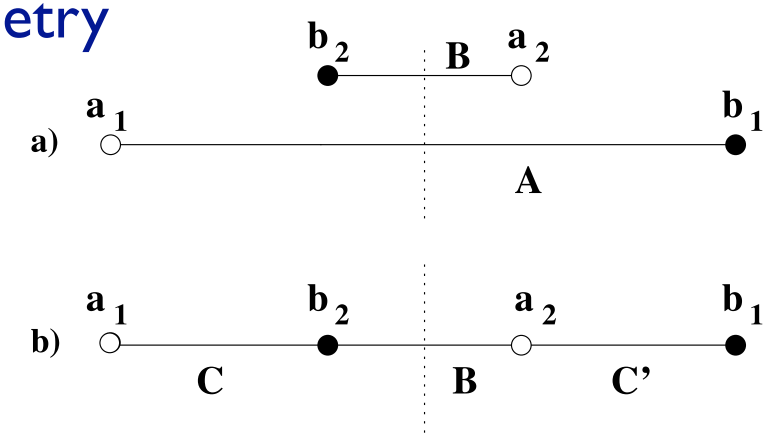
Also useful picture of different orientifold planes

(e.g. O6 on 01236 and 45 degrees in (45,89))

Non-gauge instantons

O(1) → O(1) x U(1)

Orientifolded geometry



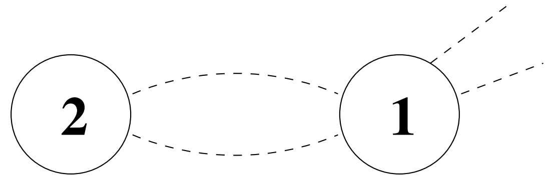
a) Generically there are two O(1) instantons, A and B

$$W = f_1 e^{-T_B} + f_2 e^{-T_A}$$

b) Line of marginal stability, in which instanton A disappears.
One O(1) instanton B and one U(1) instanton C/C'

How is $\exp(-T_A)$ generated?

2-instanton process



Zero mode analysis

Translational Goldstones x_1, x_2; "Goldstinos" $\theta_1, \tilde{\theta}_1, \theta_2$;
bi-fundamental hyperm. Φ_{12}, Φ_{21} ie $\varphi_{12}, \varphi_{21}, \chi_{12}, \chi_{21}$

For instantons separated in 4d, too many zero modes:
localization onto $x_1 = x_2$

Couplings for fermions

- $(\chi_{12}\,(\theta_1 - \theta_2))\varphi^*_{12} - (\chi_{21}\,(\theta_1 - \theta_2))\varphi^*_{21} + (\bar{\chi}_{12}\tilde{\theta})\varphi_{12} - (\bar{\chi}_{21}\tilde{\theta})\varphi_{21}$
- $\chi_{12}\varphi_{21}\chi_{12}\varphi_{21} + 2\chi_{12}\chi_{21}\varphi_{12}\varphi_{21} + \varphi_{12}\chi_{21}\varphi_{12}\chi_{21} \quad + \text{h.c.}$

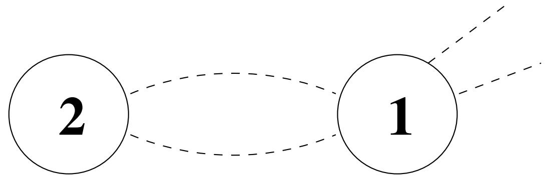
(from $W \simeq (\Phi_{12}\Phi_{21})^2$)

All fermions couple except for the overall Goldstinos $\theta_1 + \theta_2$

Pull down interactions in exp(-S_{inst}) and soak up zero modes

We recover $S_{4d} \simeq \int d^4x\, d^2\theta\, e^{-(T_B + 2T_C)} \simeq \int d^4x\, d^2\theta\, e^{-T_A}$

