WORLDSHEET INSTANTONS vs D-STRING INSTANTONS on T^4/Z_2

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Foreword

- This talk is based on work in progress with J.F. Morales, partly motivated by a very elegant test of Heterotic / Type IIA duality performed by E. Kiritsis, N. Obers and B. Pioline
- We discuss (non) perturbative corrections to hypermultiplet geometry in a T^4/Z_2 compactification to $D = 6$ with dual description: heterotic (worldsheet instantons) and Type I (unoriented D-string instantons)
- We will gain some useful guidance in identifying the rules for non-perturbative effects induced by unoriented D-branes and a *controllable* not fully exploited mechanism for moduli stabilization related to anomaly cancellation in $D = 6$

Motivations

$SO(32)$ Heterotic / Type I duality well tested in $D = 10$ and in toroidal compactifications: BPS saturated thresholds, stable non BPS states ...

Workable dual pairs with lower or no supersymmetry ... harder to find

Freely acting orbifolds [Vafa, Witten '96; ... ; Camara, Dudas, Maillard, Pradisi '07]

Two prototypical examples:

T^4/Z_2 orbifold in $D = 6$ [MB, Sagnotti '91; ...; Gimon, Polchinski '96]

T^6/Z_3 orbifold in $D = 4$ [Angelantonj, MB, Pradisi, Sagnotti, Stanev '96; ...]

World-sheet instantons in heterotic string duals help clarifying the rules for multi-instanton calculus with unoriented D-branes

(Non) perturbative corrections to Hypermultiplets on T^4/Z_2 may translate into more interesting terms in LEEA's with lower susy

Plan

- Heterotic - Type I duality in $D \leq 10$
- Compactification on T^4/Z_2
- Hypermultiplet Geometry: (non) perturbative corrections
- Heterotic description
- Type I interpretation
- Outlook

Heterotic - Type I duality in $D \leq 10$

- In $D = 10$ strong - weak coupling duality $g_s^H = 1/g_s^I$, rescaling of tension $\alpha'_H = g_s^I \alpha'_I$ [Polchinski, Witten '95; Dabholkar, '95; Hull '95]
- Anomaly related couplings and chiral spectrum
- Heterotic fundamental string $\approx$ Type I D-string
- Heterotic NS penta-brane $\approx$ Type I D5-brane
- Toroidal Compactifications: precise match of BPS saturated couplings eg F^4 and susy related terms in the LEEA [Bachas, Fabre, Kiritsis, Obers; ...]
- In D dimensions: $\phi_H = \frac{6-D}{4}\phi_I + \frac{D-2}{8} \log\det G_I$ [Angelantonj, MB, Pradisi, Sagnotti, Stanev '96; Antoniadis, Partouche, Taylor '96]
- Worldsheet instantons vs D-string instantons [Becker, Becker, Strominger '97; ...; Vandoren's talk]

Compactification on T^4/Z_2 to $D = 6$: Type I description

Parent Type IIB theory
$\mathcal{N} = (2,0)$ susy, 21 tensor multiplets, $SO(5,21)/SO(5) \times SO(21)$

- Unoriented projection (Klein bottle, Ω9- and Ω5-planes):
  $\mathcal{N} = (1,0)$ sugra, 1 tensor (volume), 20 hypers (dilaton ...)
  $SO(4,20)/SO(4) \times SO(20)$ (4 untwisted, 16 twisted)
- Open strings (Annulus and Möbius), RR tadpole cancellation
  $U(16)_9 \times U(16)_5$ gauge group,
  D9- and D5-branes with $Tr\rho^{Z_2}_{5/9} = 0$ *i.e.* $\mathbf{32} \to i(\mathbf{16} - \mathbf{16}^*)$
  hypers $\mathbf{120}_{+2} + \mathbf{120}^*_{-2}$, $\tilde{\mathbf{120}}_{+2} + \tilde{\mathbf{120}}^*_{-2}$
  half-hypers $(\mathbf{16}_{+1}, \tilde{\mathbf{16}}^*_{-1}) + (\mathbf{16}^*_{-1}, \tilde{\mathbf{16}}_{+1})$

[MB, Sagnotti '90-91; Gimon, Polchinski '96]

Compactification on T^4/Z_2 to $D = 6$: Heterotic description

Compactification without vector structure: $\tilde{\omega}^{SW}_{2,YM} \neq 0$

[Berkooz, Polchinski, Schwarz, Seiberg, Witten '96; M.B. '97; Witten '97; ... Honecker, Trapletti '05]

