Thermal Mass and Plasmino for Strongly Interacting Fermions via Holography

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July 30, 2013

Based on arXiv:1205.3377 and arXiv:1305.1446
in collaboration with Sang-Jin Sin and Yang Zhou

- Hard Thermal Loop(HTL) approximation in QCD
  - Fermion propagator

$$G(p) = \frac{1}{\gamma \cdot p - m - \Sigma(p)}$$

  - In the limit of $m \ll T, \mu$

$$\begin{aligned} G &= \frac{1}{2}(\gamma_0 - \gamma_i p^i)/\Delta_+ + \frac{1}{2}(\gamma_0 + \gamma_i p^i)/\Delta_- \ , \\ \Delta_\pm &= \omega \mp p - \frac{m_f^2}{4p}\left[\left(1 \mp \frac{\omega}{p}\right)\log\left(\frac{\omega+p}{\omega-p}\right) \pm 2\right] \end{aligned}$$

Motivation

- Hard Thermal Loop(HTL) approximation in QCD
  - Fermion propagator

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  - Effective mass is generated by thermal and medium effect

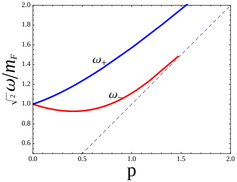
$$m_f^2 = \frac{1}{4} g^2 (T^2 + \mu^2 \pi^2)$$

  - Solving the pole of the propagator we will get two branches of dispersion curves $\omega = \omega_\pm(p)$

$$\begin{aligned} p << m_f &: \quad \omega_\pm(p) \simeq m_f \pm \frac{1}{3}p \\ p >> m_f &: \quad \omega_\pm(p) \simeq p \end{aligned}$$

Motivation

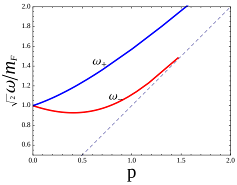
- Plasmino



- Opposite direction with helicity and chirality
- Negative slope near zero momentum region (-1/3)
- Minimum at finite momentum
- Propagating anti-quark-hole in the medium

Motivation

- Plasmino



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- Motivation
  - From direct solving Schwinger-Dyson equation, thermal mass seems to disappear at strong coupling limit arXiv:1111.0117, Nakkagawa et. al.
  - The behavior of plasmino in strong coupling limit with finite temperature or finite density is not known in field theory
  - We want to study thermal mass and plasmino in strong coupling by using AdS/CFT correspondence

- D4 brane background with D8, $\bar{D}8$ brane as probe(Sakai & Sugimoto)

	0	1	2	3	4	5	6	7	8	9
D4	•	•	•	•	•					
D8, $\bar{D}8$	•	•	•	•		•	•	•	•	•

- Background geometry
  - Deconfined phase

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(-f(U)dt^2 + d\vec{x}^2 + dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

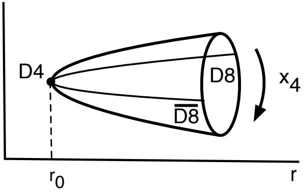
  - Confined phase

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

- Turn on $U(1)$ gauge field on the probe brane → Finite chemical potential
  - Fundamental strings in deconfined phase
  - D4 baryon vertices in confined phase
- Chemical potential

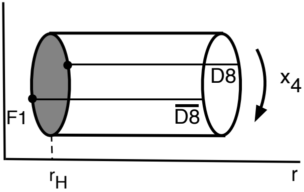
$$\mu = m_5/q + \int_{r_0}^{\infty} a_0' dr.$$

- D brane embedding
  - Confined Phase



$$m_5 = S_{D4}^{DBI}$$

  - Deonfined Phase



$$m_5 = 0$$

- Turn on fermionic fluctuation on probe brane

$$S = \int d^5x\sqrt{-g}\ \left(\bar{\psi}\Gamma^M iD_M\psi - m_5\bar{\psi}\psi\right)$$

$$D_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M$$

- Equation of motion H. Liu et al.

$$(\partial_r + m_5\sqrt{g_{rr}}\sigma^3)\Phi_\alpha = \sqrt{g_{rr}/g_{ii}}(i\sigma^2 v(r) + (-1)^\alpha k\sigma^1)\Phi_\alpha$$

$$v(r) = \sqrt{-g_{ii}/g_{tt}}(\omega + qa_0), \quad \Phi_1 = (y_1, z_1)^T,\ \Phi_2 = (y_2, z_2)^T$$

