Holographic entropy production

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The problem

- Perturb a thermodynamic system in equilibrium
  $\Longrightarrow$Various transport processes pull it back to equilibrium
  $\Longrightarrow$Production of entropy
- Perturb a (static) black hole
  $\Longrightarrow$The black hole absorbs the energy of perturbations
  $\Longrightarrow$Increase of the black-hole entropy
- Do the above two physical processes have direct relationship?

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The physical picture

- Thanks to holography!
  Bulk: a black hole that eats everything
  Boundary: transportation that smoothes everything

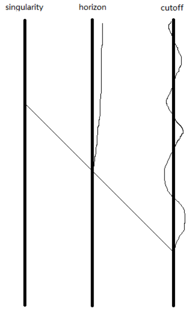

Figure : A sketch map

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1. Overview and preparation
   - Holography (bulk/boundary correspondence)
   - In equilibrium: thermodynamics (and phase transition)
2. Non-equilibrium and fluid dynamics
   - In non-equilibrium: transportation and entropy production
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Holography: a brief introduction

- Early (rough) ideas of holography
  G. 't Hooft (1993); L. Susskind (1995).
- A more precise prescription: AdS/CFT
  J. Maldacena (1998).
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  Basic principle (Euclidean):

$$Z_{B^{d+1}}[\bar{\phi}+\delta\bar{\phi}] = Z_{B^{d+1}}[\bar{\phi}] \left\langle \exp \int_{S^d} \delta\bar{\phi}\,\mathscr{O}_\phi \right\rangle_{\mathrm{CFT}}$$

- Generalization: bulk/boundary correspondence
  AdS/QCD, AdS/CMT, holographic entanglement entropy, gravity/fluid, ...

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The bulk/boundary correspondence

- bulk: not necessarily (asymptotic) AdS
  boundary: not necessarily conformal (effective FT)
  [Takayanagi et al (2010), Strominger et al (2011), Maldacena et al (2013), . . .]
- The general principle:
  $[\phi|_{\text{bdry}} \leftrightarrow$ Non-dynamical (background) field $\bar{\phi}]$

$$Z_{\text{bulk}}[\bar{\phi}] = \int D\psi \exp(-I_{\text{FT}}[\bar{\phi}, \psi]) \Longrightarrow$$

$$\begin{aligned} Z_{\text{bulk}}[\bar{\phi} + \delta\bar{\phi}] &= Z_{\text{bulk}}[\bar{\phi}] \left\langle \exp \int_{\text{bdry}} \delta\bar{\phi}\, \mathscr{O}_\phi \sqrt{\bar{g}}\, d^d x \right\rangle_{\text{FT}} \\ \mathscr{O}_\phi &= -\frac{\delta I_{\text{FT}}[\bar{\phi}, \psi]}{\sqrt{\bar{g}}\, \delta\bar{\phi}(x)} \end{aligned}$$

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The bulk/boundary correspondence

- The general principle under classical approximation of the bulk gravity:

$$\exp(-I_{\text{bulk}}[\bar{\phi}]) = \int D\psi \exp(-I_{\text{FT}}[\bar{\phi}, \psi])$$

with $I_{\text{bulk}}[\bar{\phi}]$ the on-shell action (Hamilton's principal function).

- Variation with respect to $\bar{\phi}$ gives

$$-\frac{\delta I_{\text{bulk}}[\bar{\phi}]}{\sqrt{\bar{g}}\,\delta\bar{\phi}(x)} = \langle \mathscr{O}_\phi(x) \rangle_{\text{FT}}$$

- Further variations give the correlations of $\mathscr{O}_\phi$ on the boundary.

