Instanton Recombination

based on: R. Blumenhagen, M. Cvetič,T. Weigand, R.R., 0708.0403

Robert Richter

University of Pennsylvania

Introduction

Non-perturbative effects are crucial in a proper understanding of string compactifications

- moduli stabilization
- generation of perturbatively forbidden couplings
- supersymmetry breaking
- destabilizing effects

for stringy instantons to contribute to the superpotential should exhibit only two fermionic zero modes

generic instanton carries 4 fermionic zero modes

How do we get rid of undesired zero modes?

- Orientifold projection
- fluxes
- lifting via integration over bosonic modes
- instanton recombination

U(1) Instanton zero modes

Type IIA: E2-instanton wrapping sLag 3-cycle Ξ in the CY

$\rightsquigarrow$ pointlike in space-time

uncharged zero modes E2-E2 sector

- x^μ, θ_α, $\overline{\tau}_{\dot\alpha}$
- $b_1(\Xi)$ chiral multiplets $\mathcal{C}^i = (c^i, \chi^i_\alpha)$ and their anti-chiral partners ($\overline{\mathcal{C}}^i = (\overline{c}^i, \overline{\chi}^i_{\dot\alpha})$)
- m, $\overline{m}$, μ_α, $\overline{\mu}_{\dot\alpha}$ at intersections of Ξ and Ξ'

zero mode	$(Q_E)_{Q_{ws}}$	Multiplicity
$m, \overline{m}$	$(2)_1$,$(-2)_{-1}$	$\frac{1}{2}\left(\Xi' \circ \Xi + \Pi_{O6} \circ \Xi\right)$
$\overline{\mu}^{\dot\alpha}$	$(-2)_{1/2}$	$\frac{1}{2}\left(\Xi' \circ \Xi + \Pi_{O6} \circ \Xi\right)$
μ^α	$(2)_{-1/2}$	$\frac{1}{2}\left(\Xi' \circ \Xi - \Pi_{O6} \circ \Xi\right)$

U(1) Instanton zero modes

charged zero modes E2-D6 sector

- project onto states odd under GSO-projection
- generically no bosonic zero modes
- chiral fermionic zero modes λ_a, $\overline{\lambda}_a$ at intersections of Ξ and a, a'

zero modes	Reps	number
$\lambda_{a,I}$	$(-1_E, \Box_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^+$
$\overline{\lambda}_{a,I}$	$(1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^-$
$\lambda_{a',I}$	$(-1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \Pi'_a]^+$
$\overline{\lambda}_{a',I}$	$(1_E, \Box_a)$	$I = 1, \ldots, [\Xi \cap \Pi'_a]^-$

$$Q_a(E2) = \mathcal{N}_a \, \Xi \circ (\Pi_a - \Pi'_a)$$

Orientifold action for E2's

(1) internal oscillator part same as in D6-brane case

(2) space-time excitations are transverse to the E2-instantons

(2a) additional minus sign for bosonic excitations in 4D

(2b) additional factor Γ^{1234} for fermionic zero modes
$\Gamma^{1234}\, S^{\dot\alpha}\, (S^{\alpha}) = 1(-1)$

(3) $\gamma_{\Omega\overline{\sigma},D6} = \pm\gamma^T_{\Omega\overline{\sigma},D6} \Longleftrightarrow \gamma_{\Omega\overline{\sigma},E2} = \mp\gamma^T_{\Omega\overline{\sigma},E2}$

for $O6^-$-plane: $\Omega_x = \Omega_x^T$ $\Omega_\theta = \Omega_\theta^T$ $\Omega_{\overline{\tau}} = -\Omega_{\overline{\tau}}^T$

$\Omega_{m/\overline{m}} = \Omega^T_{m/\overline{m}}$ $\Omega_\mu = -\Omega_\mu^T$ $\Omega_{\overline{\mu}} = \Omega_{\overline{\mu}}^T$

Instanton recombination

generic U(1) instanton exhibits non-desired $\overline{\tau}$-modes

by analogy with D6-branes, $\Xi \cup \Xi'$ might recombine into $\tilde{\Xi}$ invariant under Orientifold $\rightsquigarrow$ chance to get rid of $\overline{\tau}$-modes.

