Multiple Holographic Orderings

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- *Musso 1302.7205, 1304.6118*
- *Bigazzi, Cotrone, Musso, Pinzani, Seminara 1111.6601*
- *Amoretti, Braggio, Maggiore, Magnoli, Musso (work in progress)*

Outline

- *Phenomenological & Theoretical Motivations*
- *The Holographic Model*
- *Results (phase diagram, transitions,...)*
- *Conclusion and Future Prospects*

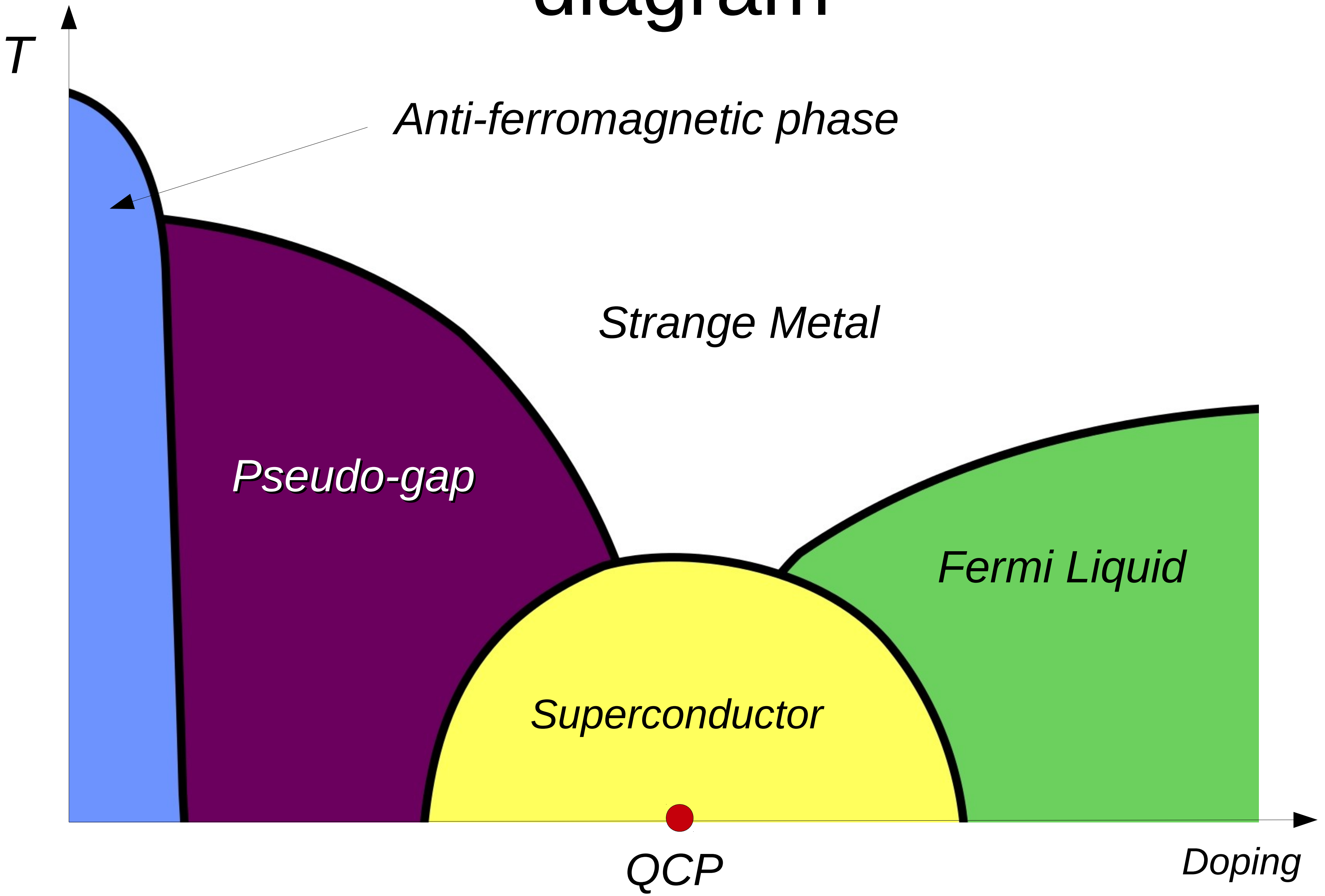
Physical Motivations

- *Fermi liquid picture vs strong coupling*
  [Basu, He, Mukherjee, Rozali, Shieh 2010]
- *Relevance in strongly correlated electrons systems*
  [Zaanen 2011, ...] (cuprates, heavy fermions, ...)
- *Emergent degrees of freedom and new instabilities*
  ("deconfined" quantum phase transitions)

