Fluid/Gravity Correspondence for general non-rotating black holes

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Motivation

- Black hole thermodynamics

- The Membrane Paradigm of black hole (Damour, Thorne, Price and Macdonald, 1986)

- Holographic principle (G. ’t Hooft, 93; L. Susskind, 95)

- AdS/CFT correspondence (J. Maldacena, E. Witten, 98)

➢ Some "Dictionary"

Fields	Bulk	Boundary
Electromagnetic	$-n_\mu F^{\mu a}\|_{\mathrm{bdry}}$	Current $\langle J^a \rangle$
Gravitational	Brown-York $t^{ab}\|_{\mathrm{bdry}}$	Stress tensor $\langle T^{ab} \rangle$

$$T_{ab} \equiv 2(\gamma_{ab} K - K_{ab}).$$

Black holes $\leftrightarrow$ Thermal field theory

Local Hawking temperature $\leftrightarrow$ Temperature

Holographic renormalization group (RG) flow

Position of the boundary $\leftrightarrow$ Energy scale

Black hole horizon $\leftrightarrow$ IR limit

➢ Problem:

Based on AdS/CFT, we have following facts:

1. Stationary black hole $\leftrightarrow$ Equilibrium state of dual theory on boundary
2. Perturbation of gravity $\leftrightarrow$ perturbation of dual theory on boundary
3. In large spatial and temporal scales the CFT dynamics reduces to hydrodynamics, and the dual gravity description consist of long wavelength, long time perturbations of the gravity

$\rightarrow$ the dynamics of the fluid (for example, Navier-Stokes equation) should be related to the dynamics of the geometry, as described by the relativistic Einstein equations, i.e. fluid/gravity correspondence.

Does such relation really exist ?

Some answers for above question:

- Dual hydrodynamic system at conformal boundary

  Black 3-brane-super Y-M case (Policastro, Son and Starinets, 2001, PRL)

  Perturbation of black brane (Bhattacharyya, Hubeny, Minwalla and Rangamani, 2008, JHEP)

- Dual hydrodynamic system on horizon

  Black brane horizon case-linear perturation (Kovtun, Son and Starinets, 2003, JHEP)

  Black brane horizon-Navier-Stokes equation (Eling, Fouxon and Oz, 2009, PLB)

- Dual hydrodynamic system on finite boundary

  Black brane case-linear perturbation (Iqbal and Liu, 2009, PRD)

  Rindle case (Bredberg, Keeler, Lysov, and Strominger, 2011; Compere, McFadden, Skenderis and Taylor, 2011, JHEP)

  non-linear perturbation of Black brane (Cai, Li and Zhang, 2011, JHEP; Niu, Tian, Wu and Ling, 2012, PLB)

Problems:

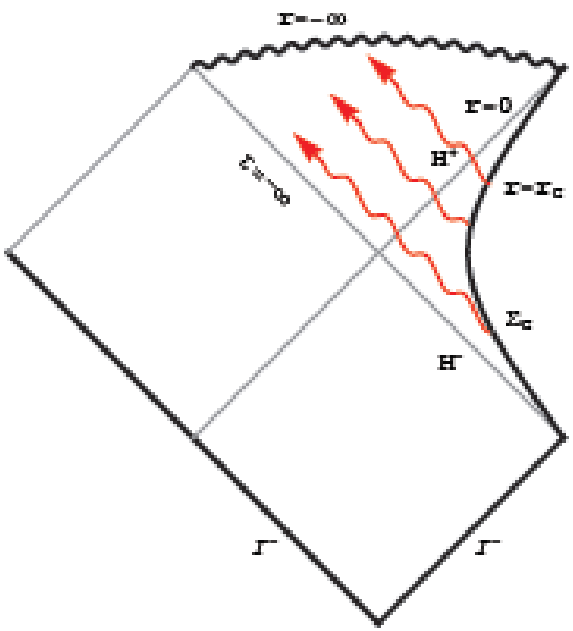
1. All methods use long wavelength limit (or derivative expansion)

$$\partial_r f \sim f, \quad \partial_t f \sim \epsilon^2 f, \quad \partial_i f \sim \epsilon f$$

2. Calculations strongly depend on the concrete form of back ground geometry(For example, regularity on horizon)

Can such correspondence be established generally ?

Reconsideration



From view point of bulk, it is a initial-boundary value problem of Einstein equations, so maybe one can choose suitable boundary condition to establish such correspondence in general cases.