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The bulk/boundary correspondence

- Important examples (with n^μ the unit normal to the boundary)

Fields	Bulk	Boundary
EM A_μ	$-n_\mu F^{\mu a}\vert_{\mathrm{bdry}}$	Current $\langle J^a\rangle$
Grav. $g_{\mu\nu}$	Brown-York $t^{ab}\vert_{\mathrm{bdry}}$	Stress tensor $\langle T^{ab}\rangle$

- Additional dictionary
  Black holes $\leftrightarrow$ Thermal field theory
  [Euclidean partition function = partition function of the (grand) canonical ensemble]
  $\Longrightarrow \begin{cases} \text{Local Hawking temperature} = \text{Temperature} \\ \text{Bekenstein-Hawking entropy} = \text{Entropy} \end{cases}$
- Macroscopic investigation: Thermodynamic and hydrodynamic descriptions under the low-frequency/long-wavelength limit, where $\langle \mathscr{O}_\phi(x)\rangle_{\mathrm{FT}}$ are just the macroscopic physical quantities.

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Why macroscopic investigation?

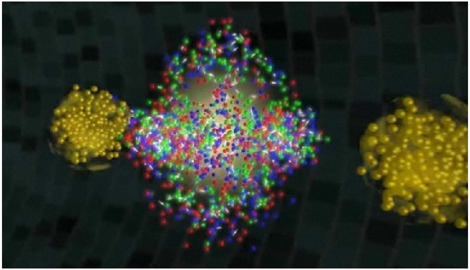

Figure : A prestigious example: Quark-gluon plasma produced in LHC

Why macroscopic investigation?

- The holographic prediction $\frac{\eta}{s} = \frac{1}{4\pi}$ for QGP in $\mathcal{N} = 4$ SYM (Policastro, Son & Starinets, 2002) is in qualitative agreement with the RHIC data.
- We hope to know how general holography can be, through macroscopic investigation.
(Tentative answer: As general as gravitation!)
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Thermodynamics

- Equilibrium thermodynamics from the bulk point of view (Brown & York, 1993)
- The Brown-York tensor for Einstein's gravity

$$t^{ab} = \frac{1}{8\pi G}(K\bar{g}^{ab} - K^{ab})$$

  in static configurations has a form

$$t^{ab} = \varepsilon u^a u^b + p h^{ab}, \qquad h^{ab} = \bar{g}^{ab} + u^a u^b$$

  of the (relativistic) perfect fluid.
- $\Longrightarrow dE + pdV = TdS + \mu dQ$ (the 1st law of thermodynamics)

Thermodynamics

- The holographic interpretation of the Brown-York thermodynamics
- In more general gravitational theories: Hamilton-Jacobi-like analyses

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Transportation in non-equilibrium thermodynamics

- Small perturbations: Linear response theory
- Example 1: Ohm's law

$$J^i = \sigma E^i$$

- Example 2: Newton's law of viscosity

$$T^{xy} = -2\eta\sigma^{xy}$$

Type	Response	Driving Force	Transport
Heat conduction	Heat flow	Temp. gradient	Energy
Shear viscosity	Momentum flow	Gradient of $\boldsymbol{v}_{\parallel}$	$\boldsymbol{p}_{\parallel}$
Bulk viscosity	Momentum flow	Gradient of $\boldsymbol{v}_{\perp}$	$\boldsymbol{p}_{\perp}$
Eletric conduction	Electric current	Potential gradient	Charge

Table : Transportation

Transportation in non-equilibrium thermodynamics

- Cross-transportation (such as the thermoelectricity phenomena):

$$J_A = \sum_B L_{AB} X_B$$

where the matrix (L_{AB}) of transport coefficients is symmetric and positive definite (Onsager's reciprocal relation).

- The holographic realization:
  - linear (grav. and material) perturbations around the static bulk spacetime (black hole)
  - ingoing boundary conditions on the horizon
  - solving the linear perturbation equations in the bulk

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Transportation in non-equilibrium thermodynamics

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}, \qquad n^{\mu} h_{\mu\nu} = 0$$

Type	Driving Force	Bulk perturbation
Heat conduction	Temp. gradient $\nabla_i \frac{1}{T}$	Grav. perturb. $\frac{1}{f_c T}\partial_t h_{ti}$
Shear viscosity	Gradient of $\boldsymbol{v}_{\parallel}$	Grav. perturb. $\frac{1}{\sqrt{f_c}}\partial_t h_{ij}$
Bulk viscosity	Gradient of $\boldsymbol{v}_{\perp}$	Grav. perturb. $\frac{1}{\sqrt{f_c}}\partial_t h_{ii}$
Eletric conduction	Potential gradient E_i	EM perturb. $\frac{1}{\sqrt{f_c}} F_{ti}(r_c)$

