Elasticity and hydrodynamics of charged black branes

Gauge/Gravity Duality

Munich, August 1, 2013

Niels Obers, NBI

1307.0504 & 1209.2127 (PRL) (with J. Armas, J. Gath)

1210.5197 (PRD) (with J. Armas)

1110.4835 (JHEP) (with J. Armas, J. Camps, T. Harmark)

+ related work:
1304.7773 (J. Armas)

Intro +overview

- long wave length perturbations of black branes
  - construction of new BH solutions in higher dimenions (ST)
  - properties of QFTs via holography

in long-wave length regime: →
black branes behave like any other type of continuous media with dynamics governed by some (specific) effective theory

- new insights into GR/geometry
- find BHs in higher dimensions and discover their properties
- effective theory that integrates out gravitational degrees of freedom
- AdS/CFT (fluid/gravity) inspired new way to look at gravity
- find universal features of black branes in long wave length regime
  described by “every day” physics
- reduce complicated gravitational physics to simple response coefficients
- cross-fertilization between classical elasticity/fluid theory and gravity
  (cf. rigorous development of fluid and superfluid dynamics using fluid/gravity correspondence)

Blackfold approach: a unified framework

two types of deformations:

- intrinsic:
time (in)dependent fluctuations along worldvolume/boundary directions

effective theory of viscuous fluid flows

Bhattacharyya,Hubeny,Minwalla,Rangamani
Erdmenger,Haack,Kaminski,Yarom/Nanerjee et al
(fluid/gravity ….)
Camps,Emparan,Haddad/Gath,Pedersen
Emparan,Hubeny,Rangamani

- extrinsic:
stationary perturbations along directions transverse to woldvolume

effective theory of thin elastic branes

Emparan,Harmark,Niarchos,NO
Armas,Camps,Harmark,NO
Camps,Emparan
Armas,Gath,NO/Armas,NO/Armas

fluids living on dynamical surfaces ("fluid branes") = **blackfold aproach**
(unified general framework of the two descriptions)

Reviews: Emparan,Harmark,Niarchos,NO
Emparan/
Harmark,NO (to appear)

Plan

- Short review of leading order blackfold (BF) approach
- Elastic properties of (charged) black branes

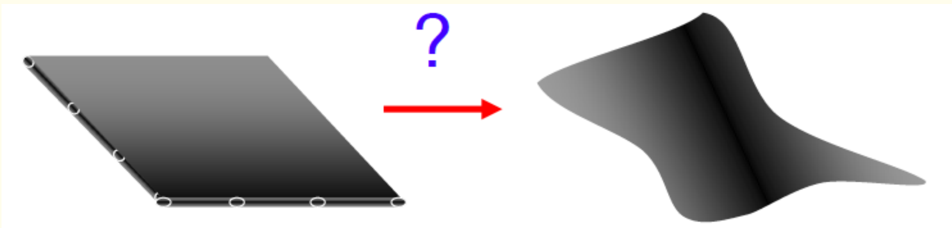
  - extrinsic perturbations:
  relativistic Young-modulus, piezo electric moduli
  - how fluids bend: elastic expansion in effective field theory
- Outlook

Punchlines

- new parallel between (electro)elasticity theory and gravitational physics
- input/insights into general effective theory of charged fluid branes
- potential applications to AdS/CFT + “flat space holography”

Blackfolds: framework for dynamics of black branes

- based on bending/vibrating of (flat) black branes



blackfold = black brane wrapped on a compact submanifold of spacetime

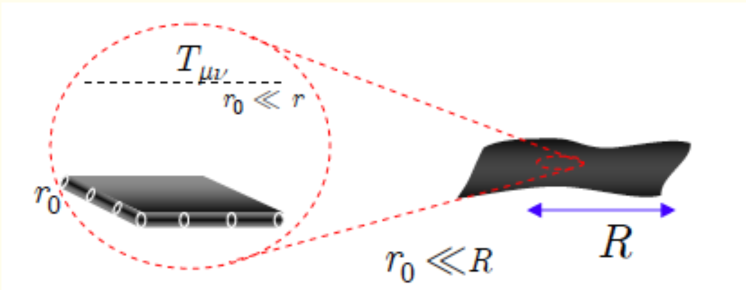
very much like other extended solitonic objects:

- Nielsen-Olesen vortices and NG strings
- open strings and DBI action

difference: - short-distance d.o.f. = gravitational short-wavelength modes
- extended objects posses black hole horizon
-> worldvolume thermodynamics

Effective worldvolume theory – leading order

widely separated scales: perturbed black brane looks locally like a flat black brane



- effective stress tensor of black branes correspond to specific type of fluid to leading order: perfect fluid

$$T^{ab} = (\varepsilon + P)u^a u^b + P\gamma^{ab}$$

for charged black branes of sugra:
novel type of (an)isotropic charged fluids

notation: spacetime $X^\mu,\ \mu,\nu\ldots = 0,\ldots,D-1$

worldvolume $\sigma^a,\ a,b\ldots = 0,\ldots,p$

$$n = D - p - 3$$

Main ingredients

- identify collective coordinates of the brane

$$X^\mu(\sigma), \varepsilon(\sigma), u^\mu(\sigma), q(\sigma), \ldots$$

positions transverse to worldvolume

energy density (horizon thickness)

local boost velocity

charge density

- blackfold equations of motion follow from conservation laws (stress tensor, currents,..)

$$\bar{\nabla}_\mu T^{\mu\nu} = 0, \bar{\nabla}_\mu J^\mu = 0, \ldots$$

⇨ effective (charged) fluid living on a dynamical worldvolume:

$$T^{\mu\nu} K_{\mu\nu}{}^\rho = 0$$ extrinsic equations (D-p-1)

$$D_a T^{ab} = 0$$ intrinsic equations (p+1)

$$D_a J^{a_1 \cdots a_{p+1}} = 0$$

leading order BF equations

BF equations

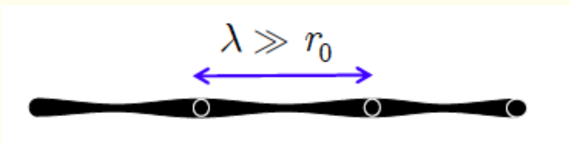
blackfold equations

Emparan,Harmark,Niarchos,NO

(liquid)

intrinsic (Euler equations of fluid
+ charge conservation)

fluid excitations (+ charge waves)



(solid)

extrinsic (generalized geodesic eqn. for
brane embedding)

elastic deformations

- gives novel stationary black holes (metric/thermo) + allows study of time evolution
- generalizes (for charged branes) DBI/NG to non-extremal solns. (thermal)
- possible in principle to incorporate higher-derivative corrections
  (self-gravitation + internal structure/multipole)
- BF equations have been derived from Einstein equations

Camps,Emparan

general emerging picture (from hydro of non-extremal D3-branes)

Membrane paradigm $\subset$ Fluid/gravity correspondence $\subset$ Blackfolds.

Emparan,Hubeny,Rangamani

Stationary solutions and 1st law of thermo

- equilibrium configurations stationary in time = stationary black holes

  fluid velocity is along worldvolume Killing direction

→ extrinsic BF equations for the embedding coordinates derivable from action

$$\tilde{I} = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \sqrt{-\gamma}\, P$$

- thermodynamics: all global quantities: mass, charge, entropy, chemical potentials by integrating suitable densities over the worldvolume

- for any embedding (not nec. solution) the "mechanical" action is proportional to Gibbs free energy:

$$\beta^{-1} I = G = M - \sum_i \Omega_i J_i - TS$$

⇒ 1st law of thermo = blackfold equations for stationary configurations

Blackfolds in supergravity and string theory

Emparan,Harmark,Niarchos,NO
Caldarelli,Emparan,v. Pol
Grignani,Harmark,Marini,NO,Orselli

- BF method originally developed for neutral BHs, but even richer dynamics when considering charged branes

- extra equations: charge conservation
  consider dilatonic black branes that solve action (includes ST black branes)

$$S = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2(q+2)!}e^{a\phi}H^2_{[q+2]}\right]$$

p-branes with q-charge: q=0, particle charge, q=1: string charge, etc.)