- Retarded Green's function

$$G_1(r) := y_1(r)/z_1(r),\ G_2(r) := y_2(r)/z_2(r)$$

$$\sqrt{\frac{g_{ii}}{g_{rr}}}\partial_r G_\alpha + 2m_5\sqrt{g_{ii}}G_\alpha = (-1)^\alpha k + v(r) + \left((-1)^{\alpha-1}k + v(r)\right)G_\alpha^2$$

- IR boundary condition can be determined by requiring regularity of equation of motion at horizon (deconfined phase) or at the tip(confined phase)

- Deconfined phase

$$G_{1,2}(r_0) = i$$

- Confined phase

$$G_\alpha(r_0) = \frac{-mR + \sqrt{m^2R^2 + k^2 - \hat{\omega}^2}}{(-1)^\alpha k - \hat{\omega}}$$

$$\hat{\omega} = \omega + m_5, \qquad m = m_5 r_0^{3/4}$$

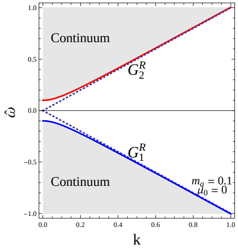
- Finite baryon mass
  - IR boundary condition

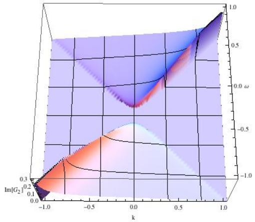
$$G_\alpha(r_0) = \frac{-m + \sqrt{m^2 + k^2 - \hat{\omega}^2}}{(-1)^\alpha k - \hat{\omega}} \, .$$

  - Continuum region

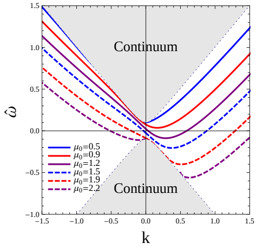
$$\hat{\omega} > \sqrt{k^2 + m^2}, \qquad \hat{\omega} < -\sqrt{k^2 + m^2}$$

  - $\mu = 0$



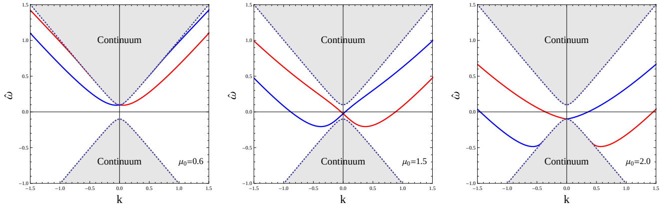


- Finite baryon mass
  - $\mu \neq 0$

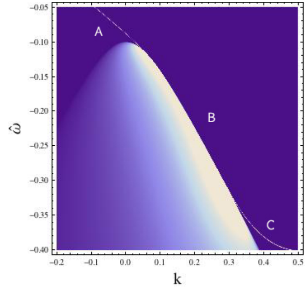


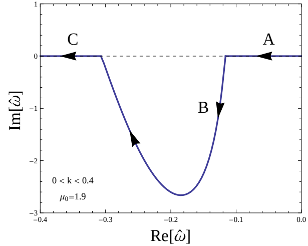
$$G_1(k) = G_2(-k)$$

  - Dispersion relations



- Complex structure of pole in continuum region

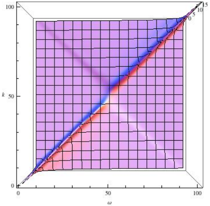


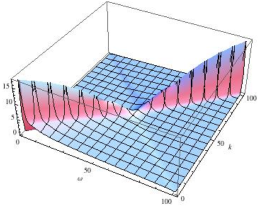


- Deconfined phase
  - $m_5 = 0$
  - IR boundary condition(Infalling condition)

$$G_{1,2}(r_0) = i$$

  - Result with $\mu = 0$

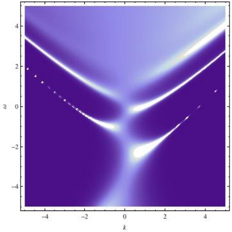




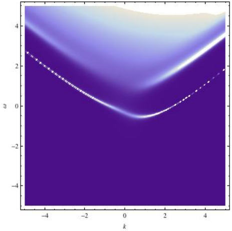
  - No thermal mass generated

$$\begin{aligned} m_T &= \frac{1}{\sqrt{6}} gT \text{ in weak coupling,} \\ m_T &= 0 \quad \text{ in strong coupling} \end{aligned}$$