Table : Holographic realization of transp. (in certain gauge)

Entropy production

- The general (non-relativistic) entropy production rate

$$\Sigma = \sum_A J_A X_A = \sum_{AB} X_A L_{AB} X_B$$

Type	Driving force	Entropy production
Heat conduction	Temperature gradient	-
Viscosity	Velocity gradient	Friction heat/ T
Electric condution	Electric field	Joule heat/ T

Table : Entropy production of transport processes

Entropy production

- The boundary side: entropy production rate

$$\Sigma = J_q \cdot \nabla \frac{1}{T} - \frac{1}{T} \Pi : \nabla \mathbf{u} + \frac{1}{T} \mathbf{J} \cdot \mathbf{E} = J_q^i \nabla_i \frac{1}{T} - \frac{1}{T} \Pi^{ij} \sigma_{ij} + \frac{1}{T} J^i E_i$$

with Π^{ij} the dissipation part of the stress tensor and

$$\sigma_{ij} = \frac{1}{2}(\nabla_i u_j + \nabla_j u_i) \qquad (\nabla \cdot \mathbf{u} = 0)$$

- The bulk side: entropy variation

$$\delta S = \frac{\delta E}{T_H}$$

Entropy production

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Entropy production

- Consider the $Q = 0$ (chargeless black hole background) case, where it turns out that there is no cross-transportation, for simplicity.
- By construction of conserved currents relating the horizon and the boundary, one can verify $\delta S = \int_{\mathrm{bdry}} \Sigma$, the equality of entropy increase of the bulk black hole and total entropy production of the boundary fluid (YT, X.-N. Wu & H.-B. Zhang, 2012).

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Discussions

- Non-equilibrium thermodynamics (boundary): energy is dissipated in irreversible processes
  The holographic point of view (bulk): energy of perturbations is absorbed by the black hole
- In cases with bulk viscosity, it seems that the spatial isotropy is required for a holographic realization of entropy production.
- The $Q \neq 0$ (charged black hole background) case (with cross-transportation)
  YT, X.-N. Wu and H.-B. Zhang, in preparation.
- Cases for more general gravitational theories with various matter content

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Towards boundary fluid dynamics: The 1st way

- Non-relativistic long-wavelength expansion
  I. Bredberg, C. Keeler, V. Lysov & A. Strominger, [arXiv:1101.2451].
  R.-G. Cai, L. Li & Y.-L. Zhang, JHEP 1107 (2011) 027 [arXiv:1104.3281].
  C. Niu, YT, X. Wu & Y. Ling, Phys. Lett. B 711 (2012) 411 [arXiv:1107.1430].
  - Works for boundary at finite distance
  - But only works for intrinsically flat boundary

Towards boundary fluid dynamics: The 2nd way

- Petrov-like boundary condition
  V. Lysov & A. Strominger, [arXiv:1104.5502].
  T. Huang, Y. Ling, W. Pan, YT & X. Wu, JHEP 1110 (2011) 079 [arXiv:1107.1464].
  T. Huang, Y. Ling, W. Pan, YT & X. Wu, Phys. Rev. D 85 (2012) 123531 [arXiv:1111.1576].
  C.-Y. Zhang, Y. Ling, C. Niu, YT & X. Wu, Phys. Rev. D 86 (2012) 084043 [arXiv:1204.0959].
  - Works for intrinsically curved boundary
  - But only works for boundary approaching the horizon

Open problems

- Holographic entropy production in the far-from-equilibrium case?
- Holographic thermodynamics and entropy production with quantum corrections to the bulk gravity
- Boundary fluid dynamics in more general cases?
- Boundary fluid dynamics with higher-order transport coefficients
- Holographic (non-linear) superfluid dynamics
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The end

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