$SO(32) \to U(16)$ via Z_4 phases $\lambda^A_{ws} \to (i)\lambda^u_{ws}, (-i)\lambda^{\bar{u}}_{ws}$
(yet, Z_2 on bilinears)

- Untwisted sector: $\mathcal{N} = (1,0)$ sugra, 1 tensor (dilaton), 4 neutral hypers (volume, ...): $SO(4,4)/SO(4) \times SO(4)$ charged hypers $\mathbf{120}_{+2} + \mathbf{120}^*_{-2}$
- Twisted sectors (16 fixed points): NO neutral hypers charged half hypers $\mathbf{16}_{-3} + \mathbf{16}^*_{+3}$

How to match with Type I?

Matching the Spectrum

Distribute one $D5$-brane per each fixed point: $U(16)_5 \to U(1)_5^{16}$
Anomaly cancellation

$$\mathcal{I}_8 = \sum_i \left(X_2^i \wedge X_6^i + X_4^i \wedge \tilde{X}_4^i \right) \quad \to \quad L_{GSS} = C_2^{RR} X_4 + \sum_f C_{0,f}^{RR} X_6^f$$

Type I photons become massive by eating twisted RR axions:
$\partial C_{0,f}^{RR} \to D C_{0,f}^{RR} = \partial C_{0,f}^{RR} + 4 A^{(9)} + A_f^{(5)}$ since $4 = 2^{\frac{(9+1)-(5+1)}{2}}$
Type I $A^I = A^{(9)} - 4 \sum_f A_f^{(5)}$ decouples from twisted closed string scalars and matches with heterotic photon A^H
Supersymmetric Higgs-like mechanism: full hypers are eaten
Efficient, not fully exploited mechanism for moduli stabilization
Remnant of $D = 6$ anomaly in $D = 4$: massive non-anomalous $U(1)$'s. [Antoniadis, Kiritsis, Tomaras '03; Anastasopoulos '04; Anastasopoulos, M.B., Dudas, Kiritsis '06]

Duality and dynamics in $D = 6$

Heterotic / Type I duality in $D = 6$, ϕ dilaton, ω volume modulus:

$$\phi_H = \omega_I \qquad , \qquad \phi_I = \omega_H$$

Supersymmetry: NO neutral couplings between vectors and hypers
Gauge couplings can only depend (linearly) on the scalar $\phi_H = \omega_I$ in unique tensor multiplet, $\phi_I = \omega_H$ in a neutral hyper
Type I gauge couplings completely determined by disk amplitudes
In heterotic string, they receives (only) a one-loop correction
Hypermultiplet geometry is tree-level exact in heterotic description.
Yet worldsheet instanton corrections $e^{-q(\Sigma)/\alpha'}$, neutral hypers q determine size of 2-cycles Σ in T^4/Z_2
Type I description, hypers receive both perturbative (string loops) and non-perturbative corrections from BPS Euclidean D-string instantons wrapping susy 2-cycles Σ in T^4/Z_2

Hypermultiplet Geometry

$\mathcal{N} = (1,0)$ susy in $D = 6$ allows for 4-hyperini contact term

$$\int d^6x \sqrt{g} \varepsilon^{abcd}_{SO(5,1)} \mathcal{W}_{IJKL}(q) \zeta^I_a \zeta^J_b \zeta^K_c \zeta^L_d$$

ζ^I_a hyperini ($I = 1,..2n_H$, $a = 1,..4$), $\mathcal{W}_{IJKL}(q)$ totally symmetric (encoding $Sp(2n_H)$ curvature), q^i hyper-scalars ($i = 1,..4n_H$)
Full $Sp(2) \times Sp(2n_H)$ curvature (Riemann) tensor

$$\mathcal{R}_{ij,kl} h^k_{Ir} h^l_{Js} = \varepsilon_{rs} \mathcal{H}_{ij,IJ} + \mathcal{E}_{IJ} \mathcal{K}_{ij,rs}$$