- Deconfined phase
  - $m_5 = 0$, $\mu \neq 0$



  - $m_5 \neq 0$, $\mu \neq 0$



- Action

$$S = \int d^{d+1}x\sqrt{-g}\left(\frac{R-2\Lambda}{16\pi G_N} - \frac{1}{4e^2}F^2 + i(\bar{\psi}\Gamma^M D_M\psi - m\bar{\psi}\psi)\right)$$

  - We fix background geometry
  - Fermions coupled with gauge field

- Equation of motion
  - Geometry: AdS soliton geometry in 5 dimension

$$ds^2 = r^2(-dt^2 + d\vec{x}^2 + f(r)dx_3) + \frac{1}{f(r)r^2}dr^2 \ , \quad f(r) := 1 - \frac{r_0^4}{r^4}$$

  - Equation of motion for fermion

$$(\partial_r + \frac{m}{r\sqrt{f}}\sigma^3)\Phi_\alpha = \frac{1}{r^2\sqrt{f}}(i\sigma^2(\omega + eA_t) + (-1)^\alpha k\sigma^1)\Phi_\alpha$$

  - Equation of motion for gauge field

$$(\sqrt{-g}g^{tt}g^{rr}\phi'(r))' - \sqrt{-g}g^{tt}\langle\psi^\dagger\psi\rangle \ = 0$$

- Lutinger theorem

$$< \psi\psi^\dagger >= \sum_l \int \frac{d^2k}{(2\pi)^2} \Phi^\dagger_{lk}(r)\Phi_{lk}(r)\theta(-\omega_l(k))$$

- Equation of motion

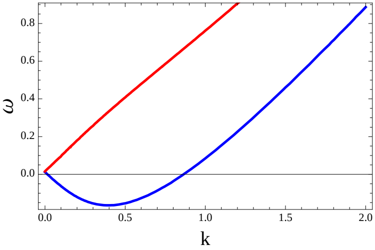
$$(\sqrt{-g} g^{tt} g^{rr} \phi'(r))' - e^2 \sqrt{\frac{g^{tt}}{g^{rr}}} \sum_l \int \frac{d^2k}{(2\pi)^2} \Phi^\dagger_{lk}(r)\Phi_{lk}(r)\theta(-\omega_l(k)) = 0$$

- Solve coupled equation of motion by using iteration method

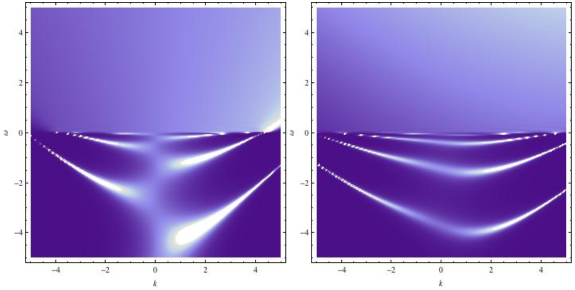
- IR boundary condition

$$G_\alpha(r_0) = \frac{-mR + \sqrt{m^2R^2 + k^2 - \omega^2}}{(-1)^\alpha k - \omega}$$

- Dispersion relation



- In deconfined phase, all dynamical fermions fall into the black hole
- Background becomes RN-AdS black hole
- We put probe fermion in the bulk
- Spectral density



Herzog et. al

Summary

- The condition for existence of plasmino mode

		Top down		Bottom up	
		Confining	Deconfining	Confining	Deconfining
m_q	$= 0$				
	> 0		$\circledcirc$		$\circledcirc$
μ	$< \mu_c$				
	$> \mu_c$	$\circledcirc$	$\circledcirc$	$\circledcirc$	$\circledcirc$

- Rashiba effect in bulk

$$H_{\pm} = \frac{k^2}{2m_{eff}(r)} + \alpha E(r) \times \sigma \cdot k + \ldots,$$

  - The field theory dual of spin-orbit coupling in bulk can be a density generated plasmino

$$\omega_{\pm} \sim \alpha k^2 \pm \beta\mu \cdot k - \mu$$

Conclusion and Discussion

- We calculate fermion Green's function by using AdS/CFT correspondence
- In deconfined phase, there is no thermal mass generation
- In confined phase, plasmino excitations are present in certain window of chemical potential
- We speculate that the spin-orbit coupling in bulk is dual of plasmino mode in boundary theory

Thank you !!!