# Some recent developments in heterotic compactifications

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There's been a lot of interest in the recent past in the landscape program.

One of the problems in the landscape program,
is that those string vacua are counted by low-energy effective field theories,
and it is not clear that all of those have consistent UV completions -- not all of them may come from an underlying quantum gravity.

(Banks, Vafa)

One potential such problem arises in heterotic $E_8 \times E_8$ strings.

The conventional worldsheet construction builds each $E_8$ using a ($\mathbf{Z}_2$ orbifold of a) set of left-moving fermions.

$$L = g_{\mu\nu} \partial \phi^\mu \partial \phi^\nu + i g_{i \bar{\jmath}} \overline{\psi}_+^{\bar{\jmath}} D_- \psi_+^i + h_{a \bar{b}} \overline{\lambda}_-^{\bar{b}} D_+ \lambda_-^a + \cdots$$

The fermions realize a Spin(16) current algebra at level 1, and the $\mathbf{Z}_2$ orbifold gives Spin(16)/$\mathbf{Z}_2$.

Spin(16)/$\mathbf{Z}_2$ is a subgroup of $E_8$,
and we use it to realize the $E_8$.

In more detail:

Adjoint rep of $E_8$ decomposes into adjoint of $Spin(16)/\mathbf{Z}_2$ + spinor:

$$\mathbf{248} = \mathbf{120} + \mathbf{128}$$

left R sector (→ 128)

left NS sector (→ 120)

So we take currents transforming in adjoint, spinor of $Spin(16)/\mathbf{Z}_2$, and form $E_8$ via commutation relations.

More, in fact: all $E_8$ d.o.f. are realized via $Spin(16)/\mathbf{Z}_2$

This construction has served us well for many years,
but,
in order to describe an $E_8$ bundle w/ connection,
that bundle and connection must be reducible to
$Spin(16)/\mathbf{Z}_2$.

After all, all info in kinetic term $h_{\alpha\beta}\lambda_-^\alpha D_+\lambda_-^\beta$

Can this always be done?

Briefly: Bundles -- yes (in dim 9 or less)
Connections -- no.

Heterotic swampland?

(R. Thomas)

# Reducibility of connections

On a p-pal G bundle,
even a trivial p-pal G bundle,
one can find connections with holonomy that fill out all of G,
and so cannot be understood as connections on a p-pal H bundle for H a subgroup of G:
just take a connection whose curvature generates the Lie algebra of G.

Thus, just b/c the bundles can be reduced, doesn't mean we're out of the woods yet.

(R. Thomas)