$h^i_{Ir}(q)$ quaternionic vielbein ($r = 1,2$), $Sp(2)$ curvature

$$\mathcal{K}_{ij,rs} = \kappa^2 (h_{i,Ir} h_j{}^I{}_s - h_{j,Ir} h_i{}^I{}_s)$$

and, at last (!), $Sp(2n_H)$ curvature

$$\mathcal{H}_{ij,IJ} = \kappa^2 (h_{i,Ir} h^r_{j,J} - h_{j,Ir} h^r_{i,J}) + h_i{}^{Kr} h_j{}^L{}_r \mathcal{W}_{IJKL}$$

Computational strategy

- Focus on a specific 4-hyperini amplitude

$$\mathcal{A}_{4hyper} = \langle V^{\zeta}_{\mathbf{16}} V^{\zeta}_{\mathbf{16}^*} V^{\zeta}_{\mathbf{16}} V^{\zeta}_{\mathbf{16}^*} \rangle$$

  absent at tree level for particular choice of fixed points
- Compute $\mathcal{A}_{4hyper}$ in the limit of vanishing momenta (subtracting IR divergences due to massless exchange, if necessary)
- Start with heterotic string, where tree level exact and deduce worldsheet instanton corrections
- Translate into Type I language and interpret the result in terms of perturbative and non-perturbative contributions
- Learn new rules for unoriented multi D-brane instantons

Heterotic 4-hyperini amplitude

Hyperini vertex operators

$$V^{\zeta}_{\mathbf{16}/\mathbf{16}^*} = \zeta^{u/\bar{u}}_{f,a}(p) S^a e^{-\varphi/2}(z) \tilde{\Sigma}^{u/\bar{u}}(\bar{z}) \sigma_f e^{ipX}(z, \bar{z})$$

σ_f bosonic Z_2-twist field ($h = 1/4$), $\tilde{\Sigma}^{u/\bar{u}} =: e^{\pm i\tilde{\phi}_u} \prod_v e^{\mp i\tilde{\phi}_v/4}$:
twisted ground-states ($h = 3/4$) for heterotic fermions $\tilde{\lambda}^{u/\bar{u}}$
Use $SL(2, C)$ on sphere to set $z_1 \to \infty$, $z_2 \to 1$, $z_3 \to z$, $z_4 \to 0$
with cross ratio $z = z_{12}z_{34}/z_{13}z_{24}$
Bosonic conformal blocks

$$\langle c\tilde{c}(z_1) c\tilde{c}(z_2) c\tilde{c}(z_4)\rangle = |z_{12}z_{14}z_{24}|^2 \to |z_\infty|^4$$

$$\langle \prod_{i=1}^{4} e^{ip_i X}(z_i)\rangle = \prod_{i<j} z_{ij}^{2\alpha' p_i p_j} \to |z|^{\alpha' s} |1-z|^{\alpha' t}$$

Fermionic conformal blocks

Spacetime fermions and superghosts

$$\langle \prod_{i=1}^{4} S^{a_i} e^{-\varphi/2}(z_i) \rangle = \epsilon^{a_1 \ldots a_4} \prod_{i<j} z_{ij}^{-1/2} \longrightarrow \epsilon^{a_1 \ldots a_4} z_\infty^{-3/2} (1-z)^{-1/2} z^{-1/2}$$

Heterotic internal fermions

$$\langle \prod_{i=1}^{4} \tilde{\Sigma}^{u_i}(\bar{z}_i) \rangle = \left(\frac{\bar{z}_{13}\bar{z}_{24}}{\bar{z}_{12}\bar{z}_{14}\bar{z}_{23}\bar{z}_{34}} \right)^{1/2} \left(\frac{\delta^{u_1 \bar{u}_2} \delta^{u_3 \bar{u}_4}}{\bar{z}_{12}\bar{z}_{34}} + \frac{\delta^{u_1 \bar{u}_4} \delta^{u_3 \bar{u}_2}}{\bar{z}_{14}\bar{z}_{23}} \right)$$

$$\longrightarrow \bar{z}_\infty^{-3/2} (1-\bar{z})^{-1/2} \bar{z}^{-1/2} \left(\frac{\delta^{u_1 \bar{u}_2} \delta^{u_3 \bar{u}_4}}{\bar{z}} + \frac{\delta^{u_1 \bar{u}_4} \delta^{u_3 \bar{u}_2}}{1-\bar{z}} \right)$$

