HETEROTIC FLUXES

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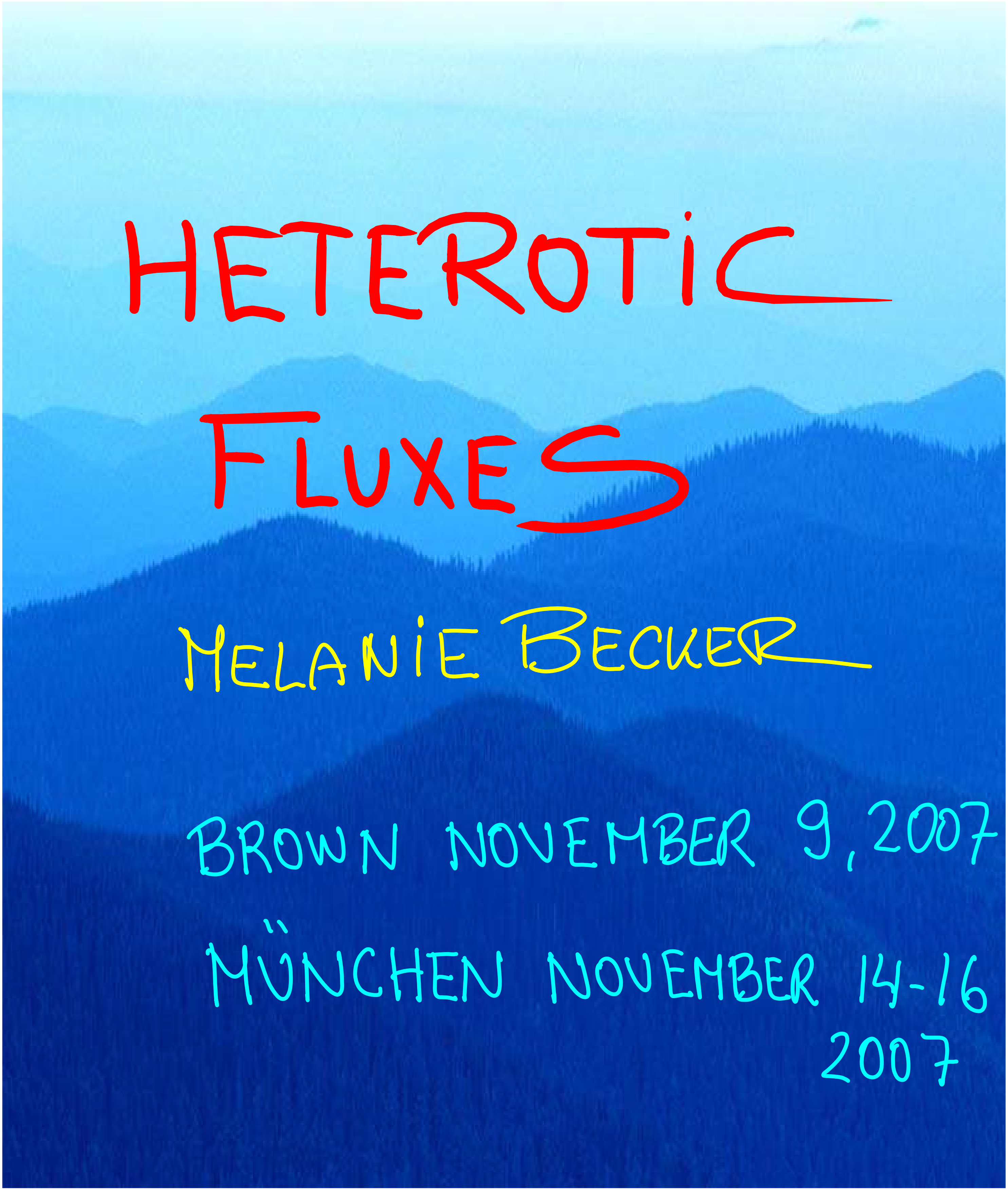


HETEROTIC FLUXES

M.B, L.S. TSENG + S.T YAU, in progress

MB, L.S TSENG + S.T. YAU, arXiv: 0706.4290 [hep-th]