Non-perturbative lifting of fermion zero modes



The O(1) instanton leads to a term in the 4d action, but also leads to a modification of the zero mode action of the U(1) instanton

$$\Delta S_{\rm inst1} = \int d^2\theta_2\, d^4\chi\, d^4\varphi\, \exp[\,(\theta_1-\theta_2)\,\varphi\,\chi \,+\, \tilde\theta_1\,\bar\varphi\,\bar\chi + \chi^2\,\varphi^2 \,+\, V(\varphi)\,]$$

Sketchily, upon integration over zero modes of O(1) instanton

$$\Delta S_{\rm inst1} \,\simeq\, e^{-T_B}\,\tilde\theta_1\tilde\theta_1$$

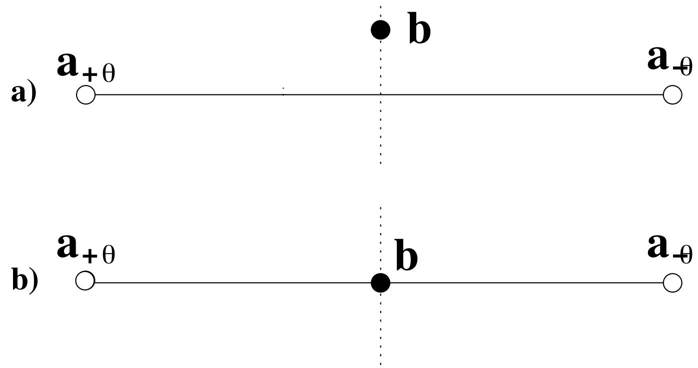
Extra zero modes of U(1) instanton are lifted, it contributes to W

$$S_{4d} \;\simeq\; \int d^4x\, d^2\theta\, d^2\tilde\theta\, \exp\,[\,-2T_C \,-\, e^{-T_B}\,\tilde\theta\tilde\theta)\,]$$

$$= \int d^4x\, d^2\theta\, e^{-T_B}\, e^{-2T_C} \,=\, \int d^4x\, d^2\theta e^{-T_A}$$

Non-gauge instantons

2nd example: O(1) → U(1)



Orientifold geometry

$$xy = u(u + \alpha v)(u - \alpha v)$$

O(1) instanton splits as U(1) and image

There is a contribution from the U(1) instanton

All fermion zero modes (except Goldstinos) couple and can be soaked up (purely perturbatively)

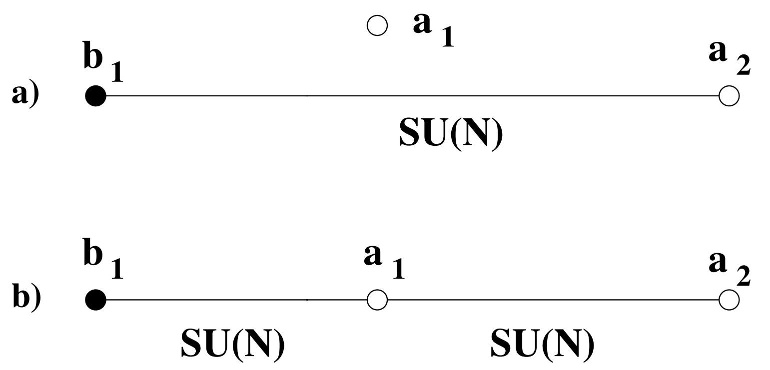
Important role of interactions among 11' modes

$$W \simeq (\,\Phi_{11'}\Phi_{1'1}\,)^2$$

Evades assumptions in Blumenhagen, Cvetic, Richter, Weigand

Gauge instantons

A SQCD superpotential example



a) SU(N) pure SYM

$$W = N\Lambda^3 \simeq (e^{-T})^{\frac{1}{N}}$$ (from 1/N-instanton)

b) unHiggsses to $SU(N)_1 x SU(N)_2$ with bifundamentals Q_{12}, Q'_{21} and adjoint Φ_2 with (perturbative) $W=Q_{12}\Phi_2 Q'_{21}$

Is complete superpotential continuous?