Twist field correlator

Z_2-twist field correlator [Dixon, Friedan, Martinec, Shenker '87]

$$\langle \prod_{i=1}^{4} \sigma_{f_i}(z_i, \bar{z}_i) \rangle \to |z_\infty|^{-1} \mathcal{C}_{qu}(z, \bar{z}) \, \mathcal{C}_{cl} \begin{bmatrix} \vec{f}_{12} \\ \vec{f}_{13} \end{bmatrix} (z, \bar{z})$$

$\mathcal{C}_{qu}$ quantum part independent of twist-field locations *i.e.* choice of 4 out 16 fixed points $\vec{f}_i = 1/2(\epsilon_i^1, \epsilon_i^2, \epsilon_i^3, \epsilon_i^4)$ with $\epsilon_i^a = 0, 1$ and $\sum_i \vec{f}_i = \vec{0} \bmod \Lambda(T^4)$.

$\mathcal{C}_{cl}$ classical part accounting for worldsheet instantons depending on $\vec{f}_{ij} = \vec{f}_i - \vec{f}_j$

Map to torus, doubly covering sphere with two Z_2 branch cuts

$$z = \vartheta_3^4(\tau) / \vartheta_4^4(\tau)$$

Full amplitude

The quantum and classical parts of 4-twist correlator

$$\mathcal{C}_{qu}(z,\bar{z}) = 2^{-8/3}\,|z(1-z)|^{-1/3}\,\tau_2^{-2}|\eta(\tau)|^{-8}$$

$$\mathcal{C}_{cl}\begin{bmatrix}\vec{f}_{12}\\ \vec{f}_{13}\end{bmatrix}(z,\bar{z}) = \sum_{\vec{m},\vec{n}} e^{-\frac{\pi}{\tau_2(z)}(\vec{m}+\vec{n}\tau+\vec{f}_{13}+\vec{f}_{12}\tau)\cdot(G+B)\cdot(\vec{m}+\vec{n}\bar{\tau}+\vec{f}_{13}+\vec{f}_{12}\bar{\tau})}$$

G_{ij} metric, B_{ij} antisymmetric tensor of T^4/Z_2 (neutral hypers)
Write z-integral as integral over torus modulus τ (for $s,t\to 0$)

$$\mathcal{A}^{f_1,f_2,f_3,f_4}_{u_1\bar{u}_2u_3\bar{u}_4} = \mathcal{V}(T^4)\int_{\mathcal{F}_2}\frac{d^2\tau}{\tau_2^2}\left(\frac{\bar{\vartheta}_4^4}{\bar{\vartheta}_3^4}\delta_{u_1\bar{u}_2}\delta_{u_3\bar{u}_4} - \frac{\bar{\vartheta}_4^4}{\bar{\vartheta}_2^4}\delta_{u_1\bar{u}_4}\delta_{u_3\bar{u}_2}\right)\mathcal{C}_{cl}\begin{bmatrix}\vec{f}_{12}\\ \vec{f}_{13}\end{bmatrix}$$

$\mathcal{F}_2$ fundamental domain of Γ_2, index 6 subgroup of $SL(2,Z)$
leaving invariant ϑ_{even} [Kiritsis, Obers, Pioline '00]

Integral folding

Map integral over $\mathcal{F}_2$ into integral over $\mathcal{F}$ fundamental domain of $SL(2,Z)$ summing over 6 images under $SL(2,Z)/\Gamma_2$ (modular invariance)

$$\int_{\mathcal{F}_2} \frac{d^2\tau}{\tau_2^2} \Phi(\tau,\bar{\tau}) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \sum_{s=1}^{6} \Phi(\tau_s,\bar{\tau}_s)$$

$\tau_s = \gamma_s(\tau)$, $\gamma_s = \{1, S, T, TS, ST, TST\}$,
For 4-hyperini amplitude

$$\Phi(\tau,\bar{\tau}) = \left(\frac{\bar{\vartheta}_4^4}{\bar{\vartheta}_3^4} \delta_{u_1\bar{u}_2}\delta_{u_3\bar{u}_4} - \frac{\bar{\vartheta}_4^4}{\bar{\vartheta}_2^4} \delta_{u_1\bar{u}_4}\delta_{u_2\bar{u}_3} \right) \mathcal{C}_{cl} \begin{bmatrix} \vec{f}_{12} \\ \vec{f}_{13} \end{bmatrix}$$

Special cases

Special cases where 4-hyperini amplitudes receive contribution only from BPS-like modes as in Type I (see later)