$U(1)_E$-charge: $\sum Q_E(\lambda^i) = \sum_a N_a \, (\Pi_a + \Pi_{a'}) \circ \Xi = 4\, \Pi_{O6} \circ \Xi$

for global consistency, in case $\Pi_{O6} \circ \Xi \neq 0$ need additional charged zero modes

$\rightsquigarrow$ ensures $U(1)_E$ invariant instanton measure

Instanton recombination

simplest case: $\Xi \circ \Xi' = \Pi_{O6} \circ \Xi = 1$

$$\int d\mathcal{M} = \int d^4x\, d^2\theta d^2\,\overline{\tau}\; dm\; d\overline{m}\; \underbrace{d^2\overline{\mu}^{\dot\alpha}}_{Q_E=4}\; \underbrace{\prod_b d\overline{\lambda}_b \prod_a \lambda_a}_{Q_E=-4}$$

Key couplings: $S_{E2} = (2m\overline{m} - \xi)^2 + m\overline{\tau}_{\dot\alpha}\overline{\mu}^{\dot\alpha}$

for $\xi > 0$ modes m tachyonic $\rightsquigarrow$ instanton recombination

bosonic modes constraint by D-term $m\overline{m} = \frac{1}{2}\xi$

with $m\overline{\tau}_{\dot\alpha}\overline{\mu}^{\dot\alpha}$ we can always pair up $\overline{\tau}$ and $\overline{\mu}$ modes

$\int d\mathcal{M} = \int d^4x\, d^2\theta\, \prod_a d\lambda_a \prod_b d\overline{\lambda}_b \int\, d|m|\, |m|^3\, \delta\left(|m|^2 - \xi/2\right)$

no way to soak up the 4 additional $\overline{\lambda}_b$ modes

Instanton recombination

non-chiral $E2 - E2'$ intersection $\rightsquigarrow$ no access of λ's required

$[\Xi' \cap \Xi]^+ = [\Xi' \cap \Xi]^- = 1, \quad [\Pi_{O6} \cap \Xi]^+ = [\Pi_{O6} \cap \Xi]^- = 1$

zero mode	$(Q_E)_{Q_{ws}}$	zero mode	$(Q_E)_{Q_{ws}}$
$m, \overline{m}$ $\overline{\rho}^{\dot\alpha}$	$(2)_1$,$(-2)_{-1}$ $(-2)_{1/2}$	$n, \overline{n}$ $\overline{\nu}^{\dot\alpha}$	$(-2)_1$,$(2)_{-1}$ $(2)_{1/2}$

$$\int d\mathcal{M} = \int d^4x\, d^2\theta d^2\,\overline{\tau}\; dm\;\, d\overline{m}\, d^2\overline{\rho}\, dn\;\, d\overline{n}\, d^2\overline{\nu} \prod_b d\overline{\lambda}_b \prod_a d\lambda_a$$

$$S_{E2} = (m\overline{m} - n\overline{n})^2 + \overline{\tau}_{\dot\alpha}\left(m\,\overline{\rho}^{\dot\alpha} - n\,\overline{\nu}^{\dot\alpha}\right)$$

no coupling $\overline{m}\,\overline{m}\,\overline{\nu}\,\overline{\nu}$! opposite to Uranga's case

D-term constraints $m = n$

$$\overline{\tilde\mu}^{\dot\alpha} = \overline{\rho}^{\dot\alpha} - \overline{\nu}^{\dot\alpha} \quad \text{and} \quad \overline{\mu}^{\dot\alpha} = \overline{\rho}^{\dot\alpha} + \overline{\nu}^{\dot\alpha}$$

$$\int d\mathcal{M} = \int d^4x\, d^2\theta d^2\,\overline{\tau}\; dm\;\, d\overline{m}\, d^2\overline{\mu} \prod_b d\overline{\lambda}_b \prod_a d\lambda_a$$

Instanton recombination

$\int d\mathcal{M} = \int d^4x\, d^2\theta d^2\,\overline{\tau}\; dm\; d\overline{m}\, d^2\overline{\mu} \prod_b d\overline{\lambda}_b \prod_a d\lambda_a$