M.B, L.S.TSENG + ST YAU, hep-th/0612290

K.BECKER, MB, J.X FU, L.S.TSENG + ST YAU
hep-th/0604137

K.BECKER, MB, K.DASGUPTA + P.S. GREEN
hep-th/0301161

K.BECKER, M.B. K.DASGUPTA, P.GREEN + E.SHARPE
hep-th/0310058

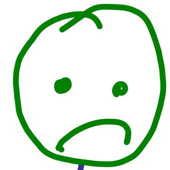
HETEROTIC FLUXES

Flux compactifications have received a great deal of attention in recent years :

PHYSICS : Moduli space problem ?
Standard model ?

MATH : torsional manifolds / Hitchin

Flux compactifications are much harder to analyze than their Calabi-Yau counterparts, especially in the heterotic theory where only a very limited number of examples is known!



And even the best known examples turn out to be remarkably hard to understand...

String duality nevertheless taught us that string theories on different geometrical spaces can be dual to each other!

Calabi-Yau $\longleftrightarrow$ Non-Kähler duality

It would be very interesting if a duality between a non-Kähler manifold with torsion and a conventional CY compactification of the heterotic theory existed!

GOAL Show that such a duality is indeed possible in the context of $\mathcal{N}=2,\ D=4$ compactifications of the heterotic string!

Adams, Ernebjerg, Lapan, Sethi, M.B, Tseng, Yau, K. Becker, Dasgupta, Rajesh, Aspinwall, Kallosh, Green, Sharpe ....

$$\begin{tikzcd} & \boxed{\dfrac{\text{M-theory}}{K3\times K3}} \arrow[dl, leftrightarrow, "\text{U(1) gauge bundle}"'] \arrow[dr, leftrightarrow, "\text{H-flux}"] & \\ \boxed{\dfrac{\text{Heterotic}}{K3\times T^2}\ ;\ F\neq 0\ \ \text{Kähler}} \arrow[rr, leftrightarrow] & & \boxed{\dfrac{\text{Heterotic}}{K3\ltimes T^2}\ ;\ H\neq 0\ \ \text{Non-Kähler}} \end{tikzcd}$$

Both backgrounds give $\mathcal{N}=2,\ D=4$

(thus solve susy constraints)

Strominger
De Wit et al

HETEROTIC TORSIONAL CONSTRAINTS

The bosonic part of the action of the 10D heterotic string contains $(g_{\mu\nu}, \phi, H, F)$

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{H^2}{2} - \frac{\alpha'}{4} \mathrm{tr} F^2 \right]$$

$$\delta\psi_M = \nabla_M \eta + \frac{1}{8} H_{MNP} \gamma^{NP} \eta$$

$$\delta\lambda = \gamma^M \partial_M \phi\, \eta + \frac{1}{12} H_{MNP} \gamma^{MNP} \eta$$

$$\delta\chi = \gamma^{MN} F_{MN} \eta$$

Demanding invariance under these transformations gives constraints ...

These constraints are formulated in terms of

$$J^{(1,1)}_{mn} = -i\,\eta^{+}\gamma_{mn}\eta$$ hermitian 2-form

$$\Omega^{(3,0)}_{mnp} = e^{-2\phi}\,\eta^{T}\gamma_{mnp}\eta$$ holomorphic 3-form

Calabi-Yau	Non-Kähler
$dJ = 0$	$d(\Vert\Omega\Vert^2 J\wedge J) = 0$
$d\Omega = 0$ Kähler	balanced $d\Omega = 0$
$H = 0$	$H = i(\bar\partial - \partial)J$
$\phi = \text{const}$	$e^{-2(\phi-\phi_0)} = \Vert\Omega\Vert$
	H,ϕ from geometry!