Ferromagnetic superconductors
[Huy, Gasparini, Nijs, Huang, Klaasse, Gortenmulder, Visser, Hamann, Gorlach, Lohneysen cond-mat 0708.1388]

- *QCP ferromagnetic/paramagnetic transition*
- *“cooperative” orderings (SC driven by critical magnetic fluct.)*
- *itinearant ferromagnetism*

High Tc Superconductor phase diagram



The Specific Problem

- Multiple orderings at strong coupling
  - *possibility of coexisting order parameters*
  - *phase structure and transitions*
- Competition/enhancement of multiple order parameters
  - *dynamics and interactions between the order parameters*

Theoretical Framework

- Gauge/gravity correspondence
  - *Strong/weak map & dual semi-classical treatment*
  - *Thermodynamics*
  - *Linear response*
- Multi-haired black holes
  - *non-trivial black hole configurations*
  - *multiple instabilities*
- Bottom-up models

The Models (1): *Holographic Superconductors*

Paradigmatic example of holographic model featuring spontaneous symmetry breaking (with charged, scalar order parameter)

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

[Gubser '08 – Hartnoll, Herzog, Horowitz '08]

Ansatz: $A = \phi(r)dt\,, \qquad \psi = \psi(r)$

Near-boundary asymptotics and the *“holographic dictionary”*:

$$\phi = \mu - \frac{\rho}{r} + \cdots \qquad\qquad \psi = \frac{\psi^{(1)}}{r} + \frac{\psi^{(2)}}{r^2} + \cdots$$

The Models (2): *Unbalanced Holographic Superconductors*

$$\mathcal{R} + \frac{6}{L^2} - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{4}Y_{ab}Y^{ab} - V(|\psi|) - |\partial\psi - iqA\psi|^2$$

[Bigazzi, Cotrone, Musso, Pinzani, Seminara '11]

Field strength:

$$Y = dB$$

Ansatz:

$$B_a dx^a = v(r)dt$$

Boundary expansion:

$$v(r) = \delta\mu - \frac{\delta\rho}{r} + \dots$$

“phenomenological” interpretation: Condensate of Cooper pairs in the singlet state

State of the Art

- Iqbal, Liu, Mezei, Si - *Quantum phase transitions in holographic models of magnetism and superconductors*, 1003.0010 [hep-th]
- Huang, Lin, Maity - *Holographic Multi-Band Superconductor*, 1102.0977 [hep-th]
  Krikun, Kirilin, Sadofyev - *Holographic model of S^{+/-} multiband superconductor*, 1210.6074 [hep-th]
- Basu, He, Mukherjee, Rozali, Shieh - *Competing Holographic Orders*, 1007.3480 [hep-th]
- ...

"holographic orders tend to compete for attractive bulk interactions, including gravity, and to cooperate, or be mutually enhancing, for repulsive bulk interactions between the corresponding order parameters"

New Investigations

- *Neat test of the competition/enhancement conjecture*
- *Two scalar order parameters*
- *Effects of direct interactions and characterization of the equilibrium*
- *Probe and a look beyond*

The Two Scalar Model

Action:

$$S = \frac{1}{2\kappa_4^2}\int dx^4 \sqrt{-g}\Big[R + \frac{6}{L^2} - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{4}Y_{ab}Y^{ab} - |\partial\psi - \mathrm{i}q_\psi A\psi|^2 - m_\psi^2\psi^\dagger\psi - |\partial\lambda - \mathrm{i}q_\lambda B\lambda|^2 - m_\lambda^2\lambda^\dagger\lambda - \gamma\psi^\dagger\psi\lambda^\dagger\lambda\Big]$$

Ansatz:

$$\psi = \psi(r)$$
$$A_a dx^a = \phi(r)dt$$
$$\lambda = \lambda(r)$$
$$B_a dx^a = v(r)dt$$

- *Direct quartic interaction for the scalars (with and without gravitational interactions)*
- *Minimal couplings with the gauge fields*
- *Equal charges*
- *Equal scalar masses*
- *Swap symmetry between the A and B sectors* $(A_\mu, \psi) \leftrightarrow (B_\mu, \lambda)$

