Single Hadron Production in DIS

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- Factorization in short and long distance parts

- Leading order calculation $\mathcal{O}(\alpha_s)$

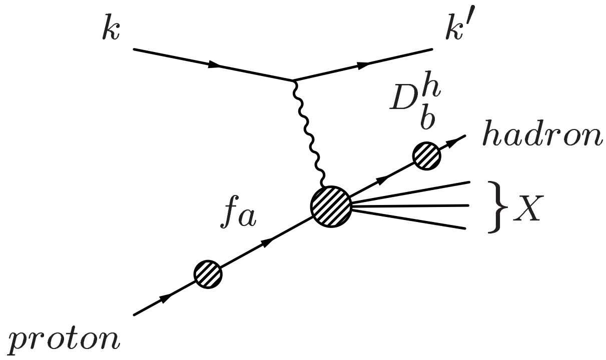
- Next-to-leading order calculation $\mathcal{O}(\alpha_s^2)$

- Phase space slicing versus subtraction

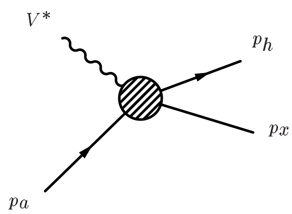
- Dipole subtraction formalism

Introduction

- Single hadron production in DIS



◇ Beyond Bjorken approximation
◇ Transversal hadron momentum $p_t^h > 0$
◇ At least 2 final state partons



◇ Literature
[Méndez, 1978] Born approximation
[Graudenz, 1994] NLO jet rates in DIS
[Büttner, 1998] NLO Meson production in DIS, approximation of transversal photons

◇ Test of QCD/Factorization
◇ Test of parton densities and fragmentation functions
◇ QCD corrections typically large
◇ Reduction of scale dependence

Factorization

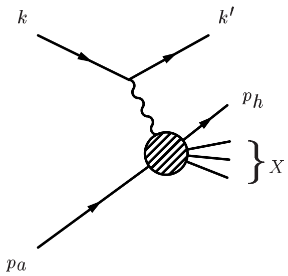
- Basis of perturbative QCD [Collins, Soper, Sterman, 1989]

  $$\frac{d\sigma^h}{d\overline{x}dyd\overline{z}d\phi} = \sum_{ab} \int_{\overline{x}}^{1} \frac{dx}{x} \int_{\overline{z}}^{1} \frac{dz}{z}\ f_a(\tfrac{\overline{x}}{x}, Q^2)\ \frac{d\sigma^{ab}}{dxdydzd\phi}\ D_b^h(\tfrac{\overline{z}}{z}, Q^2)$$

  (DIS variables: $\overline{x}, \overline{y}, \overline{z}$, parton variables: x, y, z)

  ◇ Bjorken variables $x := \frac{Q^2}{p_a q}$, $y := \frac{p_a q}{p_a k}$, $z := \frac{p_a p_b}{p_a q}$

  ◇ partonic cross section



  $$\frac{d\sigma^{ab}}{dxdydzd\phi} = C \cdot \alpha^2 \frac{1}{4 p_a k} \left(\frac{1}{Q^2}\right)^2 l^{\mu\nu}\ H^{ab}_{\mu\nu}$$

  ◇ Integrating over azimuthal dependence

  $l^{\mu\nu} = A\ (-g^{\mu\nu}) + B\ p_a^{\mu} p_a^{\nu}$

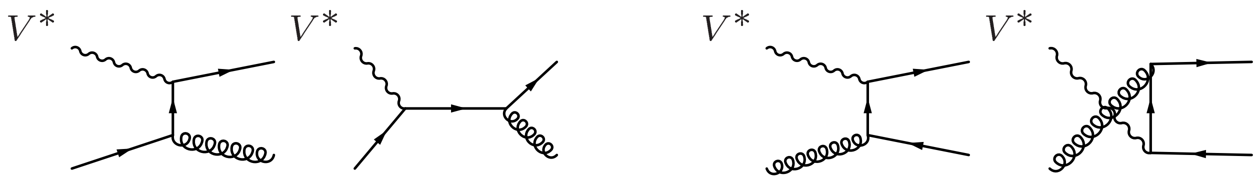
- PDF from the CTEQ collaboration

- Fragmentation functions for pions and kaons

  [Binnewies, Kniehl, Kramer, 1994]