- All 4 hyperini located at the same fixed point $\vec{f}_{12} = \vec{f}_{13} = (\vec{0})$,
  The instanton sum $\mathcal{C}_{cl}\begin{bmatrix}\vec{0}\\ \vec{0}\end{bmatrix}$ is modular invariant.
  Sums over images produce

$$\sum_{s=1}^{6} \frac{\bar{\vartheta}_4^4}{\bar{\vartheta}_3^4}(\bar{\tau}_s) = 3 \quad , \quad \sum_{s=1}^{6} \frac{\bar{\vartheta}_4^4}{\bar{\vartheta}_2^4}(\bar{\tau}_s) = -3$$

  Final expression for the amplitude with $\vec{f}_1 = \vec{f}_2 = \vec{f}_3 = \vec{f}_4$

$$\mathcal{A}^{f_1,f_1,f_1,f_1}_{u_1\bar{u}_2u_3\bar{u}_4} = 3(\delta_{u_1\bar{u}_2}\delta_{u_3\bar{u}_4} + \delta_{u_1\bar{u}_4}\delta_{\bar{u}_2u_3})\,\mathcal{V}(T^4)\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\,\mathcal{C}_{cl}\begin{bmatrix}\vec{0}\\ \vec{0}\end{bmatrix}$$

Special cases ('ctd)

Hyperini located in pairs at two different fixed point

- $\vec{f}_{12} = \vec{f}_{13} = \vec{f}$ for $\delta_{u_1 \bar{u}_2} \delta_{u_3 \bar{u}_4}$ structure in 4-hyperini amplitude

$$\mathcal{A}^{f_1, f_2, f_2, f_1}_{u_1 \bar{u}_1 u_3 \bar{u}_3} = \mathcal{V}(T^4) \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \left(\mathcal{C}_{cl} \begin{bmatrix} \vec{f} \\ \vec{f} \end{bmatrix} + \mathcal{C}_{cl} \begin{bmatrix} \vec{f} \\ \vec{0} \end{bmatrix} + \mathcal{C}_{cl} \begin{bmatrix} \vec{0} \\ \vec{f} \end{bmatrix} \right)$$

- $\vec{f}_{12} = \vec{0}, \vec{f}_{13} = \vec{h}$ for $\delta_{u_1 \bar{u}_4} \delta_{u_3 \bar{u}_2}$ structure in 4-hyperini amplitude

$$\mathcal{A}^{f_1, f_1, f_3, f_3}_{u_1 \bar{u}_2 u_2 \bar{u}_1} = \mathcal{V}(T^4) \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \left(\mathcal{C}_{cl} \begin{bmatrix} \vec{0} \\ \vec{h} \end{bmatrix} + \mathcal{C}_{cl} \begin{bmatrix} \vec{h} \\ \vec{0} \end{bmatrix} + \mathcal{C}_{cl} \begin{bmatrix} \vec{h} \\ \vec{h} \end{bmatrix} \right)$$

Same integrals as for BPS saturated thresholds to F^4 in T^4 compactifications (with shifts) [Kiritsis, Obers, Pioline '00]

Shifted lattice sums

Poissòn resummation over $\vec{m}$

$$\mathcal{C}_{cl}\begin{bmatrix}\vec{0}\\ \vec{f}\end{bmatrix} = \frac{\tau_2^2}{\mathcal{V}(T^4)}\sum_{\vec{k},\vec{n}}(-)^{2\vec{f}\cdot\vec{k}} q^{p_L^2/2}\bar{q}^{p_R^2/2}$$

where $\vec{p}_{L/R} = \frac{1}{\sqrt{2}}(E^{-1}\vec{k} + E^t\vec{n})$, $EE^t = G$ and $B = 0$ for simplicity
Recognize shift orbifold partition function

$$\mathcal{C}_{cl}\begin{bmatrix}\vec{0}\\ \vec{f}\end{bmatrix}(G) + \mathcal{C}_{cl}\begin{bmatrix}\vec{f}\\ \vec{0}\end{bmatrix}(G) + \mathcal{C}_{cl}\begin{bmatrix}\vec{f}\\ \vec{f}\end{bmatrix}(G) = 2\mathcal{C}_{cl}\begin{bmatrix}\vec{0}\\ \vec{0}\end{bmatrix}(G_{(\vec{f})}) - \mathcal{C}_{cl}\begin{bmatrix}\vec{0}\\ \vec{0}\end{bmatrix}(G)$$