Handling the system

Probe approx. → *AdS-Schwarzschild background*

$$g(r) = r^2 \left(1 - \frac{r_h^3}{r^3}\right) \qquad r_h = \frac{4}{3}\pi T$$

A system of 4 second order equations → 8 parameters

… consistency constraints: 8 → 4 parameters [Horowtiz 1002.1722]

$$\phi'(r_h)\ , \quad v'(r_h)\ , \quad \lambda(r_h)\ , \quad \psi(r_h)$$

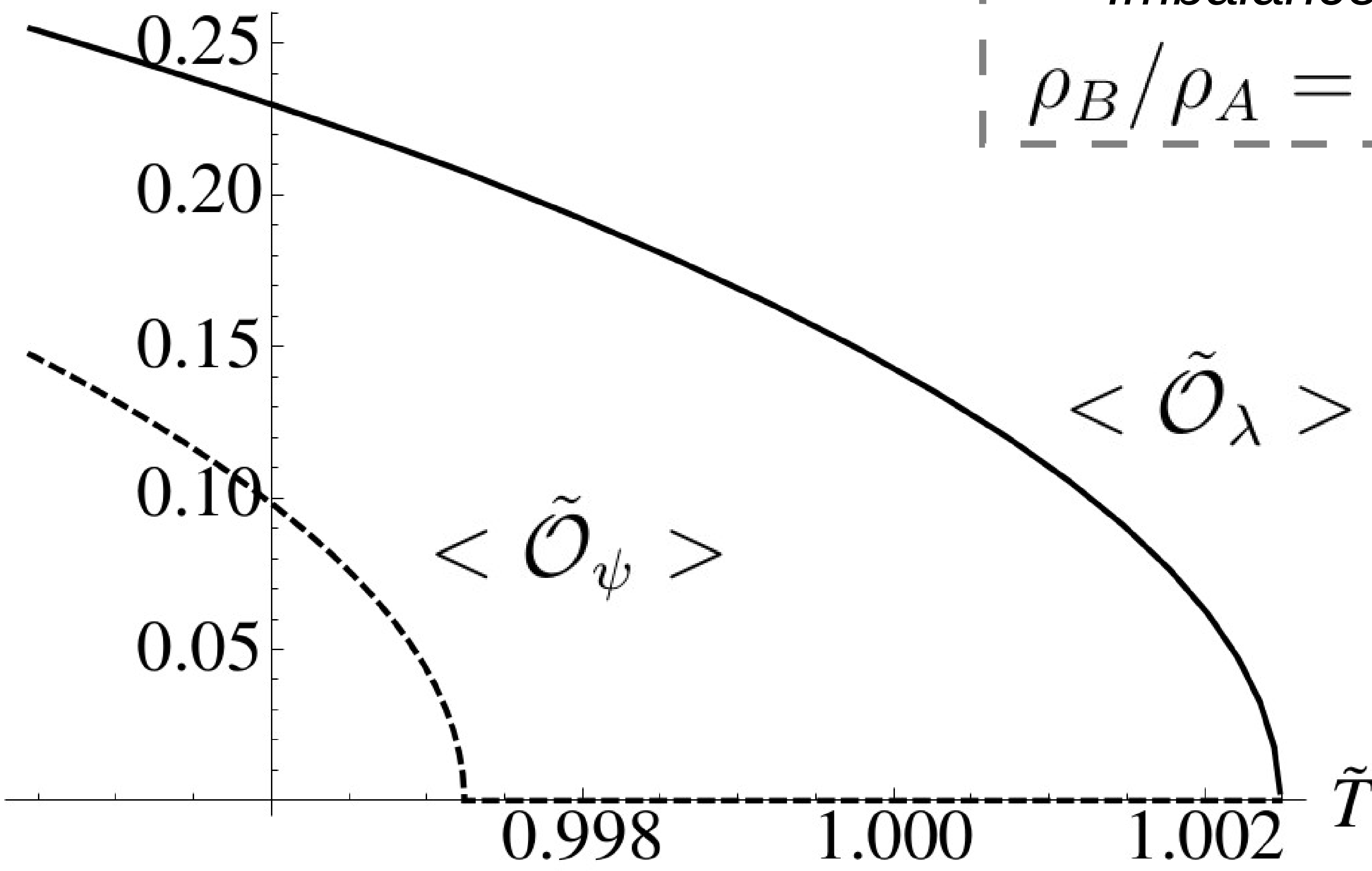
… physical requirements: 4 → 2 parameters

$$\tilde{T} = \frac{T}{\sqrt{\rho_A + \rho_B}} \qquad \rho_A/\rho_B$$

Coexistence

Imbalance:

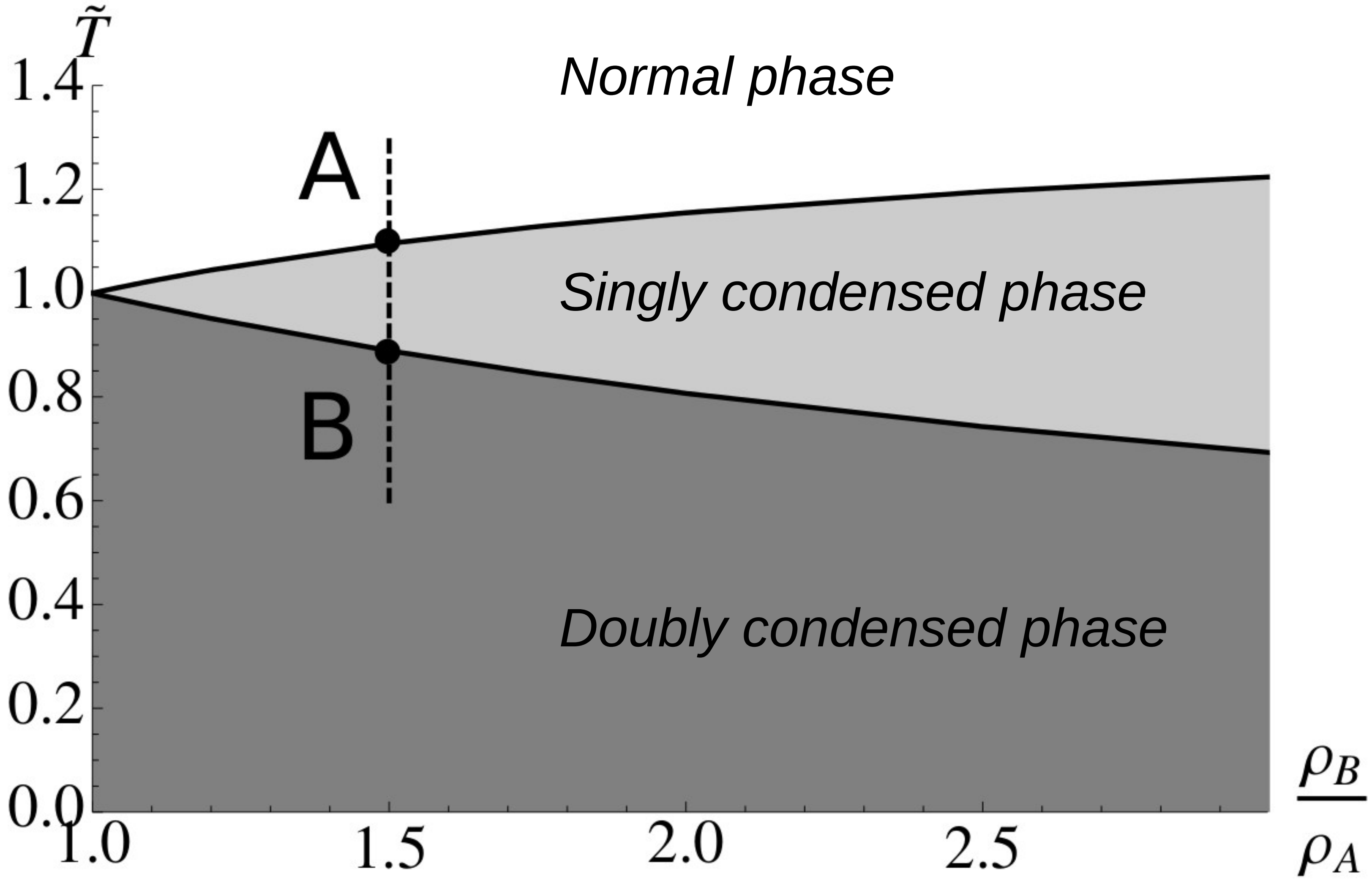
$$\rho_B/\rho_A = 1.01$$