# Reducibility of connections

We'll build an example of an anomaly-free gauge field satisfying DUY condition

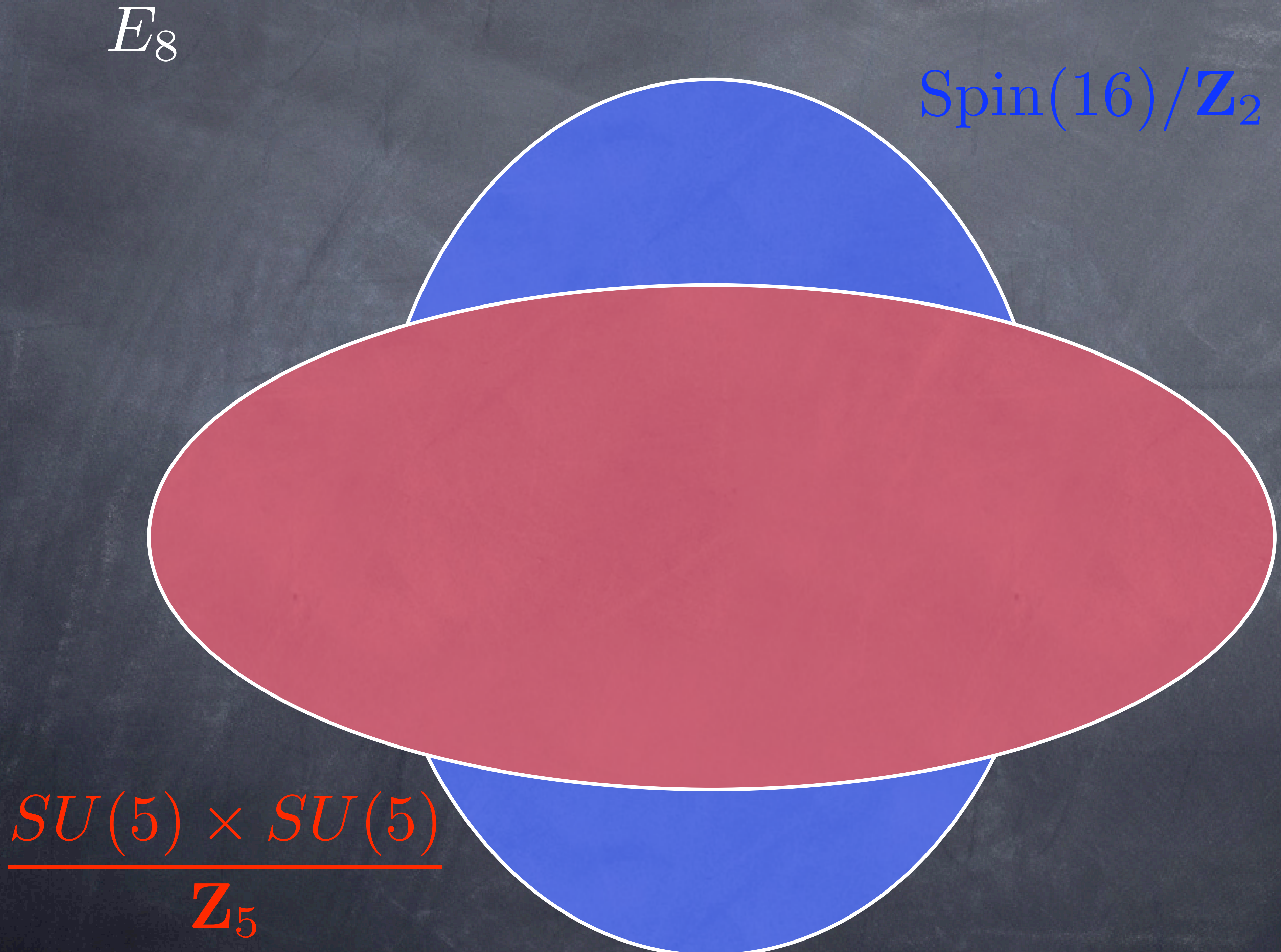
that does not sit inside Spin(16)/$\mathbf{Z}_2$.

The basic trick is to use the fact that $E_8$ has an

(SU(5)xSU(5))/$\mathbf{Z}_5$ subgroup

that does not sit inside Spin(16)/$\mathbf{Z}_2$.

We'll build an (SU(5)xSU(5))/$\mathbf{Z}_5$ connection.


$E_8$
$\mathrm{Spin}(16)/\mathbf{Z}_2$
$\frac{SU(5)\times SU(5)}{\mathbf{Z}_5}$

# Reducibility of connections

Build a stable SU(5) bundle on an elliptically-fibered K3 using Friedman-Morgan-Witten technology.

Rk 5 bundle with $c_1=0$, $c_2=12$ has spectral cover in linear system $|5\sigma + 12f|$, describing a curve of genus

$$g = 5c_2 - 5^2 + 1 = 36$$

together with a line bundle of degree

$$-(5 + g - 1) = -40$$

(R. Thomas)

# Reducibility of connections

Result is a (family of) stable SU(5) bundles with $c_2=12$ on K3.