HERMITIAN YANG-MILLS

Both theories satisfy

$$F_{2,0} = F_{0,2} = 0 \quad ; \quad F_{m\bar{n}} J^{m\bar{n}} = 0$$

holomorphic (primitive)

ANOMALY CANCELLATION

$$dH = 2i\,\partial\bar{\partial} J = \frac{\alpha'}{4}\left(\mathrm{tr}\, R\wedge R - \mathrm{tr}\, F\wedge F\right)$$

Relates the gauge bundle with geometry.

The "standard" embedding is not allowed!

Models with $H \neq 0$ exhibit some rather interesting gauge symmetry breaking patterns! (more later)

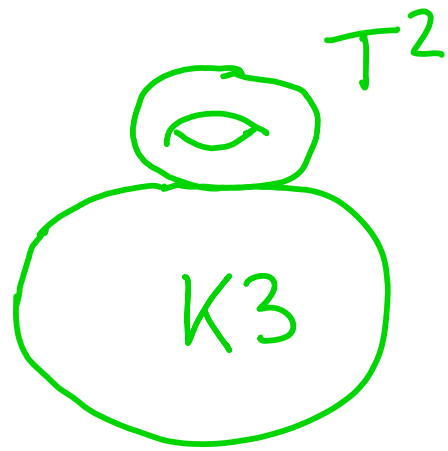
Let's look at a non-Kähler example...

EXAMPLE : T^2-BUNDLE OVER K3

· Rajesh, Dasgupta, Sethi

K. Becker, M.B, P. Green, Sharpe
Goldstein, Prokushkin

Fu, Yau, Tseng, Becker^2
Adams, Ernebjerg, Lapan



FSY GEOMETRY

The geometry is defined in terms of

$$J = e^{2\phi(y)} J_{K3} + \frac{i}{2}\, \theta \wedge \bar{\theta}$$

base

$$\Omega = \Omega_{K3} \wedge \theta \quad \text{with} \quad d\Omega = 0$$

$$\theta = dz + \alpha(y)$$

fiber

(1,0) form on K3

is a globally defined (1,0) form

θ is constrained by susy

$\mathcal{N}=1$ susy in $D=4$ allows two components for the "twist" ω

$$\omega = \omega_1 + i\omega_2 \equiv d\Theta = d\alpha = \omega_S^{(2,0)} + \omega_A^{(1,1)}$$

Susy demands the twist to be primitive

$$\omega_{mn} J_{K3}^{mn} = 0 \quad \text{Primitivity}$$

Check : conformally balanced condition

$$d(e^{-2\phi} J \wedge J) = i\, d(\Theta \wedge \bar{\Theta}) \wedge J_{K3} \overset{!}{=} 0$$

$\mathcal{N}=2$ susy further imposes

$$\omega_S^{(2,0)} = \bar{\omega}_S^{(0,2)} = 0$$

In summary ...

To summarize, $\mathcal{N}=2$ susy demands

$$\boxed{\omega_s^{(2,0)} = \omega_s^{(0,2)} = 0 \quad \Big| \quad \omega_{mn} J_{K3}^{mn} = 0}$$

The twist ($\omega = \omega_1 + i\omega_2$) is quantized!

$$\frac{\omega_i}{2\pi\sqrt{\alpha'}} \in H^2(K3, \mathbb{Z})$$

metric is globally defined

These equations are the <u>same</u> equations satisfied by the gauge bundle!

Not a coincidence and due to a rather interesting string duality?

Let's look at the gauge bundle ...

GAUGE BUNDLE

What are the eqs satisfied by F?

Dirac Quantization : $\boxed{\frac{i}{2\pi} F^{(a)} \in H^2(K3, \mathbb{Z})}$

NEXT NOTICE:

Stable bundles on the 6D manifold can be obtained by lifting stable bundles on $S = K3$ (which are well understood)

$$\boxed{F^S_{m\bar{m}} J_S^{m\bar{m}} = 0} \quad \boxed{F^{(2,0)}_{m\bar{m}} = F^{(0,2)}_{m\bar{m}} = 0}$$

CHECK: These are exactly eqs for ω!

$$F^S_{m\bar{m}} J^{m\bar{m}} = *\left(F^S_\wedge J \wedge J\right) =$$

$$= *\left(F^S_\wedge \left(e^{4\phi}(J_S \wedge J_S) + i e^{2\phi} J_S \wedge \theta \wedge \bar{\theta}\right)\right) = 0$$

Gauge groups?