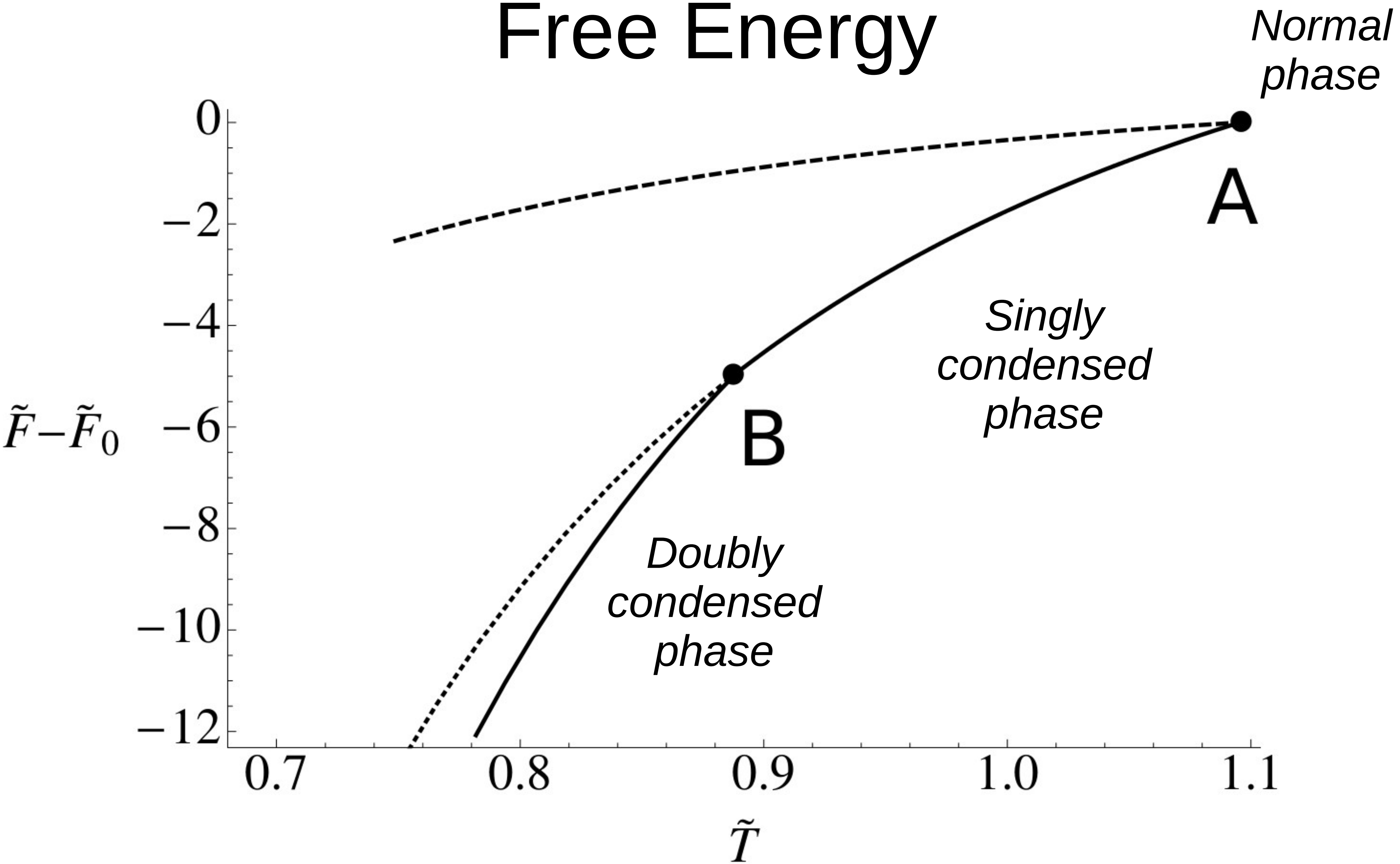
The temperature is normalized with respect to the balanced case