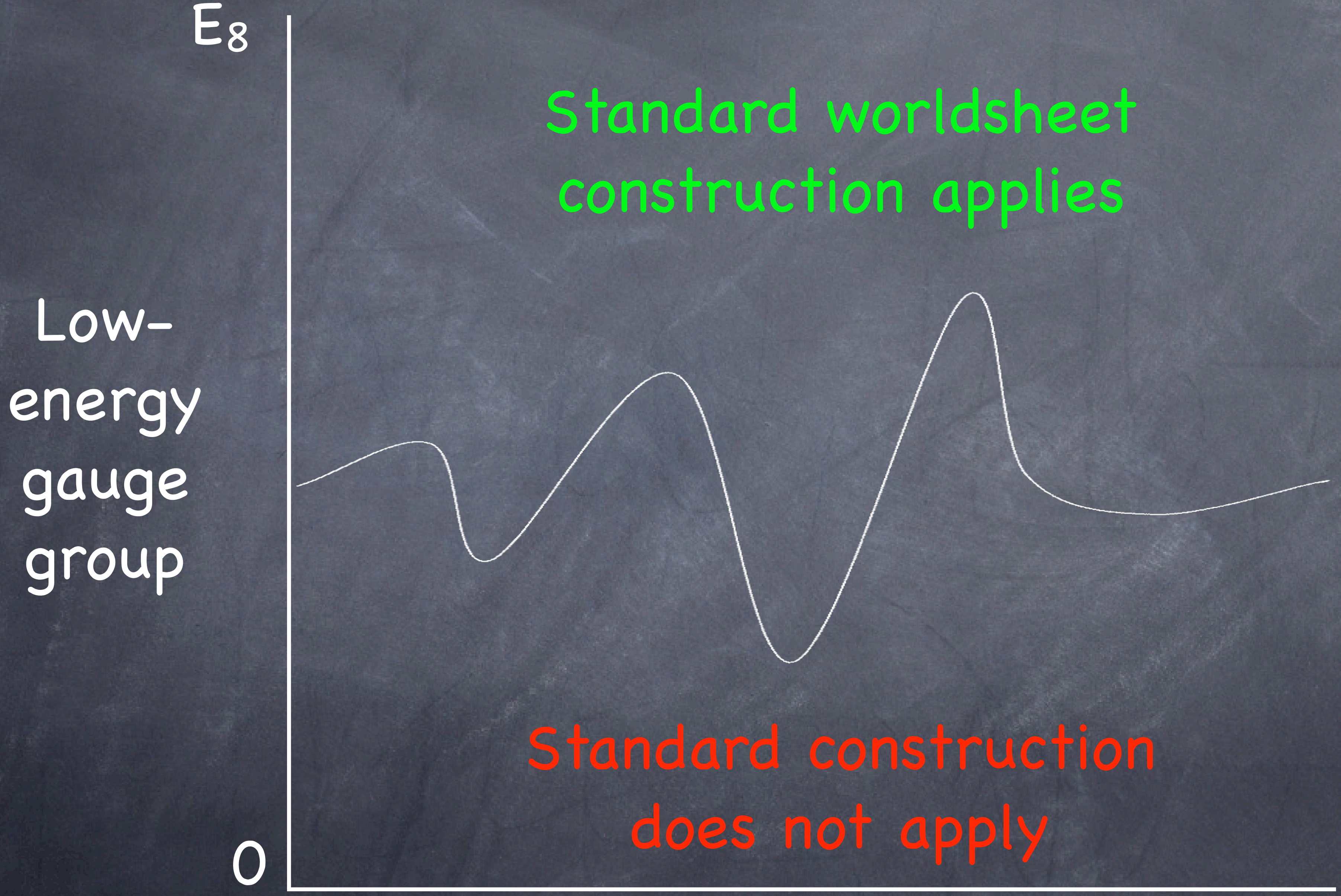
Holonomy generically fills out all of SU(5).

Put two together,

and project to $\mathbf{Z}_5$ quotient,

to get $(SU(5)\times SU(5))/\mathbf{Z}_5$ bundle w/ connection that satisfies anomaly cancellation + DUY.

# Lessons for the Landscape




Statistics on trad'l CFT's artificially favors large gps

# Alternative $E_8$ constructions

Next, we'll describe alternative worldsheet constructions, starting with 10d flat space.

**Idea:**

Replace $Spin(16)/\mathbf{Z}_2$ with other maximal-rank subgroups of $E_8$, realized as orbifolds of abstract current algebras
(since only U(n), Spin(n) have level 1 free field rep's)

To be specific, we'll describe $( SU(5) \times SU(5) )/\mathbf{Z}_5$

# Alternative $E_8$ constructions

Take current algebras for two copies of SU(5) at level 1, and orbifold by a $\mathbf{Z}_5$

Check: central charge of each SU(5) = 4, so adds up to 8 = central charge of $E_8$ ✓

A more convincing check: characters / left-moving partition function....