Leading order calculation

- LO diagrams

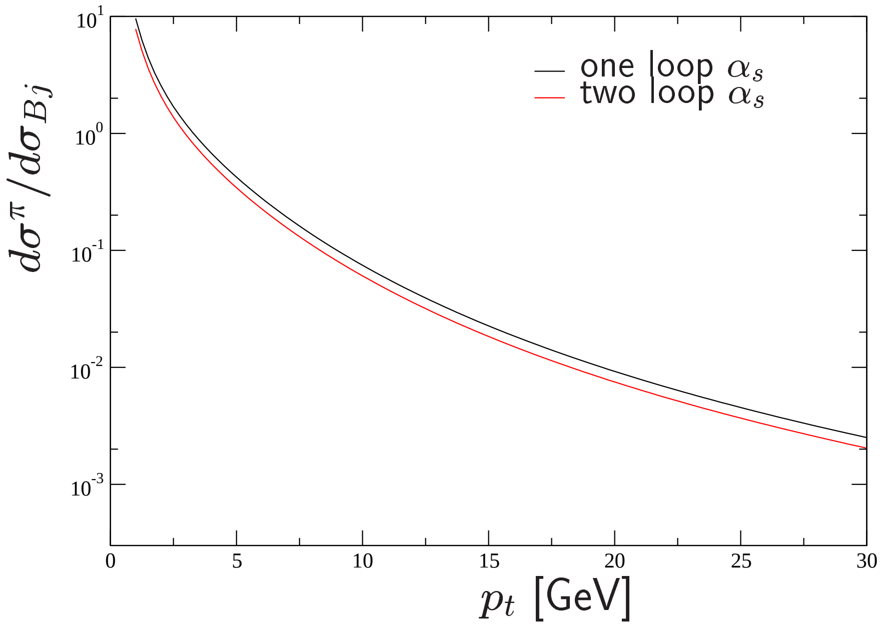


◇ C-invariance
◇ Calculation in Form and C
◇ neglecting masses
◇ four flavors
◇ combinations of parton densities and fragmentation functions:

$$\sigma^{qq,\bar{q}\bar{q}} \quad e_u^2(f_u D_u + f_{\bar{u}} D_{\bar{u}}) + e_d^2(f_d D_d + f_{\bar{d}} D_{\bar{d}})$$
$$\sigma^{qg,\bar{q}g} \quad \{e_u^2(f_u + f_{\bar{u}}) + e_d^2(f_d + f_{\bar{d}})\} \cdot D_g$$
$$\sigma^{gq,g\bar{q}} \quad f_g \cdot \{e_u^2(D_u + D_{\bar{u}}) + e_d^2(D_d + D_{\bar{d}})\}$$

- $d\sigma^{\pi}(p_t)/d\sigma_{Bj}$ distribution at LO

◇ $Q^2 = 360 GeV^2$, $\bar{x} = 0.002$, $y = 0.1$, $\bar{z} = 0.3$



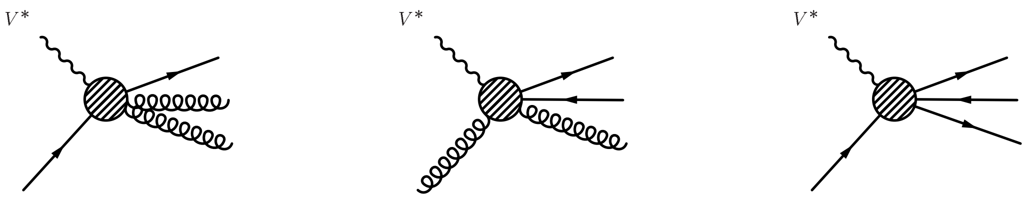
◇ normalized to differential Bjorken limit cross section $\mathcal{O}(\alpha_S^0)$

Next-to-leading order calculation

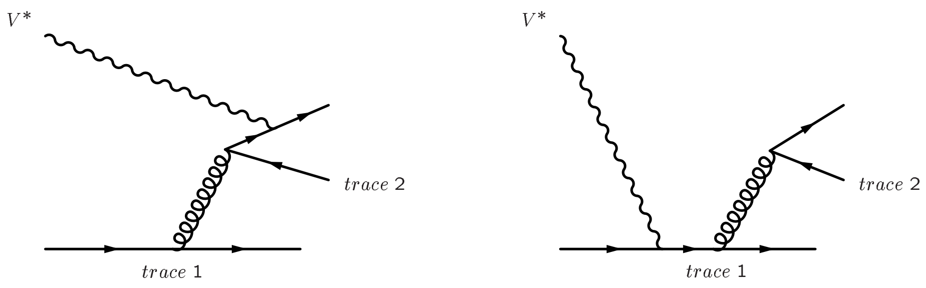
- NLO cross section

$$\sigma^{NLO} = \int_{m+1} d\sigma^{real} + \int_{m} d\sigma^{virt}$$

- Real diagrams

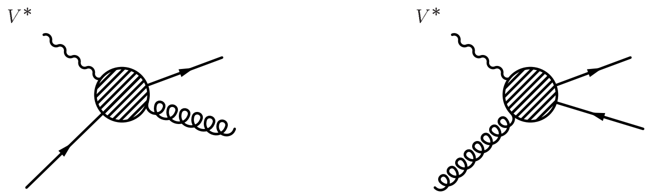


◇ many combinations of parton densities and fragmentation functions!



