Universal scaling properties of zero-temperature holographic phases

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Based on [1212.2625] with E. Kiritsis and ongoing work

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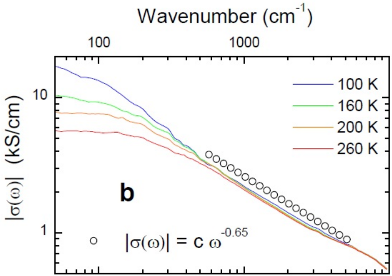
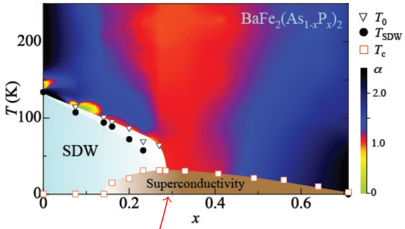
Outline

- Motivation
- Philosophy of the classification
- Classes of solutions and select examples
- Optical conductivity
- Fractionalisation transitions
- Holographic entanglement entropy, and a refinement sensitive to the presence of flux

Scaling in high critical temperature superconductors



[van der Marel&al'03]



[Kasahara&al'10]

- **Non-Fermi Liquid excitations** (no weakly-coupled quasiparticles).
- Interesting scaling behaviour of transport coefficients:

$$\sigma_{DC}^{-1} \sim T \quad (T \ll \mu)\,, \qquad Re\,(\sigma_{AC}) \sim \omega^{-2/3} \quad (T \lesssim \omega \ll \mu)$$

- A Quantum Critical Point at strong coupling is conjectured to be responsible for these scaling properties.
  Can holography help to understand the nature of QCPs at strong coupling and their transport properties?

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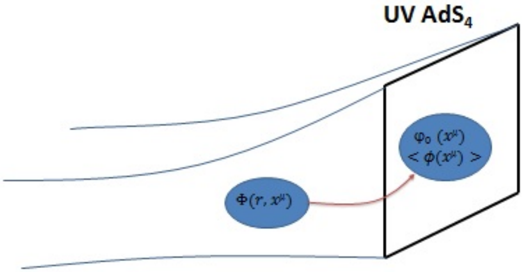
Mapping the holographic quantum critical landscape

Holography enables us to describe strongly-coupled phases of matter with a UV conformal fixed point (though strong effort to generalise to other UV asymptotics).

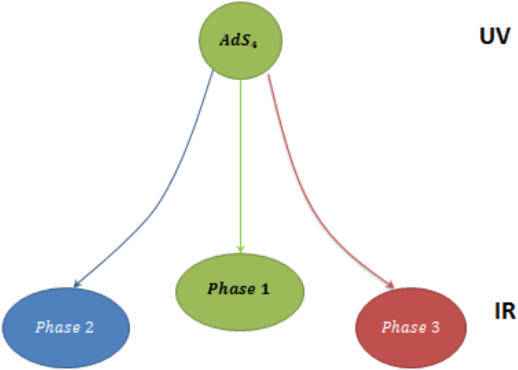
It gives a clear prescription to do so: asymptotics of bulk fields are mapped to sources and vevs of the dual field theory.



This picture however does not constrain the IR of the theory much, which might display **universal scaling behaviour** and give us information about the ground state.

Mapping the holographic quantum critical landscape

In particular, **many phases might be competing** in the IR.



How do we characterise these phases?
How do we determine the dominant one, **the ground state**?

To determine the ground state, it is particularly important to have a reliable map of the possible IR phases, the flows to the UV as well as possible quantum phase transitions.

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Mapping the holographic quantum critical landscape

One tool we can use is the concept of **effective holographic theories**, [Charmousis, Goutéraux, Kim, Kiritsis & Meyer '10]:

- introduce a minimal set of operators irrelevant in the UV but which will drive the IR dynamics (vector, scalar, etc.);
- write down an effective action describing these IR dynamics;
- figure out possible (extremal) IR phases, according to their symmetries: zero temperature, scaling backgrounds;
- work out the nature of the deformations around them (relevant or irrelevant);
- construct flows to the UV (sometimes analytically, more often numerically);
- determine the most stable thermodynamically in various regions of the phase diagram (thermal/quantum phase transitions with/without explicit/spontaneous symmetry breaking).

