Holographic thermalization at intermediate coupling

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R. Baier, S. Stricker, O. Taanila, AV, 1205.2998 (JHEP), 1207.1116 (PRD)
D. Steineder, S. Stricker, AV, 1209.0291 (PRL), 1304.3404 (JHEP)
S. Stricker, 1307.2736
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Thermalization in heavy ion collisions

1. Early dynamics of a heavy ion collision
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Holographic Green’s functions at finite coupling

1. Green’s functions as a probe of thermalization
2. Some computational details

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Motivation: Puzzles in heavy ion physics

Surprises from comparison of hydro predictions with RHIC/LHC data:

- Extremely early onset of hydrodynamic behavior: \( \tau_{\text{hydro}} \approx 0.5 \text{ fm/c} \), close to causal limit
- Small, yet nonzero shear viscosity \( \eta/s = \mathcal{O}(0.1) \)

Clear discrepancy with perturbative predictions: (All) features of the system not consistent with weakly coupled quasiparticle picture
Motivation: Puzzles in heavy ion physics

Particularly intriguing challenge: Understand the dynamics that take the system from complicated, far-from-equilibrium initial state to a near-thermal ‘hydrodynamized’ plasma

System and its characteristic energy scales evolve fast ⇒ Its description requires both perturbative and nonperturbative machinery
At RHIC/LHC energies, initial state well understood: Color Glass Condensate (CGC), characterized by

- One hard scale: Saturation momentum $Q_s \gg \Lambda_{QCD}$
- Overoccupation of gluons: $f(q < Q_s) \sim 1/\alpha_s$
- High anisotropy: $q_z \ll q_\perp$
Early dynamics of a collision

When describing early (weak coupling) dynamics of a collision, need to take into account:

- Longitudinal expansion of the system
- Elastic and inelastic scatterings
- Plasma instabilities

Traditional field theory tools available:

1. Classical (bosonic) lattice simulations — work as long as occupation numbers large\(^1\)
2. Parametric weak coupling estimates; however, even obtaining correct scaling of \(\tau_{\text{therm}}\) in powers of \(\alpha_s\) highly nontrivial\(^2\)

\(^1\)Berges et al., 1303.5650
\(^2\)Baier et al., hep-ph/0009237; Kurkela, Moore, 1107.5050; Blaizot et al., 1107.5296
Weak coupling thermalization — no expansion

Current understanding for homogenous and isotropic systems with initial overoccupation: *Thermalization proceeds as a turbulent cascade with self-similar evolution, associated with presence of a non-thermal fixed point* \(^3\)

In expanding systems, competition between interactions and longitudinal expansion, and between different thermalization mechanisms

\(^3\)Schlichting, 1207.1450
Weak coupling thermaliz. — expanding system

Inelastic scatterings drive bottom-up evolution
- Soft modes quickly create thermal bath
- Hard splittings lead to $q \sim Q_s$ particles being eaten by the bath

Numerical evolution of expanding SU(2) YM plasma seen to always lead to Baier-Mueller-Schiff-Son type scaling exponents at late times (Berges et al., 1303.5650)

Ongoing debate over role of instabilities in hard interactions, argued to lead to slightly faster thermalization: $\tau_{KM} \sim \alpha_s^{-5/2}$ vs. $\tau_{BMSS} \sim \alpha_s^{-13/5}$
Impressive results for the (very) early dynamics of a high energy collision. However, extension to full thermalization process in real-life heavy ion collisions problematic:

- Dynamics assumed classical in lattice simulations — works only at earliest times
- System clearly not asymptotically weakly coupled $\Rightarrow$ Parametric $\alpha_s$ scalings of limited use, and going beyond them very hard

In absence of first principles field theory techniques, clearly room for input from holographic methods

Cleanest setup: Strong coupling limit of large-$N_c$ $\mathcal{N} = 4$ Super Yang-Mills theory
Thermalization in heavy ion collisions

Thermalization at strong(er) coupling

**Strong coupling thermalization**

Important lessons from gauge/gravity calculations at infinite coupling:

- Thermalization always of top-down type (causal argument)
- Thermalization time naturally short, $\sim 1/T$
- Hydrodynamization $\neq$ Thermalization, isotropization

Chesler, Yaffe, 1011.3562
Bridging the gap

Obviously, it would be valuable to bring the two limiting cases closer to each other — and to a realistic setting. Is it possible to:

- Extend weak coupling picture to lower energies, with \( \alpha_s \) of \( \mathcal{O}(1) \)?
- Marry CGC description of initial state and early dynamics with strong coupling evolution of the system?
- Bring gauge/gravity calculations closer to real QCD?
  - Finite coupling & \( N_c \), SUSY and conformal invariance breaking,...