anistropic (charged) fluids

q=1

$$T_{ab} = \varepsilon u_a u_b + P_\perp\left(\gamma_{ab} + u_a u_b - v_a v_b\right) + P_\parallel v_a v_b$$

$$J^{(0)}_{ab} = Q u_{[b} v_{a]}$$

spacelike vector v along the directions of the 1-charge (string)

Black branes as fluids and elastic materials

Goal: show that asymptotically flat (charged) black branes have both elastic and fluid properties

Method: perturb -> consider derivative corrections

two ways:

- intrinsic perturbations parallel to the worldvolume (wiggle)

  viscosities (shear, bulk)
  charge diffusion
  gives connection to GL instability, fluid/gravity, …

  Camps,Emparan,Haddad
  Gath,Pedersen
  Emparan,Hubeny,Rangamani,

  can be used in AdS/Ricci flat map

  Caldarelli,Camps,Gouteraux,Skenderis

- extrinsic perturbations transverse to the worldvolume (bend)

  response coefficients are inputs to effective theory

* generalizes Polyakov QCD string + actions considered in theoretical biology

Elasticity: Fine structure corrections to blackfolds

Armas, Camps, Harmark, NO

- can explore corrections in BF approach that probe the fine structure: go beyond approximation where they are approximately thin

$$\hat{T}^{\mu\nu}(x^\alpha) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma\sqrt{-\gamma}\left[T^{\mu\nu}_{(0)}(\sigma^a)\frac{\delta^D(x^\alpha - X^\alpha(\sigma^a))}{\sqrt{-g}} - \nabla_\rho\left(T^{\mu\nu\rho}_{(1)}(\sigma^a)\frac{\delta^D(x^\alpha - X^\alpha(\sigma^a))}{\sqrt{-g}}\right) + \ldots\right]$$

Vasilic, Vojinovic

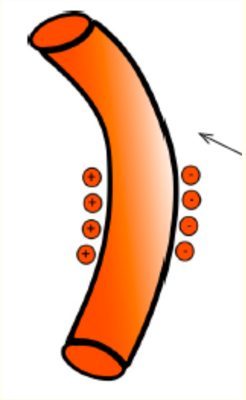
$$T^{\mu\nu\rho}_{(1)} = u_b{}^{(\mu}j^{\nu)\rho b} + u^\mu_a u^\nu_b d^{ab\rho} + u^\rho_a T^{\mu\nu a}_{(1)}$$

$$u^\mu_a \equiv \partial_a X^\mu$$

accounts for:

- dipole moment of wv stress energy = bending moment (density)

$$D^{ab\rho} = \int_\Sigma d^{D-1}x\sqrt{-g}\,\hat{T}^{ab}x^\rho = \int_{\mathcal{B}_p} d^p\sigma\sqrt{-\gamma}\,d^{ab\rho}$$



- internal spin degrees of freedom (conserved angular momentum density)

$$J^{\mu\nu}_\perp = \int_\Sigma d^{D-1}x\sqrt{-g}\left(\hat{T}^{\mu 0}x^\nu - \hat{T}^{\nu 0}x^\mu\right) = \int_{\mathcal{B}_p} d^p\sigma\sqrt{-\gamma}\,j^{0\mu\nu}$$

Fine structure: Charged branes

Armas,Gath,,NO

- branes charged under Maxwell fields: multipole expansion of current

$$\hat{J}^{\mu}(x^{\lambda}) = \int_{\mathcal{W}_{p+1}} dV \left[\frac{J^{\mu}_{(0)}(\sigma^a)}{\sqrt{-g}} \delta^{(D)}(x^{\lambda} - X^{\lambda}(\sigma^a)) - \nabla_{\rho} \left(\frac{J^{\mu\rho}_{(1)}(\sigma^a)}{\sqrt{-g}} \delta^{(D)}(x^{\lambda} - X^{\lambda}(\sigma^a)) \right) + \ldots \right]$$