$G_{(\vec{f})}$ toroidal metric 'halved' along the direction $\vec{v} = 2\vec{f}$ by $SO(d,d)$ transformation

Thresholds in toroidal compactifications

Heterotic threshold corrections to F^4 terms on T^d

$$\mathcal{I}_d[\Phi] = \mathcal{V}_d \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \sum_M e^{2\pi i \mathcal{T}(M)} e^{-\frac{\pi Im\mathcal{T}(M)}{\tau_2 Im\mathcal{U}(M)}|\tau - \mathcal{U}(M)|^2} \Phi(\tau)$$

$$= \mathcal{I}_d^{triv}[\Phi] + \mathcal{I}_d^{deg}[\Phi] + \mathcal{I}_d^{ndeg}[\Phi]$$

where $M = (\vec{n}, \vec{m})$ and

$$\mathcal{U}(M) = \frac{1}{\mathcal{G}_{11}}(\mathcal{G}_{12} + i\sqrt{\det \mathcal{G}}) \quad , \quad \mathcal{T}(M) = \mathcal{B}_{12} + i\sqrt{\det \mathcal{G}}$$

with $\mathcal{G} = M^t G M \quad , \quad \mathcal{B} = M^t B M$

[Dixon, Kaplunovsky, Louis '89; Bachas, Fabre '97; + Kiritsis, Obers '98, Lerche, Stieberger '99]

Orbits

Trivial orbit $M=0$, $\mathcal{I}^{triv}_{d,d}[\Phi]=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\Phi\to\mathcal{I}^{triv}_{d,d}[1]=\frac{\pi^2}{3}\mathcal{V}_d$

Degenerate orbits $M\neq 0$, $\det(M^{i,j})=n^i m^j-n^j m^i=0\ \ \forall i,j$,
Choose $\vec{n}=0$ representative and unfold $\mathcal{F}$ to strip
$\mathcal{S}=\{|\tau_1|<1/2,\tau_2>0\}$
$\mathcal{I}^{deg}_{d,d}[\Phi]=\mathcal{V}_d\int_{\mathcal{S}}\frac{d^2\tau}{\tau_2^2}\sum_{\vec{m}\neq\vec{0}}e^{-\frac{\pi}{\tau_2}\vec{m}^tG\vec{m}}\Phi\to\mathcal{I}^{deg}_{d}[1]=\mathcal{V}_d\mathcal{E}^{SL(d)}_{\mathbf{d}}(G)$

Non degenerate orbits at least one $\det(M_{ij})=n^i m^j-n^j m^i\neq 0$,
Choose representative $\vec{n}^\alpha=0$ for $\alpha=1,..,k$, $m^\alpha\neq 0$,
$n^{\bar\alpha}>m^{\bar\alpha}\geq 0$ and unfold $\mathcal{F}$ to upper half plane $\mathcal{H}^+$

$$\mathcal{I}^{ndeg}_{d,d}[\Phi]=\mathcal{V}_d\int_{\mathcal{H}^+}\frac{d^2\tau}{\tau_2^2}\sum_{(n^{\bar\alpha},0;m^{\bar\alpha},m^\alpha)}e^{2\pi i\mathcal{T}(M)}e^{-\frac{\pi Im\mathcal{T}(M)}{\tau_2 Im\mathcal{U}(M)}|\tau-\mathcal{U}(M)|^2}\Phi$$

$$\to\mathcal{I}^{deg}_{d,d}[1]=\mathcal{V}_d\mathcal{E}^{SO(d,d)}_{\mathbf{V},s=1}(G,B)$$

Generalized Eisenstein series

Generalized Eisenstein series for $h \in G(R)/K$ in the representation $\mathcal{R}$ at level s

$$\mathcal{E}^{G(Z)}_{\mathcal{R},s}(h) = \sum_{m \in \Lambda_{\mathcal{R}}/\{0\}} \delta_{BPS}(m \wedge m)[m^t \mathcal{R}^t(h)\mathcal{R}(h)m]^{-s}$$

E.g. for $SL(d,Z)$ standard Kaluza-Klein or winding modes

$$\mathcal{E}^{SL(d,Z)}_{\mathbf{d},s}(G) = \sum_{\vec{n} \neq \vec{0}} [\vec{n}^t G \vec{n}]^{-s}$$