Rest of the talk: Attempt to relax the \( \lambda = \infty \) approximation in studies of holographic thermalization
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Holographic Green’s functions

In- and off-equilibrium correlators offer useful tool to study thermalization:

- Poles of retarded thermal Green’s functions give dispersion relation of field excitations: Quasiparticle / quasinormal mode spectrum
  - Describe response of the system to infinitesimal perturbations
- Time dependent off-equilibrium Green’s functions probe how fast different energy (length) scales equilibrate
- Related to measurable quantities, e.g. particle production rates
Holographic Green’s functions

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Example 1: EM current correlator $\langle J_{\mu}^{EM} J_{\nu}^{EM} \rangle$

- Obtain by adding to SYM theory a U(1) vector field coupled to a conserved current corresponding to a subgroup of $SU(4)_R$
- Excellent phenomenological probe of thermalization because of photons’ weak coupling to plasma constituents
Holographic Green’s functions

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Example 2: Energy momentum tensor correlators $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$ related to e.g. shear and bulk viscosities and dual to metric fluctuations $h_{\mu\nu}$
- Scalar channel: $h_{xy}$
- Shear channel: $h_{tx}, h_{zx}$
- Sound channel: $h_{tt}, h_{tz}, h_{zz}, h_{ii}$
Model of thermalization

- Simplest way to describe thermalizing system: Begin with a thin massive shell at $r = r_s > r_h$ and let it collapse towards $r_s = r_h$

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<thead>
<tr>
<th>center</th>
<th>horizon</th>
<th>shell</th>
<th>boundary</th>
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<tbody>
<tr>
<td>$r = 0$</td>
<td>$r = r_h$</td>
<td>$r = r_s$</td>
<td>$r = \infty$</td>
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- Metric piecewise defined:

  $$ds^2 = -r^2 f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 dx^2, \quad f(r) = \begin{cases} f_-(r) = 1, & \text{for } r < r_s \\ f_+(r) = 1 - \frac{r_h^4}{r^4}, & \text{for } r > r_s \end{cases}$$

- Shell fills all of 3-space $\Rightarrow$ System translationally and rotationally invariant

- In *quasistatic limit* — valid at large $\omega$ — Green’s functions available with a minor modification to standard (Son-Starinets) recipe
  - Bulk fields matched at the shell using junction conditions
Beyond infinite coupling: $\alpha'$ corrections

Recall key relation from AdS/CFT dictionary: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$, with $\alpha'$ the inverse string tension

- To go beyond $\lambda = \infty$ limit, need to add $\alpha'$ terms to supergravity action, i.e. first non-trivial terms in a small-curvature expansion
- Leading order corrections $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

End up dealing with $\mathcal{O}(\alpha'^3)$ improved type IIB sugra action

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left( R_{10} - \frac{1}{2} (\partial \phi)^2 - \frac{F_5^2}{4 \cdot 5!} + \gamma e^{-\frac{3}{2} \phi} (C + T)^4 \right),$$

$$T_{abcdef} \equiv i \nabla_a F^+_{bcdef} + \frac{1}{16} \left( F^+_{abcmn} F^+_{def}^{\ mn} - 3 F^+_{abfmn} F^+_{dec}^{\ mn} \right),$$

$$F^+ \equiv \frac{1}{2} (1 + *) F_5, \quad \gamma \equiv \frac{1}{8} \zeta(3) \lambda^{-3/2}$$

$\Rightarrow \gamma$-corrected metric and EoMs for different fields
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Quasinormal mode spectra at finite coupling

Solving for the poles of retarded thermal Green’s functions gives the *dispersion relation* of field excitations,

\[ \omega_n(k) = E_n(k) + i\Gamma_n(k), \]

with \(E_n\) the energy and \(\Gamma_n\) the width of the mode

- At weak coupling expect long-lived quasiparticles with \(\Gamma_n \ll E_n\)
- At strong coupling expect quasinormal mode spectrum

\[ \hat{\omega}_n|_{k=0} = \frac{\omega_n|_{k=0}}{2\pi T} = n(\pm 1 - i) \]

Magnitude of \(\Gamma_n\) related to thermalization pattern: At strong coupling, the highest energy modes decay fastest. What happens at intermediate coupling?
QNMs at finite coupling: Photons

Effect of decreasing $\lambda$: System flows towards quasiparticle spectrum already at relatively large couplings