Spacetime analysis

$SU(N)_1$ has $N_f=N_c$, has quantum deformed moduli space for its mesons $M=Q'_{21}\, Q_{12}$ and barions B

(from Beasley-Witten instanton on 1)

$$W = \Lambda_1 \Phi M + \Lambda_1^{-2N+2} X \left(\det M - B\tilde{B} - \Lambda_1^{2N}\right)$$

Mesons are massive, baryons decouple, leaving $SU(N)_2$ pure SYM with dynamical scale

$$\Lambda_2'^{3N} = \Lambda_2^N \Lambda_1^{2N} = e^{-T_2} e^{-T_1}$$

The non-perturbative superpotential

(from 1/N-fractional instanton on 2)

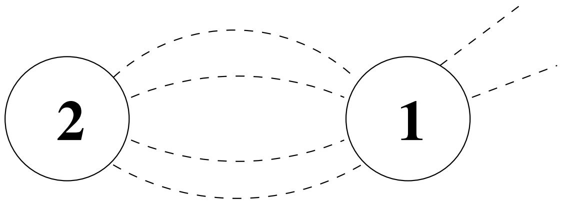
$$W = N \Lambda_2'^3 \simeq \left(e^{-T}\right)^{\frac{1}{N}}$$

Nice agreement!

What is the microscopic instanton description?

Microscopic interpretation

Superpotential is reconstructed by a combined effect of a Beasley-Witten instanton on 1, and a (fractional) instanton on 2



Skip discussion of fermion zero modes, and their coupling, similar to previous examples

Non-perturbative zero mode lifting revisited

Cleaner viewpoint on non-perturbative zero mode lifting

- Gauge instanton zero mode action by substituting instanton field configuration (as function of zero modes) in 4d action
- Modifications of 4d action induce modifications of zero mode action
- Effect of instanton 1 on instanton 2

Yet another view, inspired in [Beasley,Witten]

Consider an instanton with 2n extra fermion zero modes

$$\int d^4x\, d^2\theta\, \mathcal{O}_w \;=\; \int d^4x\, d^2\theta\; w_{\bar{i}_1\bar{j}_1\ldots\bar{i}_n\bar{j}_n}(\Phi)\, \mathcal{O}^{\bar{i}_1\bar{j}_1}\ldots\mathcal{O}^{\bar{i}_n\bar{j}_n} \qquad \mathcal{O}_{\bar{i}\bar{j}} \;=\; \bar{D}\bar{\Phi}^{\bar{i}}\,\bar{D}\bar{\Phi}^{\bar{j}}$$

Addition of a superpotential to W 4d action modifies to

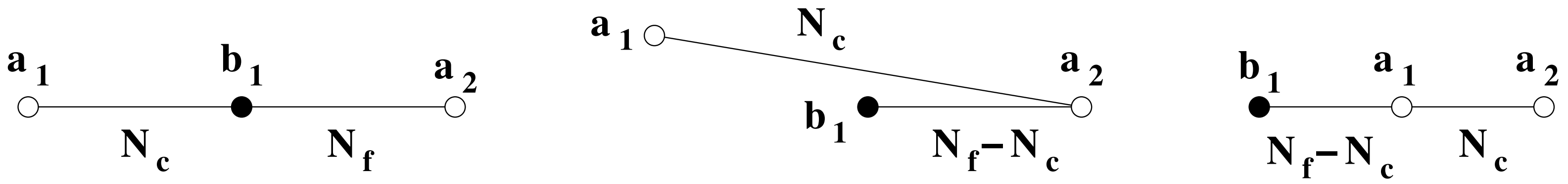
$\tilde{\mathcal{O}}^{\bar{i}\bar{j}} \;=\; \bar{D}\bar{\Phi}^{\bar{i}}\,\bar{D}\bar{\Phi}^{\bar{j}} + W^{\bar{i}\bar{j}}$ Pairs of zero modes soaked up by W

- If W arises from a 2nd instanton, it is a 2-instanton effect
- Generalize to addition of 4d F-terms (tree or from 2nd instanton)

Gauge instantons (cont.)