Phase Diagram

Temperature vs imbalance plane



Free Energy

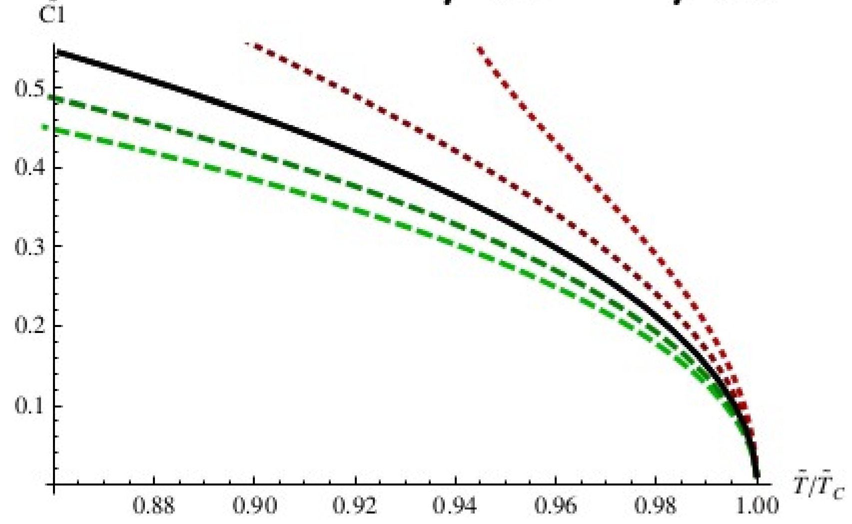
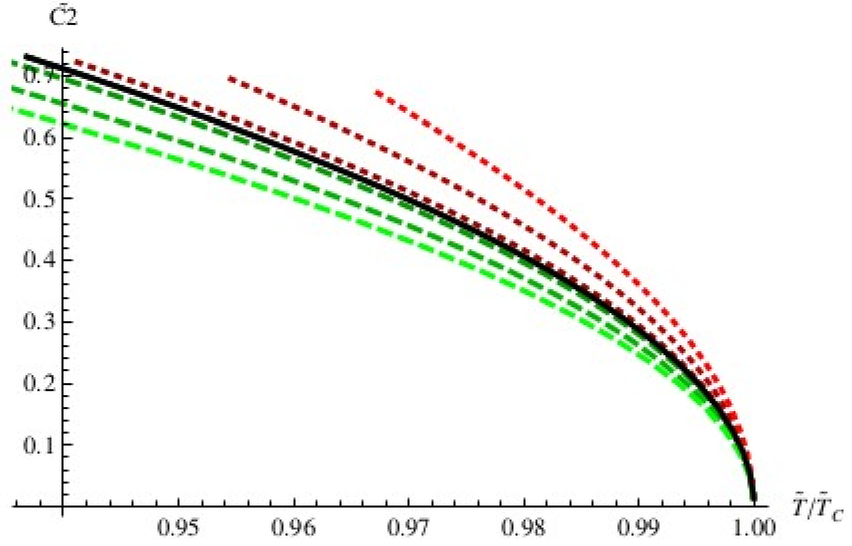


Free energy decrement with respect to the value at the first condensation *(i.e. point A in the plot)*

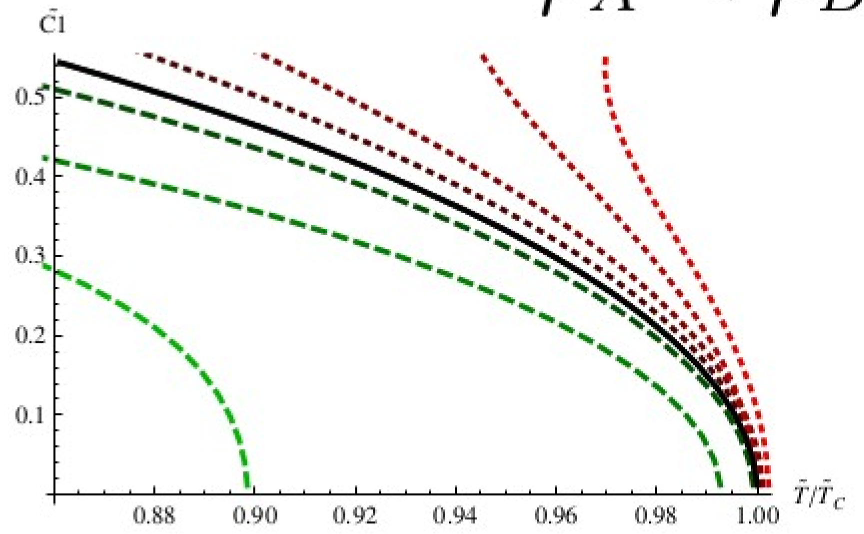
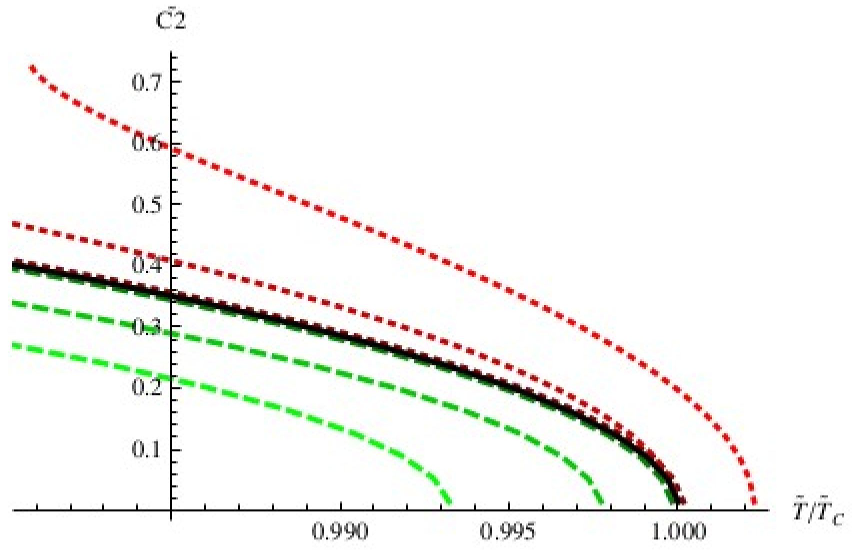
Condensates