# Alternative $E_8$ constructions

For Spin(16)/$\mathbf{Z}_2$, corresponding to the decomposition

**248 = 120 + 128**

there is a decomposition of characters/left-moving partition f'ns:

$$\chi_{E_8}(\mathbf{1}, q) = \chi_{Spin(16)}(\mathbf{1}, q) + \chi_{Spin(16)}(\mathbf{128}, q)$$

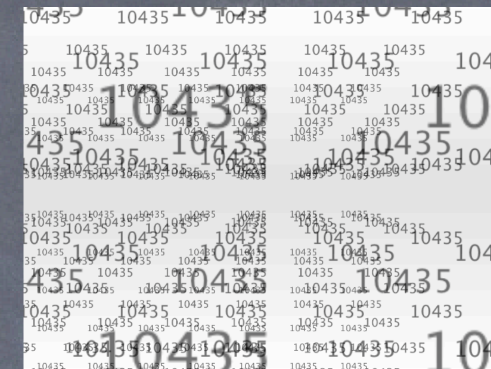
For $SU(5)^2/\mathbf{Z}_5$, from the decomp' of adjoint

**248 = (1,24) + (24,1) + (5,10*) + (5*,10) + (10,5) + (10*,5*)**

get a prediction for characters:

$$\chi_{E_8}(\mathbf{1}, q) = \chi_{SU(5)}(\mathbf{1}, q)^2 + 4\,\chi_{SU(5)}(\mathbf{5}, q)\,\chi_{SU(5)}(\mathbf{10}, q)$$

# Alternative $E_8$ constructions




Check: characters

$$\chi_{SU(5)}(\mathbf{1},q) = \frac{1}{\eta(\tau)^4}\sum_{\vec{m}\in\mathbf{Z}^4} q^{\left(\sum m_i^2 + (\sum m_i)^2\right)/2}$$

$$\chi_{SU(5)}(\mathbf{5},q) = \frac{1}{\eta(\tau)^4}\sum_{\vec{m}\in\mathbf{Z}^4,\sum m_i\equiv 1\ mod\ 5} q^{\left(\sum m_i^2 - \frac{1}{5}(\sum m_i)^2\right)/2}$$

$$\chi_{SU(5)}(\mathbf{10},q) = \frac{1}{\eta(\tau)^4}\sum_{\vec{m}\in\mathbf{Z}^4,\sum m_i\equiv 2\ mod\ 5} q^{\left(\sum m_i^2 - \frac{1}{5}(\sum m_i)^2\right)/2}$$

Can show (E. Scheidegger) (Kac, Sanielevici)

$$\chi_{E_8}(\mathbf{1},q) = \chi_{SU(5)}(\mathbf{1},q)^2 + 4\,\chi_{SU(5)}(\mathbf{5},q)\,\chi_{SU(5)}(\mathbf{10},q)$$

so $E_8$ worldsheet d.o.f. can be replaced by $SU(5)^2$

# Alternative $E_8$ constructions

In the statement for Spin(16)/$\mathbf{Z}_2$

$$\chi_{E_8}(\mathbf{1},q) \,=\, \chi_{Spin(16)}(\mathbf{1},q) \,+\, \chi_{Spin(16)}(\mathbf{128},q)$$

there is a $\mathbf{Z}_2$ orbifold implicit --

the **1** character is from untwisted sector,

the **128** character is from twisted sector.

Sim'ly, in the expression

$$\chi_{E_8}(\mathbf{1},q) \,=\, \chi_{SU(5)}(\mathbf{1},q)^2 \,+\, 4\,\chi_{SU(5)}(\mathbf{5},q)\,\chi_{SU(5)}(\mathbf{10},q)$$

there is a $\mathbf{Z}_5$ orbifold implicit. Good!: $SU(5)^2/\mathbf{Z}_5$

# Alternative $E_8$ constructions

Another max-rank subgroup: $SU(9)/\mathbf{Z}_3$.

Check: central charge = 8 = that of $E_8$.

$E_8$ conformal family decomposes as

**[1] = [1] + [84] + [84*]**

Can show

$$\chi_{E_8}(\mathbf{1}, q) = \chi_{SU(9)}(\mathbf{1}, q) + 2\,\chi_{SU(9)}(\mathbf{84}, q)$$

(Note $\mathbf{Z}_3$ orbifold implicit.)