Initial-Boundary value problem of Einstein Equations

1. H.Friedrich and G. Nagy, Commun. Math. Phys. 201(1999)619
2. H. O. Kreiss, O. Reula, O. Sarbach and J. Winicour, Commun. Math. Phys., 289(2009)1099
3. O. Reula and O. Sarbach, arxiv : 1009.0589

Key observation :

1. The free boundary data of IBVP of Einstein equations are components of Weyl curvature.
2. For black brane case, long wavelength perturbation satisfies

$$W_{\mu\nu\rho[\sigma}k_{\lambda]}k^{\nu}k^{\rho} = 0.$$

Which is called Petrov-like boundary condition.

Main Idea of Strominger’s new method

1. Fix the inner geometry of boundary
2. Perturb the extrinsic curvature of boundary
3. Rescale time coordinate and take near horizon limit to get non-relativistic low energy limit of perturbation.
4. Impose the Petrov-like boundary condition

Then the Gaussian-Coddazi equation of boundary will give the Navier-Stokes equation and the dispersion relation of the dual system.

A naive counting of degrees of freedom

Independent components of perturbation : (p+1)(p+2)/2

constrains caused by Petrov-like boundary condition : (p+1)p/2-1

free components of perturbation : (p+1)(p+2)/2- (p+1)p/2+1-1=p+1
$\rightarrow (v_i , P)$

Achievements of Strominger's new method

- ✓ Rindle case – incompressible N-S equation in flat space (Lysov and Strominger, 2011)
- ✓ Examples of curved space-time - incompressible N-S equation in curved space (Huang, Ling, Pan, Tian and Wu, 2011)
- ✓ Examples of AdS-black holes (Huang, Ling, Pan, Tian and Wu, 2012)
- ✓ Example of charged black hole (Zhang, Ling, Niu, Tian and Wu, 2012)

How about general case ?

General definition of black hole horizon

(A. Ashtekar, 1999; Korzynski et. al. , 2005)

Definition 1 *(Weakly Isolated Horizon in $(p+2)$-dimensional space-time)* *Let (M, g) be a $(p+2)$-dim Einstein manifold with or without a cosmological constant. $\mathcal{H}$ is a $(p+1)$-dim null hypersurface in M and l is the null normal of $\mathcal{H}$. $\mathcal{H}$ is called a weakly isolated horizon in M if*

(1). there exists an embedding : $S \times [0,1] \to M$, $\mathcal{H}$ is the image of this map, S is a p-dimensional compact, connected manifold and for every maximal null curve in $\mathcal{H}$ there exists $x \in S$ such that the curve is the image of $x \times [0,1]$;

(2). the expansion of l vanishes everywhere on $\mathcal{H}$;

(3). $R_{ab} l^a l^b |_{\mathcal{H}} = 0$;

(4). let $\mathcal{D}$ denote the induced connection on $\mathcal{H}$, $[\mathcal{L}_l, \mathcal{D}] l = 0$ holds on $\mathcal{H}$.

All stationary black holes satisfy above definition.

- Control geometry near horizon

$$
\begin{aligned}
n &= \partial_r, \\
l &= \partial_t + U\partial_r + X^i\partial_i, \\
E_I &= W_I\partial_r + e_I^i\partial_i, \qquad I, i = 1, 2, \cdots, p,
\end{aligned}
$$

$$
\pi_I := \langle E_I, \nabla_l n\rangle, \quad \varepsilon := \langle n, \nabla_l l\rangle, \quad \theta'_{IJ} := \langle E_I, \nabla_J n\rangle
$$

$$
\begin{aligned}
\partial_r W_I &= -\pi_I - \theta'_{IJ} W_J, \\
\partial_r e_I^i &= -\theta'_{IJ} e_J^i, \\
\partial_r U &= \varepsilon - \pi_I W_I, \\
\partial_r X^i &= -\pi_I e_I^i, \\
\partial_r \pi_I &= R_{nlIn} - \theta'_{IJ}\pi_J, \\
\partial_r \theta'_{IJ} &= R_{nIJn} - \theta'_{IK}\theta'_{KJ} \\
\partial_r \varepsilon &= R_{nlnl} + \sum_I \pi_I^2,
\end{aligned}
$$