These eqs are identical to the eqs satisfied by the twist ! (Duality ?)

NOTICE : Some rather interesting gauge groups emerge for more general SU(N) bundles

$$E_8 \to SU(3) \times E_6 \quad \text{standard embedding}$$

$$E_8 \to SU(4) \times \boxed{SO(10)}$$

$$E_8 \to SU(5) \times \boxed{SU(5)}$$

GUT groups !

NOTICE Fluxes emerge even without singularities or explicit sources like 5-branes (no standard embedding)

Next : Constraints from anomaly cancellation

Fu + Yau
Becker[2], Tseng

INTERLUDE: ANOMALY EQN AND BACKGROUND GEOMETRY

So far an infinite number of background geometries is allowed labelled by (Φ, ω, F) which are not determined.

Anomaly cancellation provides this important information!

$$dH = 2i\,\partial\bar{\partial} J = \frac{\alpha'}{4}\left(\mathrm{tr}\, R\wedge R - \mathrm{tr}\, F\wedge F\right)$$

Eqn. was solved by Fu + Yau.

Sublety: 3 different connections
Choose hermitian connection (2,2)

Fu + Yau evaluated the $\mathrm{Tr}\, R \wedge R$, $\mathrm{Tr}\, F \wedge F$ terms for our background and derived a determining eqn. for the dilaton:

$$\boxed{D_2(\phi) = \text{source term}}$$

This eqn can be solved if (and only) if topological constraint is satisfied:

$\mathcal{N}=2$

$$-\frac{\overset{>0}{p_1(F)}}{2} + \int_{K3} \left(\|\omega_s\|^2 + \|\omega_A\|^2 \right) \frac{J_s^2}{2} = 24$$

$$\omega_S = m\, \Omega_{K3} \quad ; \quad \omega_A = \sum_{I=1}^{19} m_I K^I$$

Finite # solutions!

Next: M-theory lift

M-THEORY: KÄHLER/NON-KÄHLER

The M-theory lift of the torsional geometry reveals that indeed a very interesting duality is present!

Adams
MB Tseng Yau
Sethi
Aspinwall

RECALL: DUALITY CHAIN

$$
\begin{tikzcd}
\boxed{\begin{array}{c}\text{M-theory}\\ K3\times K3\end{array}} \arrow[r, Rightarrow, "\text{Orbifold}"] & \boxed{\begin{array}{c}\text{M-theory}\\ K3\times T^4/\mathbb{Z}_2\end{array}} \arrow[r, Rightarrow, "T^2\to 0"] & \boxed{\begin{array}{c}\text{Type IIB}\\ K3\times T^2/\mathbb{Z}_2\end{array}} \arrow[d, Rightarrow, "\text{T-duality}"'] \\
 & \boxed{\begin{array}{c}\text{Heterotic}\\ K3\times T^2\end{array}} & \boxed{\begin{array}{c}\text{Type I}\\ K3\times T^2\end{array}} \arrow[l, Rightarrow, "S"']
\end{tikzcd}
$$

NON-ZERO G-FLUX

This duality can also incorporate a non-zero 4-form G-flux in M-theory.

To preserve $N=4$ susy in $D=3$ the M-theory G-flux is required to be a primitive $(2,2)$ form

$$G^{(2,2)} \wedge J = 0 \quad \text{Primitive}$$

$$G^{(4,0)} = G^{(0,4)} = G^{(1,3)} = G^{(3,1)} = 0$$

In addition $G \in H^4(Y,\mathbb{Z})$; $Y = K3 \times K3$ is quantized and satisfies the tadpole condition

$$\frac{1}{2}\int_Y G \wedge G = \frac{\chi(Y)}{24} \quad \text{(no M2's)}$$

GOAL: Construct G !

G can be taken to be the exterior product of two $(1,1)$ forms, one for each K3.