Legenda:
Dotted → *repulsive/enhancing*
Dashed → *attractive/competing*

Balanced case $\rho_A = \rho_B$

Unbalanced case $\rho_A < \rho_B$

Competition/Enhancement

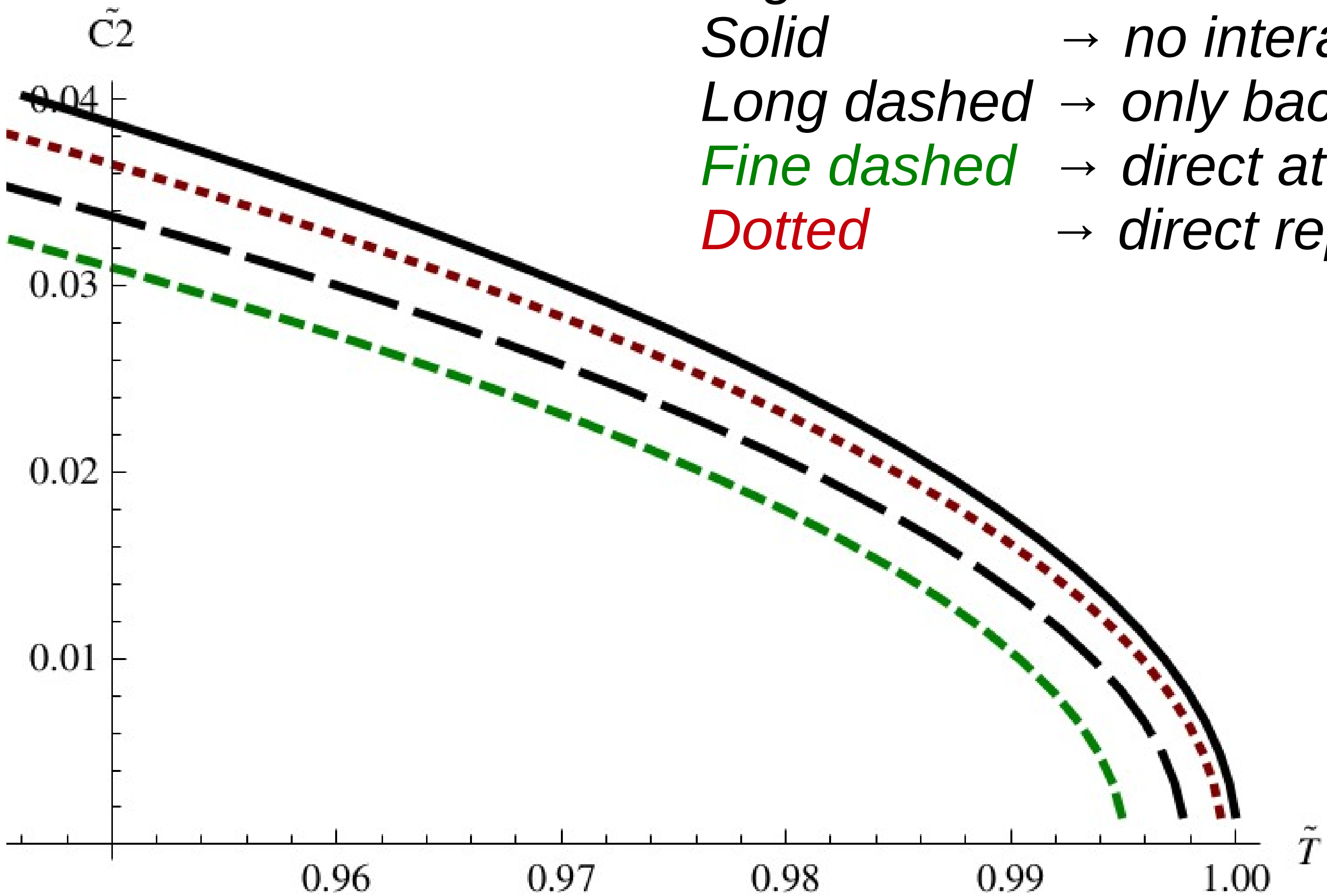
- *effective account of mediator exchange*
- *The quartic interaction can be interpreted straightforwardly as a contribution to the effective mass of the scalars*
- *The sign of such contribution corresponds to the repulsive/attractive character of the interaction*

Scalar equation of motion

$$\psi'' + \left(\frac{2}{r} + \frac{g'}{g}\right)\psi' + \frac{q_A^2}{g^2}\phi^2\psi - \frac{m_\psi^2 + \gamma\lambda^2}{g}\psi = 0$$

Adding the Back-reaction

Legenda:
Solid → no interaction at all
Long dashed → only backreaction
Fine dashed → direct attractive int. + backreact.
Dotted → direct repulsive int. + backreact.