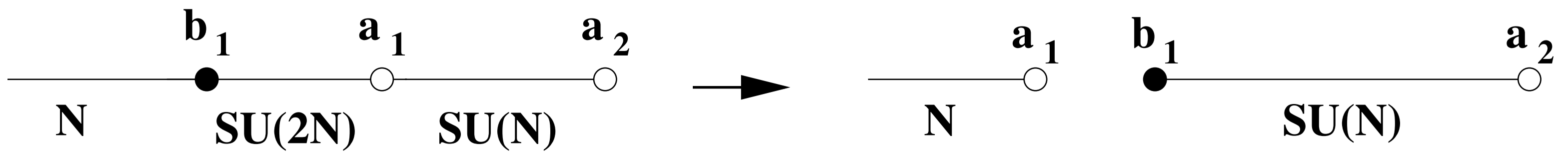
Seiberg duality

Some motions in moduli space correspond to Seiberg duality on 4d gauge D-branes



Continuity of superpotential related to matching of scales Λ in the field theory

Concrete examples similar to previous discussions, e.g. Nc=Nf



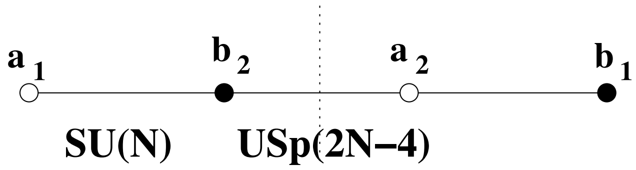
+ Many more from D-brane realization of Seiberg duality

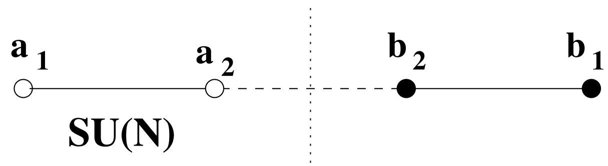
Gauge & non-gauge instantons

Gauge and non-gauge D-brane instantons are not so different!

In fact, they turn into each other upon motion in moduli space

Example:





USp confines and develops
W=Pf M

O(1) instanton couples to
adjoints M and induces
W=Pf M

(simplified discussion for sake of clarity, see paper for details)

Related discussion in a duality cascade by Aharony, Kachru

Systems of D3's at singularities [Ibanez, AU]

Multi-instanton processes can take place independently of the issue of lines of marginal stability

They are in fact present in some simple toroidal orientifolds!

In a recent paper, studied effects of D3-brane instantons on 4-cycles in systems of D3/D7-branes at singularities

Idea is to consider local D3/D7-brane models, and study D3-instanton on non-compact 4-cycles
Zero modes and their interactions easily computed

Interesting effects:

- mu-terms, lepton Yukawas in semirealistic models of D3/D7's at Z3 orbifold by [Aldazabal, Quevedo, Ibanez, AU]
- Multi-instanton effects

2-instanton processes on T6/Z3 orientifold [Ibanez, AU]

Take T6 /Z3 with O3-planes, with 8 D3's at origin, and 24 elsewhere

U(4) with three 6's from D3's at C3/Z3 orientifold

- O(1) fractional D(-1)-instanton gives an order six superp.
- D(-1)-brane gauge instantons, ADS or no supo, depending on branch [Seiberg, Intriligator]
- O(1) D3-branes on 4-cycles z_i=0 induce [Bianchi, Kiritsis; Bianchi, Fucito, Morales]