◇ consideration of crossed diagrams depends on flavors in traces.
◇ gauge boson may couple to trace 1 or trace 2.

- Virtual diagrams



◇ UV- and IR-singularities

◇ dimensional regularization
◇ Passarino/Veltman tensor reduction to scalar integrals.
◇ Analytical calculation of 3- and 4-point integrals.
◇ Renormalization in CT approach to absorb UV-singularities.

IR-singularities

- soft and collinear singularities from virtual correction.
  ◇ cancelation against real phase space IR-singularities
  ◇ remaining IR-singularities absorbed into redefined parton densities and fragmentation functions $\overline{MS}$.

- Analytic integration of $\int_{m+1} d\sigma^{real}$ impossible.

- In general two methods:
  ◇ phase-space slicing
  ◇ subtraction

- phase space slicing
  ◇ real phase space sliced into soft, collinear, and remaining hard region.
  ◇ introduce unphysical parameter x_{cut} to separate regions.
  ◇ soft and collinear regions regularized dimensional.
  ◇ precise calculations require small values of x_{cut}.
  ◇ Parts of numerical integration grow like $\ln(x_{cut})$.
  ◇ Much effort to calculate singular terms that cancel in the final result anyhow!

- Apply subtraction method which is numerical very stable.
  [Catani, Seymour, 1997]

Dipole subtraction

- General idea of subtraction method is to subtract and to add the same quantity.

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^{real} - d\sigma^{A}\right] + \int_{m+1} d\sigma^{A} + \int_{m} d\sigma^{virt}$$

- Cross section contribution $d\sigma^{A}$ has to fulfill the properties:

  – same *point-wise* singular behavior as $d\sigma^{real}$ itself.

$$\sigma^{NLO\{m+1\}} = \int_{m+1} \left[d\sigma^{real} - d\sigma^{A}\right] = \int_{m+1} \left[(d\sigma^{real})_{\epsilon=0} - (d\sigma^{A})_{\epsilon=0}\right]$$

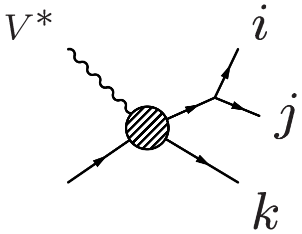
  – $d\sigma^{A}$ analytically integrable over the one-parton subspace leading to soft and collinear divergences.

$$\sigma^{NLO\{m\}} = \int_{m+1} d\sigma^{A} + \int_{m} d\sigma^{virt} = \int_{m} \left[d\sigma^{virt} + \int_{1} d\sigma^{A}\right]_{\epsilon=0}$$

◇ Cancelation of singularities before integration over phase space!

- dipole formalism in case of *identified* partons

- partons may become collinear to an *identified* parton



◇ additional dipole terms dV'_{dipole}
◇ additional integrated dipole terms

$$d\sigma_{ab}^{NLO}(p_a; p_b) =$$

$$\int_{m+1} [d\sigma_{ab}^{real}(p_b; p_a)]_{\epsilon=0} - [\sum_{dipoles} d\sigma_{ab}^{Born}(p_a; p_b) \otimes (dV_{dipole} + dV'_{dipole})]_{\epsilon=0}$$
$$+ \int_m [d\sigma_{ab}^{virt}(p_a; p_b) + d\sigma_{ab}^{Born}(p_a; p_b) \otimes \mathbf{I}]_{\epsilon=0}$$
$$+ \sum_a \int_0^1 dx \int_m d\sigma_{ab}^{Born}(p_a; p_b) \otimes (\mathbf{K}^{ab}(x) + \mathbf{P}^{ab}(x))$$
$$+ \sum_b \int_0^1 \frac{dz}{z^2} \int_m d\sigma_{ab}^{Born}(p_a; p_b) \otimes (\mathbf{H}_{ab}(z) + \mathbf{P}_{ab}(z))$$

Conclusion and Outlook

- Single hadron production at $\mathcal{O}(\alpha_s^2)$
- Solve problems with dipole formalism
- Compare theoretical predictions with experimental data
- consider massive quarks