Symmetries

- In this talk: we keep **translation and rotation invariance**.
- Break Poincaré symmetry, retain scaling symmetries $t \to \lambda^z t$, $x^i \to \lambda x^i \Rightarrow$ Hyperscaling solutions:

$$ds^2 = -r^{-2z}dt^2 + L^2 r^{-2}dr^2 + r^{-2}d\vec{x}^2_{(d)}$$

which are supported by p-forms, massive vector fields or runaway scalars [Kachru&al'08, Taylor'08, Goldstein&al'09].
$z = 1$: AdS_4; $z \to +\infty$: $AdS_2 \times R^2$; $z < +\infty$ Lifshitz.

- Break scale invariance in the metric Ansatz $\equiv$ hyperscaling violation [Goutéraux&Kiritsis'11, Huijse&al'11, Dong&al'12]

$$ds^2 = r^{\frac{2}{d}\theta}\left(-r^{-2z}dt^2 + L^2 r^{-2}dr^2 + r^{-2}d\vec{x}^2_{(d)}\right)$$

There is an **effective spatial dimensionality** $d_{eff} = d - \theta$:
$S \sim T^{\frac{d-\theta}{z}}$
Using KK lifts, d_{eff} can be traced to the higher-dimensional spacetime.

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The effective holographic action

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \partial\phi^2 - Z(\phi)F^2 + V(\phi) + A_\mu J^\mu_{eff}(\phi)\right]$$

- Contains gravity, a gauge field (finite density) and a neutral scalar [Charmousis, Goutéraux, Kim, Kiritsis & Meyer'10].
- Effective source to the right hand side of Gauss's law:

  $$\nabla_\mu\left(Z(\phi)F^{\mu\nu}\right) = J^\nu_{eff}(\phi)$$

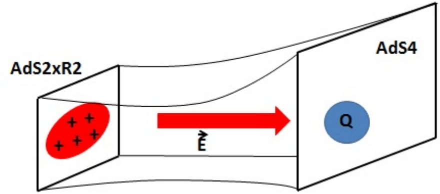
  which might break or not the U(1) symmetry.
  - $A_\mu J^\mu_{eff}(\phi) \sim W(\phi)A^2$, massive vector fields, effective description of **holographic superfluids** [Goutéraux&Kiritsis'12];
  - $A_\mu J^\mu_{eff}(\phi) \sim -p(\mu_{loc})$ describing a **charged ideal fluid of fermions** in the Thomas-Fermi limit [Hartnoll&al'10];
  - $A_\mu J^\mu_{eff}(\phi) \sim \vartheta(\phi)F\wedge F$, **Chern-Simons coupling** [Donos&Gauntlett'11]
- The effective scalar potential has several competing terms

  $$V_{eff}(\phi) = V(\phi) - Z(\phi)F^2 + A_\mu J^\mu_{eff}(\phi)$$

Cohesion/Fractionalisation in Holography

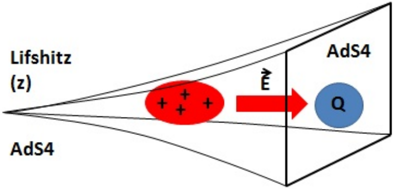
Zero density, [WITTEN'98]: **Event horizon** $\Leftrightarrow$ **Deconfinement**
Finite density, [HARTNOLL'11]: Charged horizon $\Leftrightarrow$ **Fractionalisation**

Separate contributions to the boundary charge density



Reissner-Norström black hole [MIT, LEIDEN'09]

Fractionalised phase



Electron star [HARTNOLL&AL'10], Superfluid [GUBSER&NELLORE, HOROWITZ&ROBERTS'09]

Cohesive phase

Let us introduce a 'cohesion' exponent:

$$\int_{\mathbf{R}^d} Z(\phi) \star F \underset{IR}{\sim} r^{\xi}$$

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IR dynamics: relevant vs irrelevant operators

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \partial\phi^2 - Z(\phi)F^2 + V(\phi) + A_\mu J^\mu_{eff}(\phi)\right]$$

The behaviour of the IR phase under scaling actions is determined by the dimension of the IR operators

- A relevant current breaks Poincaré symmetry $\Rightarrow$ time and space are anisotropic $z \neq 1$
  In [Gubser&Nellore'09], interplay between AdS_4 and Lifshitz asymptotics for the superfluid phase.
- A relevant scalar operator breaks scale invariance $\Rightarrow$ hyperscaling violation with $\theta \neq 0$, along with a runaway scalar in the IR.
- A relevant source for the current breaks the conservation of the electric flux $\Rightarrow$ cohesive phases with $\xi \neq 0$.