$$W = \sum_i e^{-T_i} \Phi_{6,i} \Phi_{6,i}$$

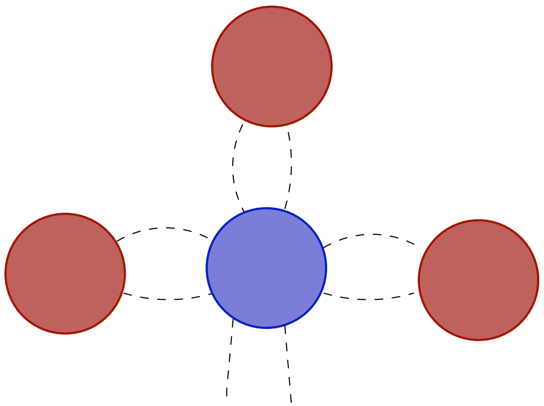
We are left with SU(4) pure SYM with scale Λ'

$$\Lambda'^{12} = \Lambda^9 m_1 m_2 m_3 = e^{-S} e^{-T_1} e^{-T_2} e^{-T_3}$$

which generates $W \sim (e^{-S} \prod_r e^{-T_r})^{\frac{1}{4}}$

Microscopic interpretation:

- Multi-instanton process
- Overall instanton has no intersection with U(4) branes
- Superpotential purely for closed moduli



F/M-theory picture and topology change

Think about the superpotential from viewpoint of M5-branes on F/M-theory on CY4

- In general, need to include contributions from M5's with extra zero modes (multiinstantons or singular divisors). Beyond $\chi(D)=1$
- IIB picture of our IIA gauge instanton transition, recombination of D7's intersecting over a curve.

We have shown the superpotential is continuous

The F-theory picture of recombination of intersecting D7's is a topology changing transition in CY4!

- Two D7's intersecting over a curve, locally $uv = 0$
- F-theory is a degenerate elliptic fibration, locally $xy = uv$
  (resolved) conifold (with vanishing 2-cycle)
- Recombined D7's are locally $uv = \epsilon$
- with F-theory lift $xy = uv - \epsilon$ deformed conifold

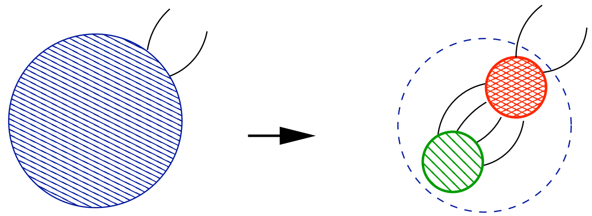
Non-perturbative superpotential continuous across transition

Non-trivial matching of W in top. different CY4's!

Conclusions

- Non-perturbative superpotential is continuous across lines of marginal stability

- Microscopically, thanks to contributions from multi-instanton processes



- Extra zero modes of an instanton can be soaked up by others in multi-instanton processes

Non-perturbative lifting of fermion zero modes

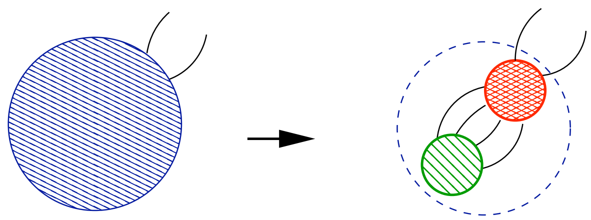
Superpotential contributions from U(1) instantons
Evades usual criteria (e.g. of arithmetic genus, etc)

- Also, purely perturbative lifting of zero modes by interactions

- Many phenomena and systems in this circle of ideas

Outlook

Universality of contributions to non-perturbative superp.



(insensitive to D-terms inside instanton world-volumes)

Presumably related to universality of category of holomorf. branes?

Lifting of zero modes in multi-instanton processes

Define index for multi-instanton systems, robust under splitting?

Revisit model building applications

eg instanton scan for neutrino masses in [Schellekens, Ibanez, AU]

Any relation to other brane splittings? multicenter black holes,... [Denef;...]

Expect many other surprises from D-brane instantons