So, can describe $E_8$ w.s. d.o.f. with $SU(9)/\mathbf{Z}_3$.

# Alternative $E_8$ constructions

To make this useful, however,
we'll need to fiber the current algebras over general base spaces.

To that end, we next describe fibered WZW models.

(J Distler, ES;  J Gates, W Siegel, etc)

# Fibered WZW models

First, recall ordinary WZW models.

$$S = -\frac{k}{2\pi}\int_\Sigma \mathrm{Tr}\ [g^{-1}\partial g g^{-1}\overline{\partial} g] - \frac{ik}{2\pi}\int_B d^3 y \epsilon^{ijk}\mathrm{Tr}\ [g^{-1}\partial_i g g^{-1}\partial_j g g^{-1}\partial_k g]$$

Looks like sigma model on mfld G w/ H flux.

Has a global $G_L \times G_R$ symmetry, with currents

$$J(z) = g^{-1}\partial g \qquad \overline{J}(\overline{z}) = \overline{\partial} g\, g^{-1}$$

obeying $$\overline{\partial} J(z) = \partial \overline{J}(\overline{z}) = 0$$

-- realizes G Kac-Moody algebra at level k

# Fibered WZW models

Let P be a principal G bundle over X, with connection A.

Replace the left-movers of ordinary heterotic with WZW model with left-multiplication gauged with A.

$$\frac{1}{\alpha'}\int_\Sigma \left( g_{i\bar{\jmath}} \partial_\alpha \phi^i \partial^\alpha \phi^{\bar{\jmath}} + \cdots \right)$$

← NLSM on X

$$- \frac{k}{4\pi}\int_\Sigma \mathrm{Tr}\ \left( g^{-1}\partial g g^{-1} \overline{\partial} g \right) - \frac{ik}{12\pi}\int_B d^3 y \epsilon^{ijk} \mathrm{Tr}\ \left( g^{-1}\partial_i g g^{-1} \partial_j g g^{-1} \partial_k g \right)$$

← WZW

$$- \frac{k}{2\pi}\mathrm{Tr}\ \left( (\partial \phi^\mu) A_\mu \overline{\partial} g g^{-1} + \frac{1}{2} (\partial \phi^\mu \overline{\partial} \phi^\nu) A_\mu A_\nu \right)$$

↑ Gauge left-multiplication

# Fibered WZW models

A WZW model action is invariant under gauging symmetric group multiplications,
but not under the chiral group multiplications that we have here.

Under $$g \mapsto hg$$

$$A_\mu \mapsto hA_\mu h^{-1} + h\partial_\mu h^{-1}$$

the classical action is not invariant.

As expected -- this is bosonization of chiral anomaly.

... but this does create a potential well-definedness issue in the fibered WZW construction ....

# Fibered WZW models

In add'n to the classical contribution, the classical action also picks up a quantum correction across coord' patches, due to right-moving chiral fermi anomaly.

To make the action gauge-invariant, we proceed in the usual form for heterotic strings:

assign a transformation law to the B field.

Turns out this implies

$$k\,\mathrm{ch}_2(\mathcal{E}) \;=\; \mathrm{ch}_2(TX)$$

Anom' canc' at level k

If that is obeyed, then action well-defined globally.

# Fibered WZW models

The right-moving fermion kinetic terms on the worldsheet couple to H flux:

$$\frac{i}{2} g_{\mu\nu} \psi_+^\mu D_{\overline{z}} \psi_+^\nu$$

where

$$D_{\overline{z}} \psi_+^\mu = \overline{\partial} \psi_+^\mu + \overline{\partial} \phi^\mu \left( \Gamma^\nu{}_{\sigma\mu} - H^\nu{}_{\sigma\mu} \right) \psi_+^\sigma$$

To make fermion kinetic terms gauge-invariant, set

$$H = dB + (\alpha') \left( k CS(A) - CS(\omega) \right)$$

➔ $k \,\mathrm{ch}_2(\mathcal{E}) = \mathrm{ch}_2(TX)$ Anomaly-cancellation

# Fibered WZW models

Demand (0,2) supersymmetry on base.
Discover an old faux-susy-anomaly in subleading terms in $\alpha'$

(Sen)