measure of an O(1)instanton with one deformation mode

$\rightsquigarrow$ Superpotential contributions or higher fermionic F-terms

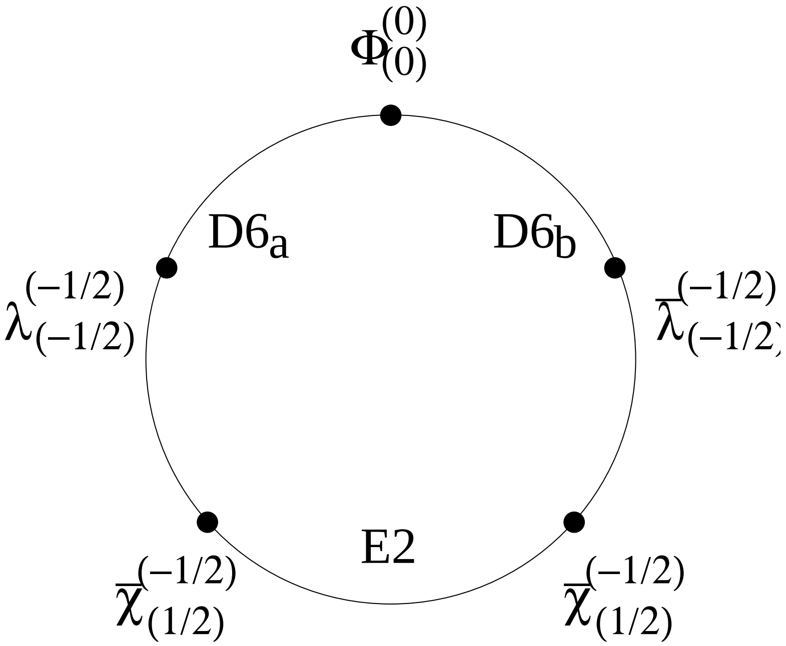
Superpotential contributions

O(1)-instanton with one deformation mode

zero modes: x^μ, θ^α, c, $\bar{c}$, $\overline{\chi}^{\dot{\alpha}}$

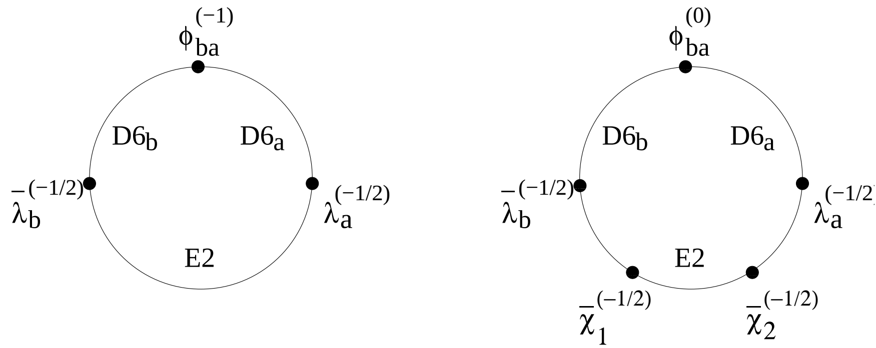
measure: $\int d^4x d^2\theta\, dc\, d\bar{c}\, d^2\overline{\chi}$

fermionic zero modes can be soaked up by $\overline{\chi\chi}\, \lambda_a \Phi \overline{\lambda}_b$



5-point amplitude

- background is $T^6 = T^2 \times T^2 \times T^2$
- non-rigid O(1) instanton



- Picture changing operator reduces the WS-charge in one torus.
- both $\overline{\chi}$ have live in different tori

Torus	$\overline{\chi}^1_{\dot{\alpha}}$	$\overline{\chi}^2_{\dot{\alpha}}$	$\overline{\lambda}_b \phi_{ba} \lambda_a$	Total $U(1)$ charge
T^2_1	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
T^2_2	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
T^2_3	$+\frac{1}{2}$	$+\frac{1}{2}$	-1	0

5-point amplitude

$$\mathcal{A} = C_{ij}^k \, (\overline{\chi}^i \overline{\chi}^j \, \overline{\lambda}_b \Phi_{ba} \lambda_a) \int_0^1 dx \int_0^1 dy \frac{x^{a_1}(1-x)^{b_1} y^{a_2}(1-y)^{b_2}}{|x-y|}$$

$$a_1 = -\theta^i_{\Xi b} - 1, \; c_1 = \theta^i_{\Xi a} - 1, \; a_2 = -\theta^j_{E2b} - 1, \; c_2 = \theta^j_{E2a} - 1$$

pole at $x \to y$ which (in case of D6-branes) corresponds to an exchange of bosonic $\overline{c}^k$

$$< \overline{\chi}^i \overline{\chi}^j \, \overline{\lambda}_b \Phi_{ba} \lambda_a > \sim < \overline{\chi}^i \overline{\chi}^j \overline{c}^k > < \overline{c}^k \overline{\lambda}_b \Phi_{ba} \lambda_a >$$

$-1 < a_i, b_i < 0 \to$ no further poles and contact term

The coupling $\mathcal{C}_{ij}^k \, \overline{\chi}^i \overline{\chi}^j \, \overline{\lambda}_b \Phi_{ba} \lambda_a$ exists !