$$J^{\mu\nu}_{(1)} = m^{\mu\nu} + u^{\mu}_{a} p^{a\nu} + J^{\mu a}_{(1)} u^{\nu}_{a}$$

dipoles of charge

electric dipole moment:

$$P^{a\rho} = \int_{\Sigma} d^{D-1}x \sqrt{-g}\, \hat{J}^{\mu} u^{a}_{\mu} x^{\rho} = \int_{\mathcal{B}_p} d^p\sigma \sqrt{-\gamma}\, p^{a\rho}$$

$$p^{a\rho} = u^{a}_{\mu} \perp^{\rho}{}_{\nu} J^{\mu\nu}_{(1)}$$

can also write generalization for p-branes carrying q-charge (omit details)

corrected pole/dipole BF equations generalize those of general relativistic (charged) spinning point particle (p=0, q=0) to extended charged objects

Relativistic Young modulus

bending moment a priori unconstrained -> assume classical Hookean elasticity theory:

$$d^{ab\rho} = \tilde{Y}^{abcd} K_{cd}{}^{\rho}$$

bending moment (not present for point particlel)

relativistic Young modulus

extrinsic curvature like Lagrangian strain (measures variation of induced metric transverse to wv.)

general structure of Y can be classified using effective action approach (done for neutral isotropic fluids):

generalization to (isotropic) case with wv. charge:

$$\tilde{Y}^{abcd} = -2\Big(\lambda_1(\mathrm{k};T,\Phi_H)\gamma^{ab}\gamma^{cd} + \lambda_2(\mathrm{k};T,\Phi_H)\gamma^{a(c}\gamma^{d)b} + \lambda_3(\mathrm{k};T,\Phi_H)\mathrm{k}^{(a}\gamma^{b)(c}\mathrm{k}^{d)} + \lambda_4(\mathrm{k};T,\Phi_H)\frac{1}{2}(\mathrm{k}^a\mathrm{k}^b\gamma^{cd} + \gamma^{ab}\mathrm{k}^c\mathrm{k}^d) + \lambda_5(\mathrm{k};T,\Phi_H)\mathrm{k}^a\mathrm{k}^b\mathrm{k}^c\mathrm{k}^d\Big) ,$$

k = Killing vector, T = global temperature, Phi = chemical potential

upshot: (charged) black branes are described by this effective theory + characterized by particular values of the response coefficients lambda

piezo electric moduli

- for piezo electric materials: dipole moment proportional to strain (q=0)

$$p^{a\rho} = \tilde{\kappa}^{abc} K_{cd}{}^{\rho}$$

relativistic generalization of piezo-electric modulus found in electro-elasticity

structure of kappa not yet classified from effective action, but from symmetries/covariance

$$\tilde{\kappa}^{abc} = -2\left(\kappa_1(\mathbf{k};T,\Phi_H)\gamma^{a(b}\mathbf{k}^{c)} + \kappa_2(\mathbf{k};T,\Phi_H)\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c + \kappa_3(\mathbf{k};T,\Phi_H)\mathbf{k}^a\gamma^{bc}\right)$$

similarly: for p-branes with q-charge:
- possible anomalous terms in Young-modulus
- piezo electric effect with new types of piezo electric moduli

(note: piezo electric effect also encountered in context of superfluids)

Erdmenger,Fernandez,Zeller

upshot: (charged) black branes are described by this effective theory + characterized by particular values of the response coefficients kappa

Measuring Young/piezo electric moduli for charged BB

- can be measured in gravity by computing the first order correction to bent charged black branes

simplest example: charged black branes of EMD theory
obtained by uplift-boost-reduce from neutral bent branes

more involved: charged black p-branes with q-charge of E[(q+1)-form]D theory
can use again same procedure to charge up branes
+ use in string theory setting U-dualities to generate higher form charge

- bending of black string (or brane) induces dipole moments of stress
  can be measured from approximate analytic solution (obtained using MAE)

$$g_{\mu\nu} = \eta_{\mu\nu} + h^{(M)}_{\mu\nu} + h^{(D)}_{\mu\nu} + \mathcal{O}\left(r^{-n-2}\right)$$

$$\nabla^2_\perp \bar{h}^{(D)}_{\mu\nu} = 16\pi G d_{\mu\nu}{}^{r_\perp}\partial_{r_\perp}\delta^{(n+2)}(r)$$