The final ingredient: the conduction exponent

Up until now, we have discussed the scaling behaviour of the metric, the scalar and the electric flux. There is a final ingredient, related to the scaling of the electric component of the vector

$$A_t \sim r^{\zeta-\xi-z}\,\mathrm{d}t$$

ζ is the conduction exponent (for reasons shortly apparent).

For fractionalised phases $\xi = 0$, it parameterises the violation of the Lifshitz scaling $t \to \lambda t^z$, $x^i \to \lambda x^i$ by A_t.

For cohesive phases $\xi \neq 0$, ξ also participates in the violation of the Lifshitz scaling.

In that sense, it has a **similar role as** θ for the metric.

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Overall classifications of solutions

Whether (partially) fractionalised ($\xi = 0$) or not ($\xi \neq 0$), hyperscaling (θ=0) or not ($\theta \neq 0$), with or without a runaway scalar, solutions organise themselves into two classes:

- The current is relevant in the IR: dynamical exponent z is arbitrary but the conduction exponent $\zeta = \theta - d$.
- The current is irrelevant in the IR: dynamical exponent $z = 1$, but the conduction exponent ζ is arbitrary.

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Fractionalised examples, $\xi = 0$ (1)

Let us start with a popular example: any theory whose scalar couplings reduce along a runaway branch to

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \partial\phi^2 - e^{\gamma\phi}F^2 + V_0 e^{-\delta\phi} + ...\right]$$

where ... design subleading terms in the IR.

Then γ, δ can be traded for θ and z, there is electric flux in the IR ($\xi = 0$) and $\zeta = \theta - d$.

If $J^\mu_{eff} \neq 0$ but irrelevant in the IR: partially fractionalised.

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Fractionalised examples, $\xi = 0$ (2)

Take the theory
[GAO&AL'04,GUBSER&ROCHA'09,HENDI&AL'10,GOUTÉRAUX&KIRITSIS'11]

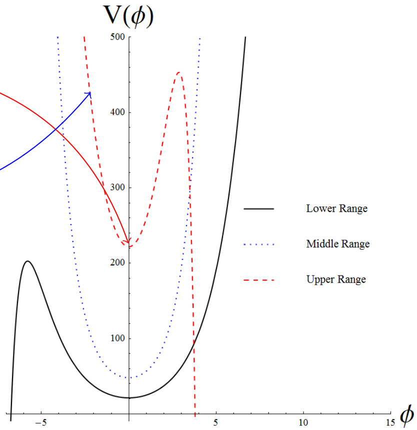
$$\mathcal{L}_{(4)} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-\frac{\phi}{\delta}}\left(F_{[2]}\right)^2 + V(\phi), \quad V = c_1 e^{-\frac{\phi}{\delta}} + c_2 e^{-\delta\phi} + c_3 e^{\frac{(1-\delta^2)}{2\delta}\phi}$$

Can be embedded into $D = 10$ SUGRA for $\delta = 1$, [CVETIC&AL'99].
There exists an asymptotically AdS solution.