Higher fermionic F-terms

simple example: $b_1(\Xi) = 1$

measure: $\int d^4x d^2\theta dc d\overline{c} d^2\overline{\chi}$

can generate higher fermionic F-terms à la Beasley Witten

$$S = \int d^4x\, d^2\theta\ w_{\overline{i}\overline{j}}(\Phi)\overline{\mathcal{D}}^{\dot\alpha}\overline{\Phi}^{\overline{i}}\overline{\mathcal{D}}_{\dot\alpha}\overline{\Phi}^{\overline{j}}$$

closed string sector: $\mathcal{T} = T + \theta^\alpha t_\alpha$ $\mathcal{U} = U + \theta^\alpha u_\alpha$

explicit CFT analysis gives couplings:

$$\theta_\alpha u^\alpha \qquad \overline{\chi}_{\dot\alpha}\overline{t}^{\dot\alpha} \qquad \theta\sigma^\mu\overline{\chi}\partial_\mu\overline{T}$$

obtain: $S = \int d^4x\, d^2\theta\ e^{-\mathcal{U}(\Xi)}\ f_{\overline{i},\overline{j}}\left(e^{-\mathcal{T}_i}, e^{-\Delta_i}\right)\overline{\mathcal{D}}^{\dot\alpha}\overline{\mathcal{T}}^{\overline{i}}\overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{T}}^{\overline{j}}$

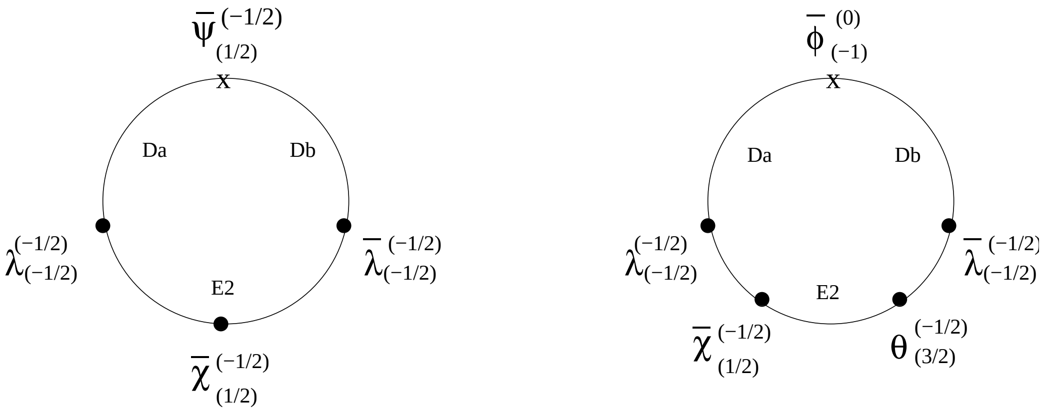
in case of additional λ-modes with coupling $< \overline{\lambda}_b\Phi_i\lambda_a >$

$$S = \int d^4x\, d^2\theta\ e^{-\mathcal{U}(\Xi)}\ \prod_i \Phi_i f_{\overline{i},\overline{j}}\left(e^{-\mathcal{T}_i}, e^{-\Delta_i}\right)\overline{\mathcal{D}}^{\dot\alpha}\overline{\mathcal{T}}^{\overline{i}}\overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{T}}^{\overline{j}}$$

Higher fermionic F-terms

can also pick up derivative terms involving open strings

relevant couplings: $\overline{\chi}\overline{\lambda}_b\overline{\psi}\lambda_a$ $\theta\sigma^\mu\overline{\chi}\,\overline{\lambda}_b\partial_\mu\overline{\phi}\lambda_a$



with the ones above $(\Phi = \phi + \theta\psi)$

$$S = \int d^4x\, d^2\theta\; e^{-\mathcal{U}(\Xi)}\; \prod_i \Phi_i\; f_{\bar{i},\bar{j}}\left(e^{-\mathcal{T}_i}, e^{-\Delta_i}\right) \overline{\mathcal{D}}^{\dot{\alpha}}\overline{\Phi}^{\bar{i}}\overline{\mathcal{D}}_{\dot{\alpha}}\overline{\Phi}^{\bar{j}}$$

Summary

- discussed the possible lifting of undesired zero modes due to instanton recombination
- in case of chiral intersection between E2 and E2' we obtain no superpotential contribution
- for non-chiral intersections between E2 and E2' we get instanton measure of a $O(1)$ instanton with $b_1(\Xi) = 1$
  - $\rightsquigarrow$ higher fermionic F-terms à la Beasley Witten
  - $\rightsquigarrow$ in generic situations get superpotential contributions