- bending of charged black string (or brane) induces dipole moments of charge
  can be measured from approximate analytic solution (obtained using MAE)

$$A_\mu = A^{(M)}_\mu + A^{(D)}_\mu + \mathcal{O}\left(r^{-n-2}\right)$$

$$\nabla^2_\perp A^{(D)}_\nu = 16\pi G p_\nu{}^{r_\perp}\partial_{r_\perp}\delta^{(n+2)}(r)\,.$$

Examples of results for new response coeffs

p-branes with 0-form (Maxwell) charge: 3+1 response coefficients

Young modulus

$$\lambda_1(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}|\mathbf{k}|^{n+2}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}}\left(\frac{3n+4}{2n^2(n+2)} - \bar{k}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)\right)$$

$$\lambda_2(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}|\mathbf{k}|^{n+2}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}+1}\frac{1}{2(n+2)} \quad ,$$

$$\lambda_3(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}|\mathbf{k}|^{n}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}} \quad ,$$

piezo electric

$$\kappa_1(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\frac{\xi_2(n)}{2}\left(\frac{n}{4\pi T}\right)^{n+2}\Phi_H|\mathbf{k}|^{n}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}}$$

similar expressions for p-branes with q-form charge: 3+1 response coefficients

Effective action for elastic expansion of branes

old physical problem: fluids living on surfaces: response to bending
(e.g. biconcave shape of red blood cells: cannot be described by standard soap bubble action, with minimal surface)

Helfrich-Canham bending energy: add
(K = mean curvature vector)

$$\mathcal{F}[X^\mu] = \alpha \int dA\, K^2$$

Carter/Capovilla,Gueven

in physics: improved effective action for QCD string (Polyakov & Kleinert)

general framework for higher order corrections (stationary brane fluids) Armas

$$I\,[X^\mu] = \int_{\mathcal{W}_{p+1}} \mathcal{L}(\sqrt{-\gamma}, \gamma_{ab}, \mathbf{k}^a, \nabla_a, K_{ab}{}^i, \omega_a{}^{ij})$$

$$K_{ab}{}^\mu = \nabla_a u_b^\mu$$

$$\omega_a{}^{ij} = -n^j{}_\mu \nabla_a n^{i\mu}$$

EM tensor

$$T^{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \gamma_{ab}}$$

dipole moment spin current

$$\mathcal{D}^{ab}{}_i = \frac{1}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta K_{ab}{}^i} \;, \quad \mathcal{S}^a{}_{ij} = \frac{1}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \omega_a{}^{ij}}$$

Leading order effective action

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \mathcal{L}\left(\sqrt{-\gamma}, \mathbf{k}\right) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \sqrt{-\gamma}\; \lambda_0(\mathbf{k})$$

gives perfect fluid

$$T^{ab} = T^{ab}_{(0)} = \lambda_0(\mathbf{k})\gamma^{ab} - \lambda_0'(\mathbf{k})\mathbf{k}u^a u^b$$

$$P = \lambda_0(\mathbf{k})\ ,\ \epsilon + P = -\lambda_0'(\mathbf{k})\mathbf{k}\ .$$

strain tensor

$$U_{ab} = -\frac{1}{2}\left(\gamma_{ab} - \bar{\gamma}_{ab}\right) \qquad dU_{ab} = N_\rho K_{ab}{}^\rho$$

can define elasticity tensor (measuring compression/stretching)

Armas,NO

$$E^{abcd} = 2\left(\lambda_0(\mathbf{k})\gamma^{a(c}\gamma^{d)b} - \left(\frac{\partial\lambda_0(\mathbf{k})}{\partial\gamma_{ab}}\right)\gamma^{cd} - 2\left(\frac{\partial^2\lambda_0(\mathbf{k})}{\partial\gamma_{ab}\partial\gamma_{cd}}\right)\right)$$

$$dT^{ab} = E^{abcd}dU_{cd}$$

extrinsic dynamics in transverse directions to the surface correspond to that of elastic brane