Orbifold limit : Kummer surface K3 $\longrightarrow T^4/\mathbb{Z}_2$

$T^4/\mathbb{Z}_2$: 19 primitive (1,1) forms $\nearrow$ $\beta_i : i = 1 \dots 16$ localized

$\searrow$ $\gamma_I : I = 1, \dots 3$ non-localized

We can then write :

integers

$$\boxed{G = C_{ij}\, \beta_i \wedge \beta_j' + C_{Ij}\, \gamma_I \wedge \beta_j' + D_{iJ}\, \beta_i \wedge \gamma_J' + D_{IJ}\, \gamma_I \wedge \gamma_J'}$$

1st K3   second K3'

T-duality is performed on K3'

$$K3' = \underset{z_3}{T^2} \times \underset{z_4}{T^2}/\mathbb{Z}_2$$

localized non-localized

$$G = C_{ij}\,\beta_i \wedge \beta_j + C_{Ij}\,\gamma_I \wedge \beta_j' + D_{iJ}\,\beta_i \wedge \gamma_J' + D_{IJ}\,\gamma_I \wedge \gamma_J'$$

Localized and non-localized contributions dualize to different types of fluxes in the heterotic theory (giving F or H)

What happens if we exchange? $K3 \longleftrightarrow K3'$.

C_{ij}, D_{IJ} terms $\longleftrightarrow$ C_{ij}, D_{IJ} terms

$$\boxed{C_{Ij} \longleftrightarrow D_{iJ}}$$

2 different heterotic theories!

Yet M-theory is invariant under

$$K3 \longleftrightarrow K3'$$

$\Rightarrow$ A transition / duality between two heterotic theories in $D=3$ emerges!!

WHAT ARE THE TWO HETEROTIC MODELS?

Consider first the G-flux non-localized on $K3'$.

$$\underbrace{\quad}_{\beta \text{ on } K3} \qquad \underbrace{\quad}_{\gamma_1, \gamma_2 \text{ on } K3'}$$

$$G = D_{i1}\beta_i \wedge \frac{1}{2}\left(dz_3 \wedge d\bar{z}_4 + d\bar{z}_3 \wedge dz_4\right) +$$

$$+ D_{i2}\beta_i \wedge \frac{1}{2i}\left(dz_3 \wedge d\bar{z}_4 - d\bar{z}_3 \wedge dz_4\right)$$

$$\gamma_3 = \frac{1}{2}\left(dz_3 \wedge d\bar{z}_3 - dz_4 \wedge d\bar{z}_4\right)$$

is not normalizable and thus not included.

Next take the periodicities of the T^8 covering space to be

$$z_K \sim z_K + 1 \sim z_K + i \,, \quad K = 1 \ldots 4$$

Re-arranging terms we get:

$$G = \frac{1}{2}(D_{i1} + i D_{i2})\,\beta_i \wedge d\bar{z}_3 \wedge dz_4 +$$

$$+ \frac{1}{2}(D_{i1} - i D_{i2})\,\beta_i \wedge dz_3 \wedge d\bar{z}_4 =$$

$$= \frac{1}{2}\left[D_3 \wedge dz_4 + \bar{D}_3 \wedge d\bar{z}_4 \right] =$$

$$= \frac{1}{2}\left[D_3 \wedge (dx_{10} + i\,dx_{11}) + \bar{D}_3 \wedge (dx_{10} - i\,dx_{11}) \right] =$$

$$= \frac{1}{2}(D_3 + \bar{D}_3) \wedge dx_{10} + \frac{i}{2}(D_3 - \bar{D}_3) \wedge dx_{11}$$

$$\boxed{G = H_3 \wedge dx_{10} + F_3 \wedge dx_{11}}$$

The heterotic H-flux comes from

$$H_3 = \frac{1}{2}\left(D_i\beta_i \wedge d\bar{z}_3 + \bar{D}_i\beta_i \wedge dz_3\right) =$$

$$= \frac{1}{2}\, d\left(\alpha \wedge d\bar{z}_3 + \bar{\alpha} \wedge dz_3\right) = dB_2$$

$$B_2 = \frac{1}{2}\left(\alpha \wedge d\bar{z}_3 + \bar{\alpha} \wedge dz_3\right)$$

T-dualize in z_3-direction

Buscher rules: $B_2 \leftrightarrow g_{ij}$ mix!