For $SO(4,4;Z)$, triality

$$\mathcal{E}^{SO(4,4;Z)}_{\mathbf{V},s=1}(G,B) = \mathcal{E}^{SO(4,4;Z)}_{\mathbf{S},s=1}(G,B) = \mathcal{E}^{SO(4,4;Z)}_{\mathbf{C},s=1}(G,B) = \frac{1}{2}\mathcal{I}_{4,4}(G,B)$$

Type I interpretation

Using heterotic Type I duality one can map the above result into a Type I amplitude including all perturbative and non-perturbative effects

$$\sqrt{\det G_H} = \phi_I \quad , \quad \phi_H = \sqrt{\det G_I}$$

and

$$B^H_{ij} = C^I_{ij}$$

Type I amplitude

Type I vertex operators, Chan-Paton Λ replace heterotic λ

$$V^{\zeta}_{\mathbf{16}} = \zeta^{f}_{a}(p) S^{a} e^{-\varphi/2} \sigma_{f} e^{ipX} \Lambda^{u}_{f} \qquad V^{\zeta}_{\mathbf{16}^*} = \zeta^{f}_{a}(p) S^{a} e^{-\varphi/2} \sigma_{f} e^{ipX} \bar{\Lambda}^{f}_{u}$$

The Chan-Paton factor projects onto $U(16) \times U(1)^{16}_{5}$ singlets

$$Tr(\Lambda^{u_1}_{f_1} \Lambda^{\bar{u}_2}_{f_2} \Lambda^{u_3}_{f_3} \Lambda^{\bar{u}_4}_{f_4}) = \delta_{f_1 f_2} \delta_{f_3 f_4} \delta^{u_1 \bar{u}_4} \delta^{u_3 \bar{u}_2} + \delta_{f_1 f_4} \delta_{f_3 f_2} \delta^{u_1 \bar{u}_2} \delta^{u_3 \bar{u}_4}$$

Perturbative contribution only when all f_i equal or $f_1 = f_2$ or $f_2 = f_3$ (opposite charge). NO perturbative contribution for f_i all different from one another or for $f_1 = f_3$ (same charge).
Perfect agreement with special cases in heterotic amplitudes exposing only BPS-like contributions

Open string conformal blocks (disk)

Insertions on the boundary of the disk, real line $z_i = x_i$

$$\langle \prod_i S^{a_i} e^{-\varphi/2}(x_i) \rangle = \frac{\varepsilon^{abcd}}{(x_{12}x_{34}x_{13}x_{24}x_{14}x_{23})^{1/2}}$$

Set $x_1 \to \infty$, $x_2 \to 1$, $x_3 \to x$ and $x_4 \to 0$ where $x = x_{12}x_{34}/x_{13}x_{24}$ $SL(2, R)$ invariant cross ratio. Jacobian from c ghost correlator

$$\langle c(x_1)c(x_2)c(x_4) \rangle = x_{12}x_{14}x_{24}$$

Open string 4-twist correlator

Z_2-twist field correlator for open strings with 4 N-D boundary conditions [Cvetic, Papadimitriou '03; Klebanov, Witten '04; Antoniadis, Benakli, Eugier '04; ...]

Quantum part, independent of location f_i of twist fields,

$$\mathcal{C}_{qu} = [x(1-x)]^{-1/3} t(x)^{-2} \eta(it)^{-4}$$

with $x = \vartheta_3^4(it)/\vartheta_4^4(it)$ for torus 4-fold covering the disk

Classical part from exchange of (massive) open string modes stretched between (different) fixed points

$$\mathcal{C}_{cl} = \delta_{f_1 f_2}\delta_{f_3 f_4} \sum_{\vec{k}} (-)^{2\vec{k}\cdot\vec{f}_{13}} e^{-\pi t \vec{k}^t G^{-1}\vec{k}} + \delta_{f_1 f_4}\delta_{f_3 f_2} \sum_{\vec{n}} e^{-\pi t (\vec{n}+\vec{f}_{12})^t G (\vec{n}+\vec{f}_{12})}$$

Amusing agreement with heterotic special cases!