Susy trans' in ordinary heterotic string:

$$\delta\lambda_- = -i\epsilon\psi_+^\mu A_\mu \lambda_-$$

-- same as a chiral gauge transformation, with parameter $-i\epsilon\psi_+^\mu A_\mu$

-- b/c of chiral anomaly, there is a quantum contribution to susy trans' at order $\alpha'$

-- appears classically in bosonized description

# Fibered WZW models

(0,2) supersymmetry:

One fermi-terms in susy transformations of:

NLSM Base: $$\frac{1}{\alpha'}\int_\Sigma (i\alpha\psi^{\bar{\imath}})\bar{\partial}\phi^\mu\partial\phi^\nu\,(H - dB)_{\bar{\imath}\mu\nu}$$

WZW fiber: $$-k\int_\Sigma (i\alpha\psi^{\bar{\imath}})\bar{\partial}\phi^\mu\partial\phi^\nu CS(A)_{\bar{\imath}\mu\nu}$$

Quantum: $$\int_\Sigma (i\alpha\psi^{\bar{\imath}})\bar{\partial}\phi^\mu\partial\phi^\nu CS(\omega)_{\bar{\imath}\mu\nu}$$

$\Rightarrow$ $$H = dB + \alpha'\,(kCS(A) - CS(\omega))$$ for susy to close

# Fibered WZW models

Massless spectrum:

In an ordinary WZW model, the massless spectrum is counted by WZW primaries, which are associated to integrable rep's of G.

Here, for each integrable rep R of the principal G bundle P,
we get an associated vector bundle $\mathcal{E}_R$.

Massless spectrum = $H^{\cdot}\left(X, \mathcal{E}_R\right)$ for each $R$.

# Fibered WZW models

Massless spectrum:

**Example**: G = SU(n), level 1

Here the integrable reps are the fundamental **n** and its exterior powers.

Massless spectrum: $H^{\cdot}\left(X, \Lambda^{\cdot}\mathcal{E}\right)$ ✓

(Distler-Greene, '88)

# Fibered WZW models

Elliptic genera:

These fibered WZW constructions realize the `new' elliptic genera of Ando, Liu.

Ordinary elliptic genera describe left-movers coupled to a level 1 current algebra;
these, have left-moving level k current algebra.

**Black hole applications?**

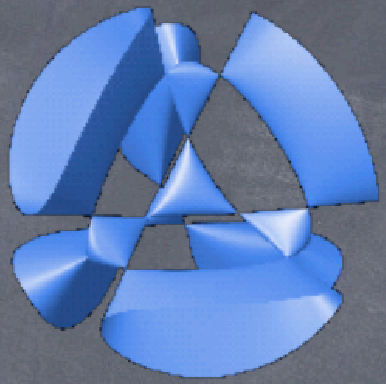
# Conclusions

There exist perturbative vacua which cannot be described by trad'l heterotic worldsheets; there exist alternative worldsheet constructions which describe them -- no swampland.

(J. Distler, E.S.)

Thank you for your time!

# Stacks



Stacks are a mild generalization of spaces.

One would like to understand strings on stacks:

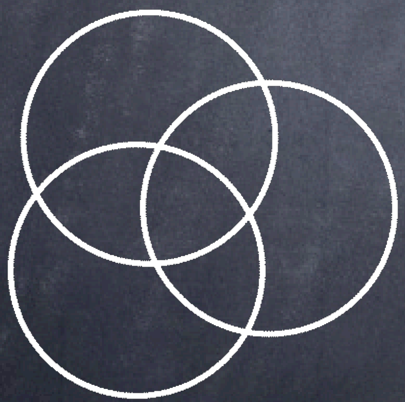
-- to understand the most general possible string compactifications

-- they often appear physically inside various constructions

# Stacks

What is a stack, roughly?

We can cover with coordinate charts, just like a manifold. So, locally, looks just like a space.



Globally, on a manifold, across triple overlaps, the coordinate charts close to 1. On a stack, across triple overlaps, the coordinate charts need only close up to an automorphism.

Locally, just like a space -- difference is global.

# Stacks

How to make sense of strings on stacks concretely?

Every* (smooth, Deligne-Mumford) stack can be presented as a global quotient

$$[X/G]$$

for $X$ a space and $G$ a group.

To such a presentation, associate a G-gauged sigma model on X.