NB: Convergence of strong coupling expansion not guaranteed, when $\hat{\omega}_n|_{k=0} = n(\pm 1 - i) + \alpha_n/\lambda^{3/2}$ shifts by $O(1)$ amount
QNMs at finite coupling: Photons

Similar shift at nonzero three-momentum: $k = 2\pi T$
QNMs at finite coupling: $T_{\mu \nu}$ correlators

Same effect also in the shear (left) and sound (right) channels of energy momentum tensor correlators (here $k = 0$)
Outside the $\lambda = \infty$ limit, the response of the strongly coupled plasma to infinitesimal perturbations appears to change, moving towards that of a weakly coupled quasiparticle system.

What happens if we take the system further away from equilibrium?
Off-equilibrium Green’s functions: Definitions

Natural correlation function to study thermalization with: Spectral density

\[ \chi(\omega, k) \equiv \text{Im } \Pi_R(\omega, k), \]

related to particle production rates when fluctuation dissipation theorem valid (large \( \omega \) in falling shell picture). For photons,

\[
k^0 \frac{d\Gamma_\gamma}{d^3k} = \frac{1}{4\pi k} \frac{d\Gamma_\gamma}{dk_0} = \frac{\alpha_{\text{EM}}}{4\pi^2} \eta^{\mu\nu} \Pi_{\mu\nu}(k_0 \equiv \omega, k) = \frac{\alpha_{\text{EM}}}{4\pi^2} \eta^{\mu\nu} n_B(\omega) \chi_\mu(\omega, k)
\]

Useful measure of ‘out-of-equilibriumness’: Deviation of spectral density from its thermal limit

\[ R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_{\text{therm}}(\omega, k)}{\chi_{\text{therm}}(\omega, k)} \]

In quasistatic approximation, approach towards equilibrium parameterized by \( r_s/r_h \to 1 \)
Spectral density and $R$ at $\lambda = \infty$: Photons

Left: Photon spectral functions for different virtualities ($c = k/\omega$) in thermal equilibrium and at $r_s/r_h = 1.1$

Right: Relative deviation $R \equiv (\chi - \chi_{\text{th}})/\chi_{\text{th}}$ for $r_s/r_h = 1.1$ and $k/\omega = 0, 0.8, 1$

Note: 1) Highly virtual field modes thermalize first

2) Clear top-down thermalization pattern (as always at $\lambda = \infty$)
Relative deviation at intermediate $\lambda$: Photons

Relative deviation $R \equiv (\chi - \chi_{th})/\chi_{th}$ for on-shell photons with $r_s/r_h = 1.01$ and $\lambda = \infty, 500, 300$ (left) and $150, 100, 75$ (right)

NB: Change of pattern with decreasing $\lambda$: UV modes no longer first to thermalize! Sign of top-down turning into bottom-up?
Relative deviation at intermediate $\lambda$: $T_{\mu\nu}$ correlators

Relative deviation $R \equiv (\chi - \chi_{\text{th}})/\chi_{\text{th}}$ in the shear (left) and sound (right) channels for $r_s/r_h = 1.2$, $\lambda = 100$, and $k/\omega = 0$ (black), $6/9$ (blue) and $8/9$ (red)
Physical interpretation of results?

So what to make of all this? Evidence for holographic plasma starting to behave like a system of weakly coupled quasiparticles at finite coupling, or simply

- ... due to the breakdown of some approximation?
  - Quasistatic limit good approximation as long as $\omega / T \gg 1$
  - Strong coupling exp. applied with care: $(\text{NLO-LO})/\text{LO} \lesssim \mathcal{O}(1/10)$

- ... a peculiarity of the channels considered, but not a fundamental feature of the plasma?
  - Interesting recent results for purely geometric probes, indicating not all correlators turn towards bottom-up behavior\(^4\)

- ... a sign of the unphysical nature of the collapsing shell model?
  - Perhaps; certainly warrants more work. However, at least QNM flow results universal

\(^4\)Galante, Schvellinger, 1205.1548
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Take home messages

1. Taking holographic (thermalization) calculations away from $\lambda = \infty$ limit not only possible, but potentially a rather fruitful exercise.

2. Indications that the holographic system obtains weakly coupled characteristics within the realm of a strong coupling expansion:
   - QNM poles flow in the direction of a quasiparticle spectrum
   - Top-down thermalization pattern weakens and shifts towards bottom-up

3. Naive(?) conclusion: To describe physical heavy ion system ($\lambda \sim 20$) using holography, accounting for strong coupling corrections important.