The scalar is running in the IR, and drives a flow to a $\theta \neq 0$ solution

$$\mathcal{L}_{(4)} = R - \frac{1}{2}(\partial\phi)^2 + c_2 e^{-\delta\phi},$$

with a subleading current: produces a $z = 1$ solution.
A happy consequence: ζ is determined by γ and generically $\zeta \neq \theta - d$



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Cohesive examples, $\xi \neq 0$ (1)

Let us start with a popular example:

$$S = \int \mathrm{d}^d x \sqrt{-g} \left[R - \partial\phi^2 - Z(\phi) F^2 - W(\phi) A^2 + V(\phi) \right]$$

Assume that the scalar field settles into a minimum of the effective potential

$$V_{eff}(\phi) = V(\phi) - Z(\phi) F^2 - W(\phi) A^2 \,, \qquad \mathrm{d}V_{eff}(\phi)/\mathrm{d}\phi|_{\phi_\star} = 0$$

We obtain a well-known Lifshitz invariant solution ($\theta = 0$) [Kachru&al'08, Taylor'08]

$$\mathrm{d}s^2 = -\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2 \mathrm{d}r^2 + \mathrm{d}\vec{x}^2}{r^2} \,, \qquad A = Q r^{-z} \mathrm{d}t \,, \quad \phi = \phi_\star$$

with $\theta = 0$ and $\zeta = \xi = -d$.

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Let us now consider:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \partial\phi^2 - Z(\phi)F^2 - W(\phi)A^2 + V(\phi)\right]$$

$$V(\phi) \sim V_0 e^{-\delta\phi}, \quad Z(\phi) \sim Z_0 e^{\gamma\phi}, \quad W(\phi) \sim W_0 e^{\epsilon\phi},$$

- Have to assume $\epsilon = \gamma - \delta$ in order for the source J^μ_{eff} to be relevant in the IR: otherwise, partially fractionalised phases.
- Then, we obtain a hyperscaling violating solution

$$\mathrm{d}s^2 = r^\theta\left[-\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2\mathrm{d}r^2 + \mathrm{d}\vec{x}^2}{r^2}\right], \quad A = Qr^{\zeta-\xi-z}\mathrm{d}t\,, \quad \phi = \kappa\ln r$$

with

$$z, \theta, \xi = F\left(\gamma, \delta, \frac{W_0}{Z_0V_0}\right), \qquad \zeta = \theta - d$$

Also appeared in [Iizuka, Kachru, Kundu, Narayan, Sircar, Trivedi & Wang'12; Gath, Hartong, Monteiro & Obers'12].

AC conductivity scaling

Assume a perturbation of one of the spatial components of the electric potential of the form:

$$A_x \sim a_x(r)e^{-i\omega t}, \qquad \vec{k} = 0$$

This usually couples to other perturbations of the metric and other fields which couple to the vector field. With a little work, the linearised, perturbed equations can be decoupled, and then solved [Horowitz&Roberts'09, Goldstein &al'09, Charmousis&al'10, Hartnoll&Tavanfar'11...]. Remembering possible delta functions $\delta(\omega)$

- $z \neq 1$, $\zeta = \theta - d$: $\qquad Re(\sigma) \sim \omega^{|3-\frac{2+\theta-d}{z}|-1}$
- $z = 1$, $\zeta \neq \theta - d$: $\qquad Re(\sigma) \sim \omega^{|1-\zeta|-1}$

It is very tempting to conjecture that

$$Re(\sigma) \sim \omega^{|3-\frac{2+\zeta}{z}|-1}$$

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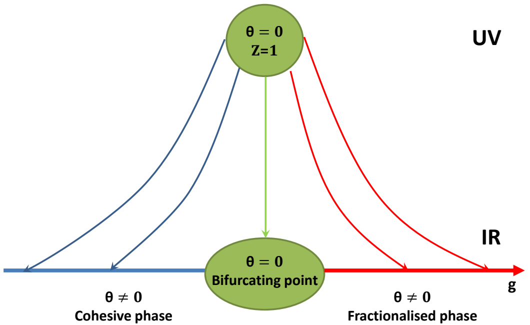
Quantum fractionalisation transitions

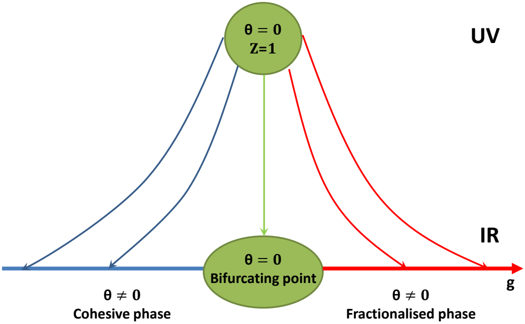
[Hartnoll&Huijse'11], [Sonner,Withers&al'12], [Goutéraux&Kiritsis'12]



If there is a **scale invariant fixed point** ($\theta = 0$) with a **relevant deformation**, there is a bifurcation in the RG flow: to reach this point from the UV, the flow must be fine tuned.