(* with minor caveats)

# Stacks

If to [X/G] we associate ``G-gauged sigma model,"
then:

$[\mathbf{C}^2/\mathbf{Z}_2]$ defines a 2d theory with a symmetry called conformal invariance

$[X/\mathbf{C}^\times]$ defines a 2d theory w/o conformal invariance

Potential presentation-dependence problem:
fix with renormalization group flow
(just as with derived categories)

# Stacks

Potential problems / reasons to believe that presentation-independence fails:

* Deformations of stacks $\neq$ Deformations of physical theories
* Cluster decomposition issue for gerbes

These potential problems can be fixed. (ES, T Pantev)

Results include: mirror symmetry for stacks, new Landau-Ginzburg models, physical calculations of quantum cohomology for stacks, understanding of noneffective quotients in physics

# Quantum cohomology

Ex: Quantum cohomology ring of $\mathbf{CP}^N$ is

$$\mathbf{C}[x]/(x^{N+1} - q)$$

Quantum cohomology ring of $\mathbf{Z}_k$ gerbe over $\mathbf{CP}^N$ with characteristic class -n mod k is

$$\mathbf{C}[x,y]/(y^k - q_2, x^{N+1} - y^n q_1)$$

-- so, even though isn't a space, one can still make sense of these sorts of notions

# Toda duals

Ex: The "Toda dual" of **CP**$^N$ is described by the holomorphic function

$$W = \exp(-Y_1) + \cdots + \exp(-Y_N) + \exp(Y_1 + \cdots + Y_N)$$

The analogous duals to **Z**$_k$ gerbes over **CP**$^N$ are described by

$$W = \exp(-Y_1) + \cdots + \exp(-Y_N) + \Upsilon^n \exp(Y_1 + \cdots + Y_N)$$

where $\Upsilon$ is a character-valued field

(ES, T Pantev, '05)

# Mirrors to stacks

More generally, there exists a notion of toric stacks (Borisov, Chen, Smith, '04) which allows us to realize sigma models on many stacks in terms of simple 2d gauge theories.

Standard mirror constructions now produce discrete-valued fields, a new effect, which ties into the stacky fan description of (BCS '04).

(ES, T Pantev, '05)

# Gerbes

We now believe that for (banded, abelian) G-gerbes on spaces X,

(2,2) strings cannot distinguish between

-- the gerbe itself

-- disjoint union of copies of X, with ``flat B fields"

$$H^2(X,G) \xrightarrow{G \rightarrow U(1)} H^2(X,U(1))$$

(Comes up in mirror constructions; meaning of ``fields valued in roots of unity;" quantum cohomology implications)

(S. Hellerman, A Henriques, T Pantev, E.S., M Ando `06)

# Gerbes

This result can be applied to understand GLSM's.

Example: $\mathbf{P}^7[2,2,2,2]$

At the Landau-Ginzburg point, have superpotential

$$\sum_a p_a G_a(\phi) = \sum_{ij} \phi_i A^{ij}(p) \phi_j$$

* mass terms for the $\phi_i$, away from locus $\{\det A = 0\}$.
* leaves just the p fields, of charge -2
* $\mathbf{Z}_2$ gerbe, hence double cover

# Gerbes

Ex, cont'd

The GLSM realizes:

$P^7[2,2,2,2]$ ⟷ branched double cover of $\mathbf{P}_3$

where RHS realized at LG point via local $\mathbf{Z}_2$ gerbe structure + Berry phase.

(A. Caldararu, J. Distler, S. Hellerman, T. Pantev, E.S., in progress)

Non-birational twisted derived equivalence

Physical realization of Kuznetsov's homological projective duality

# Gerbes


Although (2,2) models decompose into a disjoint union, (0,2) models do not seem to in general.

Prototype: $\mathcal{O}(1) \longrightarrow \mathbf{P}_{[k,k,\cdots,k]}$ "$\mathcal{O}(1/k)$"

-- understanding of some of the 2d (0,4) theories appearing in geometric Langlands program

-- genuinely new string compactifications

Another lesson for the landscape:
many more string vacua may exist than previously enumerated.

# Future directions

- (0,2) mirror symmetry
- non-Kahler compactifications
- new heterotic string compactifications (eg gerbes), & new GLSM understanding
- supercritical strings