Gravity adds an attractive contribution to the scalar interactions and then leads to competition of the two orderings

Time Reversal

Important both in relation to the "phenomenological" interpretations and in relation to possible UV embeddings

- *Electric and "magnetic" degrees of freedom*

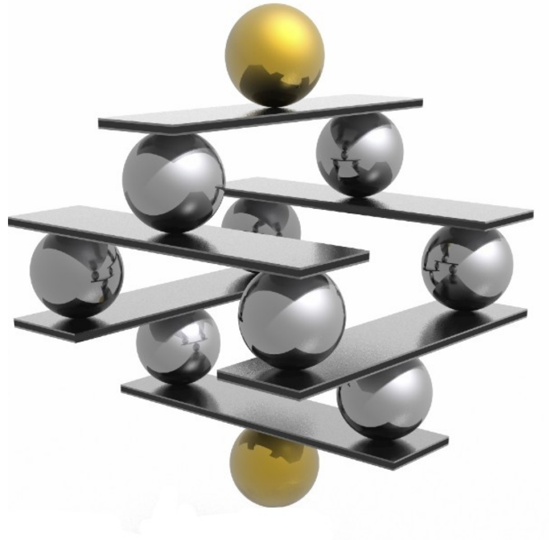
$$\mu \to \mu \,, \quad \delta\mu \to -\delta\mu$$

- *D-brane picture (two stack configuration)*

$$(A_t, A_i) \longrightarrow (A_t, -A_i)$$

$$(B_t, B_i) \longrightarrow (-B_t, B_i)$$

Comments on stability



- ***BF-like bound* for the “second” scalar?**
  - *Infrared geometry → no analytical solution for the hairy (“first” scalar) black holes*
- ***Numerical stability vs physical stability***
  - *shooting method as opposed to numerical sampling method*

2 Vector order parameters...

$$\sqrt{-g}\,\mathcal{L} = -\frac{1}{4}F^2 - \frac{1}{4}Y^2 + \frac{c}{2}FY$$

$$F = dA \qquad Y = dB$$

- *Coexistence of vectorial order parameters*
  (triplet Cooper pairing, magnetization,...)
- *Generalization of the holographic p-wave superconductor*
  [Gubser 0805.2960; Ammon, Erdmenger, Grass, Kerner, O'Bannon 0912.3515; Arias, Salazar Landea 1210.6823]
- *Field rotation*
  (identification of the chemical potentials)
- *Careful analysis of symmetry properties*
  (Abelian vs non-Abelian and the role of the ansatz,...)

Summary

- *One of the simplest models for holographic multiple orderings*
- *Coexistence of two scalar orderings in a holographic model*
- *Check of the competition/enhancement conjecture*
- *Characterization of the equilibrium of the system (phase diagram, kind of transitions, ...)*

Future Perspective

- *Systematic study of the back-reacted system*
  (low T, first order transitions ?,...)
- *Transport and momentum dependent fluctuations*
  (carrier-like behavior and two mobilities?, Negative Refraction, Additional Light Waves, ...)
- *Coexistence of vector/scalar orderings*
- *2 vector order parameters (work in progress)*
  (ferromagnetic superconductors, ...)
- *Momentum relaxation and holographic lattice*
  (Drude-like peak, mid-frequency scaling, …)
- *Spatially modulated phases*
- *...*

Related Literature

- Cai, Li, Li, Wang - *Competition and Coexistence of Order Parameters in Holographic Multi-Band Superconductors*, 1307.2768 [hep-th]
- Liu, Schalm, Sun, Zaanen - *Bose-Fermi competition in holographic metals*, 1307.4572 [hep-th]
- Dutta, Modak - *Holographic entanglement entropy in imbalanced superconductors*, 1305.6740 [hep-th]
- Sonner - *On universality of charge transport in AdS/CFT,* 1304.7774 [hep-th]
- Amado, Arean, Jimenez-Alba, Landsteiner, Melgar, Landea - *Holographic Type II Goldstone bosons*, 1302.5641 [hep-th]

Thanks!

The imbalance hinders the superconducting condensation

scalar effective mass → $$\tilde{m}_\psi^2 = m_\psi^2 - \frac{2q_A^2}{(1+\frac{\delta\mu^2}{\mu^2})}$$