Heterotic model

DRS
K. Becker + Dasgupta

$$ds^2 = e^{2\phi}\left(dz_1 d\bar{z}_1 + dz_2 d\bar{z}_2\right) + |dz_3 + \alpha|^2$$

Next: check anomaly

ANOMALY CANCELLATION

The heterotic anomaly cancellation :

$$\frac{1}{16\pi^2}\int_{K3} \mathrm{tr}\, F\wedge F - \int_{K3} \omega\wedge\bar{\omega} = 24$$

follows from the M-theory tadpole condition !

$$\frac{1}{2}\int_Y G\wedge G = \frac{\chi(Y)}{24}$$

Check :

$$24 = \frac{1}{2}\int_Y G\wedge G = -\int_{K3} \boxed{D_i\beta_i}\wedge\bar{D}_j\beta_j$$

$$= -\int_{K3} \boxed{\omega}\wedge\bar{\omega}$$

as $\omega = D_i\beta^i$; $F=0$

Next : $K3 \leftrightarrow K3'$

M-theory again : K3 $\longleftrightarrow$ K3'

The G-flux has localized components on K3'

$$G = C_{1i}\,\overbrace{\frac{1}{2}\left(dz_1\wedge d\bar{z}_2 + d\bar{z}_1\wedge dz_2\right)}^{\gamma_1}\wedge\beta_i' +$$

$$+\; C_{2i}\,\underbrace{\frac{1}{2i}\left(dz_1\wedge d\bar{z}_2 - d\bar{z}_1\wedge dz_2\right)}_{\gamma_2}\wedge\beta_i' =$$

Rearranging :

$$G = \frac{1}{2}\left[C_i\, d\bar{z}_1\wedge dz_2 + \bar{C}_i\, dz_1\wedge d\bar{z}_2\right]\wedge\beta_i'$$

$$\boxed{G = \frac{F^i}{4\pi}\wedge\beta_i'}$$

T-dualizing again in z_3 with no flux in this direction gives a direct product (Kähler) manifold :

$$\boxed{\begin{array}{l} K3 \times T^2 \\ F \neq 0 \; ; \; \omega = 0 \end{array}}$$

ANOMALY CANCELLATION :

$G = F^i \wedge \beta_i$

$$24 = \frac{1}{2}\int_Y G \wedge G =$$

$$= \frac{1}{32\pi^2}\int_{K3} F^i \wedge F^j \overbrace{\int_{K3} \beta_i \wedge \beta_j}^{= -2\delta_{ij}} =$$

$$= -\frac{1}{16\pi^2}\int_{K3} F^i \wedge F^i \quad !$$

TO SUMMARIZE :

Depending which type of flux is on K3' two different models have been found that are dual

$$\underset{K3 \times T^2}{\underline{\text{Heterotic}}} \quad \triangleq \quad \underset{K3 \times T^2}{\underline{\text{Heterotic}}}$$

$F = 0 \,;\, H \neq 0$ $\qquad$ $F \neq 0 \,;\, H = 0$

$\omega = 0$

DUALITY : $K3 \longleftrightarrow K3'$

CONCLUSION

The map to M-theory was used to connect Kähler / non Kähler geometries in the heterotic string

Turning on ω or F corresponds to turning on different M-theory G-flux.

Such a Kähler / non-Kähler duality / transition may allow us to study torsional geometries with the tools of CY geometry!

OUTLOOK

* Non-abelian F ?

  On the M-theory side M2-branes wrapping singular 2-cycles of K3 are needed

* Duality / transition for $\mathcal{N}=1$?

  Take $\omega_S^{(2,0)} \neq 0$

* 4D effective action ?
  (special geometry ?)

* More general torsional geometries (Del Pezzo base ?)

Thanks!