- Non-rotating condition

$$\pi_I \hat{=} 0.$$

- Geometry near horizon

$$\begin{aligned} g^{tr} &= 1, \qquad g^{ti} = 0, \\ g^{rr} &= 2\varepsilon r + R_{nlnl} r^2 + O\left(r^3\right), \\ g^{ri} &= R_{nInl} e_I^i r^2 + O\left(r^3\right), \\ g^{ij} &= \sim O\left(r^0\right). \end{aligned}$$

$$\begin{aligned} g_{tt} &= -2\varepsilon r - R_{nlnl} r^2 + O\left(r^3\right), \\ g_{ti} &= -R_{nInl} g_{ij} e_I^j r^2 + O\left(r^3\right), \\ g_{tr} &= 1, \qquad g_{ri} = 0. \end{aligned}$$

- Non-relativistic limit and near horizon limit

$$\tau = 2\hat{\varepsilon}\lambda^2 t \qquad r_c = 2\hat{\varepsilon}\lambda^2$$

- Near horizon behavior of extrinsic curvature

$$K^i_j = \sqrt{2\hat{\varepsilon}}\xi^i_j\lambda + O(\lambda^3) \sim O(\lambda).$$

$$K^\tau_i = 2\hat{\varepsilon}\lambda^2 K^t_i = \hat{\varepsilon}\lambda^2(h^{tt}K_{ti} + h^{kt}K_{ki}) \sim O(\lambda^4)$$

$$K^\tau_\tau = \frac{1}{2\lambda} + \beta\lambda + O(\lambda^3) \sim O(\lambda^{-1}).$$

$$K = \frac{1}{2\lambda} + \sqrt{2\hat{\varepsilon}}(\beta + \xi)\lambda + O(\lambda^3) \sim O\left(\frac{1}{\lambda}\right).$$

- Gravitational perturbation

$$t^a{}_b = \sum_{k=0}^{\infty} t^{a(k)}_{b} \lambda^k.$$

- Petrov-like boundary condition

$$C_{(\ell)i(\ell)j} \equiv \ell^\mu m_i^\nu \ell^\alpha m_j^\beta C_{\mu\nu\alpha\beta} = 0$$

$$\rightarrow \qquad t_j^{i(1)} = 2h^{ik} t_k^{\tau(1)} t_j^{\tau(1)} - 2h^{ik} \tilde{\nabla}_{(j} t_{k)}^{\tau(1)} + \frac{t^{(1)}}{p} \delta_j^i + \xi_j^i \sqrt{2\hat{\varepsilon}} - \tilde{R}^i_j.$$

- Coddazi equation

$$D_a t^a_b = 0.$$

$$\rightarrow \quad \tilde{\nabla}^i v_i = 0,$$

$$0 = \partial_\tau v^j + v^i \tilde{\nabla}_i v^j - \tilde{\Delta} v^j - \tilde{R}^j_i v^i + \tilde{\nabla}^j P + f^j + \bar{\Gamma}^\tau_{\tau\tau} v^j + \bar{\Gamma}^i_{i\tau} v^j + 2\bar{\Gamma}^j_{i\tau} v^i,$$

$$2t_j^{\tau\,(1)} = v_j, \; 2\frac{t^{(1)}}{p} = P,$$

- Gaussian equation (non-relativistic dispersion relation)

$$t_\tau^{\tau\,(1)} = \bar{R} - 2h^{ij} t_i^{\tau\,(1)} t_j^{\tau\,(1)} - \sqrt{\frac{\hat{\varepsilon}}{2}}\,\xi,$$

- Additional terms

$$\bar{\Gamma}^{\tau}_{\tau\tau} \hat{=} \frac{1}{2\hat{\varepsilon}}\partial_t \hat{R}_{nlnl},$$

$$\bar{\Gamma}^{i}_{j\tau} \hat{=} \frac{1}{2}\hat{g}_{jk}\hat{e}^{k}_{I}(\partial_t \hat{\theta}'_{IJ})\hat{e}^{i}_{J},$$

$$\bar{\Gamma}^{i}_{i\tau} \hat{=} \frac{1}{2}\partial_t (\hat{\theta}'_{IJ})\delta^{IJ}.$$

For stationary case, obviously such terms vanish.

For some “near stationary” black holes, such terms are also zero.

- Remark :

1. the fluid/gravity correspondence exists for general non-rotating stationary black holes. Cosmological constant gives no contribution.
2. Time rescaling $\leftrightarrow$ non-relativistic dispersion, near horizon limit $\leftrightarrow$ low energy limit.
3. Non-rotating condition is crucial for this proof. Does such correspondence exists for rotating black hole?
4. What is the suitable boundary condition for non-vacuum cases?
5. How about for boundary at finite distance ?
6. Any applications in hydrodynamics ?
7. How about the inverse direction, i.e. Navier-Stokes to Einstein ?

Thank you