Away from the critical value, the flow picks up the **relevant deformation** and lands into a collection of stable hyperscaling violation fixed points: **a quantum critical line.**

Quantum critical lines



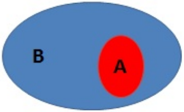
The line originates from the restoration of **scaling symmetry**:

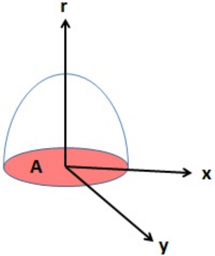
$$\phi \to \phi + \phi_0 \,, \quad (r, t, x, y) \to (e^{\#\phi_0} r, e^{\#\phi_0} t, e^{\#\phi_0} x, e^{\#\phi_0} y)$$

which automatically rescales the gauge field as well.

This is tied to the existence of an **extra scale**, originating from the size of the compact space when a lift to a scale invariant solution can be found.

Holographic entanglement entropy





To compute the entanglement entropy of region A, trace over the degrees of freedom of region B of the full Hilbert space.

$$S_E = -Tr(\rho_A)\log\rho_A\,,\qquad \rho_A = Tr_B(\rho)$$

Holographic entanglement entropy: area of the minimal bulk surface whose boundary is region A.

If $\theta = d-1$ (irrespectively of z), the area law is violated logarithmically: "hidden" Fermi surface of gauge-variant (SU(N)), charged (U(1)) dofs? [Ogawa&al'11], [Huijse&al'11]
Pb: no spectral weight at finite k and low ω [Hartnoll&Shaghoulian'12].

But $\theta = d-1$ geometries also exist in cohesive, superfluid phases [Goutéraux&Kiritsis'12]. Hidden Fermi surface of gauge-variant (SU(N)), neutral (U(1)) dofs? Normal phase on top of the superfluid phase?

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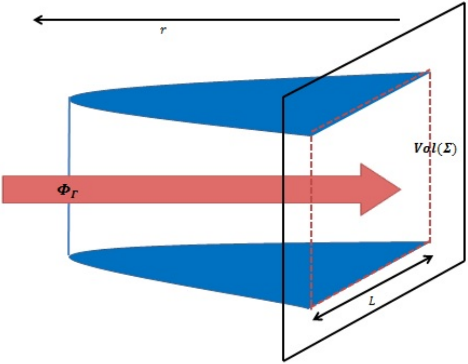
A refinement sensitive to the scaling of the electric flux

[Hartnoll&Radicevic'12] suggested to minimize

$$S_E^{\lambda} = \frac{A_\Gamma}{4G_N} + \lambda \Phi_\Gamma$$



- $\xi < \theta - d$:

$$A_\Gamma \sim Vol(\Sigma) L^{1+\theta-d}$$

$$\Phi \sim Vol(\Sigma) L^{1+\xi}$$

A_Γ always dominates Φ_Γ at large L

- $\xi > \theta - d$:

$$A_\Gamma \sim Vol(\Sigma) L^{\frac{2-\xi-3d+3\theta}{2+\theta-d-\xi}}$$

$$\Phi \sim Vol(\Sigma) L^{\frac{2-d+\theta+\xi}{2+\theta-d-\xi}}$$

Φ_Γ always dominates A_Γ at large L

Summary and outlook

- We have presented a unified framework to describe the scaling of extremal backgrounds, whether scale invariant or hyperscaling violating, introducing two novel exponents ('cohesion' and 'conduction').
- We have showed how the conduction exponent controls the scaling of the optical conductivity, while the cohesion exponent controls the scaling of a modified version of the holographic entanglement prescription.
- Can such scaling exponents show up elsewhere (DC conductivity,...)?
- Can this picture be generalised to more complicated solutions, like homogeneous breaking of translation invariance?
- Many flows to construct (numerically)