ED-string corrections

So far, no dependence on C_2^{RR} only on G and ϕ

When the four hyperini located at different f_i only non-perturbative contribution from (regular) ED-strings wrapping supersymmetric (untwisted) two cycles $\Sigma \approx T^2/Z_2 = S^2$, passing through the four fixed points

Fractional ED-strings wrapping the 16 collapsed 'rigid' 2-cycles (since corresponding moduli eaten by anomalous $U(1)_5^{16}$) may contribute to amplitudes having also perturbative contributions

Use by now well established heterotic / Type I duality to deduce 'exact' 4-hyperini amplitude

Attempt D-string instanton interpretation

ED-string spectrum

Three sectors of open string excitations:

- E1-E1 strings (2 N-N, 8 D-D)
- E1-D9 strings (2 N-N, 8 N-D),
- E1-D5 strings (2 D-D, 8 N-D)

Alternatively, after T-duality along the wrapped 2-cycle,
$E1 \to E(-1)$, $D9 \to D7_9$, $D5 \to D7_5$
Residual symmetry of spectrum and interactions $SO(9,1) \to SO(5,1) \times SU(2) \times SU(2) \to SO(5,1) \times SO(2)_E \times SO(2)$

Spectrum

- E1-E1 strings with Chan-Paton group $U(1)$:
  Bosonic zero-modes: X^I in $\mathbf{8}_v$ yield $\mathbf{6}_v$ real X^μ in $\mathbf{K}\bar{\mathbf{K}}$ and complex Z in $\mathbf{K}(\mathbf{K}-\mathbf{1})/\mathbf{2}$ and its conjugate $\bar{Z}$ in $\bar{\mathbf{K}}(\bar{\mathbf{K}}-\mathbf{1})/\mathbf{2}$.
  Fermionic zero-modes: Θ^L_A in $\mathbf{8}_s$ decompose into $\bar{\mathbf{4}}^L_{+1/2}$ in $\mathbf{K}\bar{\mathbf{K}}$ and $\mathbf{4}^L_{-1/2}$ in $\mathbf{K}(\mathbf{K}+\mathbf{1})/\mathbf{2}$. The $\mathbf{8}_c$ spinors $\Theta^R_{\dot{A}}$ yield $\bar{\mathbf{4}}^R_{-1/2}$ in $\bar{\mathbf{K}}(\bar{\mathbf{K}}+\mathbf{1})/\mathbf{2}$ and $\mathbf{4}^R_{+1/2}$ in $\mathbf{K}\bar{\mathbf{K}}$
- E1-D9 strings: R-handed spinors ν^u in $(\mathbf{16}, \bar{\mathbf{K}})$ and their conjugate ν^*_u
- E1-D5$_f$ strings: L-handed spinors μ^f in $(\mathbf{1}^+_{\mathbf{f}}, \bar{\mathbf{K}})$ and their conjugate μ^*_f

Interactions

Interactions among *zero-modes* with at least one end on E1

$$L^B_{E1-E1} = Tr([X^\mu, X^\nu]^2 + [X^\mu, Z][X_\mu, Z^*] + [Z, Z^*]^2)$$

$$L^F_{E1-E1} = Tr([X^{[ab]}, \Theta^+_a]\tilde{\Theta}^-_b + Z^*\Theta^+_a\tilde{\Theta}^{+b} + Z\Theta^{-a}\tilde{\Theta}^-_a)$$

Leave exact $\mathcal{N} = (1,0)$ supermoduli (e.g. for $K = 1$): 6 neutral X^ν and 4 charged Θ^{-a} for chiral supermeasure

$$L_{E1-D9-D5} = H_{\bar{u}f}\nu^{\bar{u}}\mu^f + ... + \Phi_{[uv]}\nu^u Z^* \nu^v + \Phi_{[fh]}\mu^f Z^* \mu^h + h.c$$

Should lift μ and ν zero-modes ...
Alternatively, non-zero mode sector (boundary conditions) of μ and ν contribute, as in BPS threshold corrections to F^4 in $D \leq 8$)

Outlook

- NO conclusions, only a fully non-perturbative amplitude
- Still some way to go ... and a lot to learn for multi D-brane instanton from duality with heterotic worldsheet instantons
- Hope to apply the same approach to models with lower susy in $D = 4$ after shift orbifold / non geometric reduction
- As a by-product of the analysis: economical mechanism of moduli stabilization: (non) anomalous $U(1)$'s. Fractional branes: twisted scalars. Magnetized, rotated or coisotropic branes: bulk scalars. [Antoniadis, Maillard '04; M.B.,Trevigne '05, ... w.i.p.]