Second order corrections

Armas

first order: $\mathbf{k}^a\nabla_a\mathbf{k}\ ,\ \nabla_a\mathbf{k}^a$. are zero

in agreement with analysis of stationary & non-dissipative fluids (expansion and shear vanish)

2nd order elastic

$$\lambda_1(\mathbf{k})K^iK_i\ ,\ \lambda_2(\mathbf{k})K^{abi}K_{abi}\ ,\ \lambda_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^bK_{ac}{}^iK^c{}_{bi}\ ,$$

$$\lambda_4(\mathbf{k})\mathbf{k}^a\mathbf{k}^bK_{ab}{}^iK_i\ ,\ \lambda_5(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\mathbf{k}^dK_{ab}{}^iK_{cdi}\ .$$

2nd order spin

$$\varpi_1(\mathbf{k})\omega^a{}_{ij}\omega_a{}^{ij}\ ,\ \varpi_2(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\omega_{aij}\omega_b{}^{ij}$$

2nd order hydrodynamic

$$\upsilon_1(\mathbf{k})\nabla_a\nabla^a\mathbf{k}\ ,\ \upsilon_2(\mathbf{k})\mathcal{R}\ ,\ \upsilon_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathcal{R}_{ab}\ ,$$

$$\upsilon_4(\mathbf{k})\nabla_{[a}\mathbf{k}_{b]}\nabla^{[a}\mathbf{k}^{b]}\ ,\ \upsilon_5(\mathbf{k})\nabla_a\mathbf{k}\nabla^a\mathbf{k}\ ,\ \upsilon_6(\mathbf{k})R^a{}_{ba}{}^b\ ,\ \upsilon_7(\mathbf{k})\mathbf{k}^a\mathbf{k}^bR^c{}_{acb}$$

coupling between elastic and hydrodynamic modes

using field redefinitions, ibp and other props:
one finds for codimension higher than 1 branes
- 3 elastic response coefficents
- 5 hydrodynamic response coefficients
- 1 spin response coefficient

but: coupling between elastic and hydro due to geometric constraints

Gauss-Codazzi $$R_{abcd} = \mathcal{R}_{abcd} - K_{ac}{}^{i} K_{bdi} + K_{ad}{}^{i} K_{bci}$$

new terms compared to stationary and non-dissipative space-filling fluids

cf. Banerjee,Bhattacharya,Bhattarcharyya,Jain,Minwalla et al
Jensen,Kaminski,Kovtun,Meyer,Ritz et al
Bhattacharya,Bhattacharyya,Rangamani

Young modulus from effective action

using 2nd order elastic corrections one finds from the action

$$\mathcal{D}^{abi} = \mathcal{Y}^{abcd} K_{cd}{}^{i}$$

with

$$\mathcal{Y}^{abcd} = 2\left(\lambda_1 \gamma^{ab}\gamma^{cd} + \lambda_2 \gamma^{a(c}\gamma^{d)b} + \lambda_3 \mathbf{k}^{(a}\gamma^{b)(c}\mathbf{k}^{d)} + \frac{\lambda_4}{2}\left(\gamma^{ab}\mathbf{k}^c\mathbf{k}^d + \gamma^{cd}\mathbf{k}^a\mathbf{k}^b\right) + \lambda_5 \mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\mathbf{k}^d\right)$$

black branes in gravity are a particular case of this !

Outlook

• systematic effective actions for elastic/hydrodynamic properties of charged fluid branes
 - obtained useful inputs/insights from gravity

Cf. development of fluids/superfluids inspired by gravity and holography

- elastic corrections for D3-branes and AdS/CFT !

- responses for spinning charged branes

- response coefficients in other backgrounds with non-zero fluxes (susceptibility, polarizability)

- Chern-Simons couplings

- multi-charge bound states

- entropy current

- effective hydrodynamics of spinning D3-branes (Dp,M)

- further explore AdS/Ricci flat connection

The end