Inhomogeneous Holographic Thermalization

Ben Craps

Vrije Universiteit Brussel &
International Solvay Institutes

Collaborators: V. Balasubramanian, A. Bernamonti, J. de Boer, L. Franti,
F. Galli, E. Keski-Vakkuri, B. Müller, A. Schäfer

INTERNATIONAL SOLVAY INSTITUTES
BRUSSELS

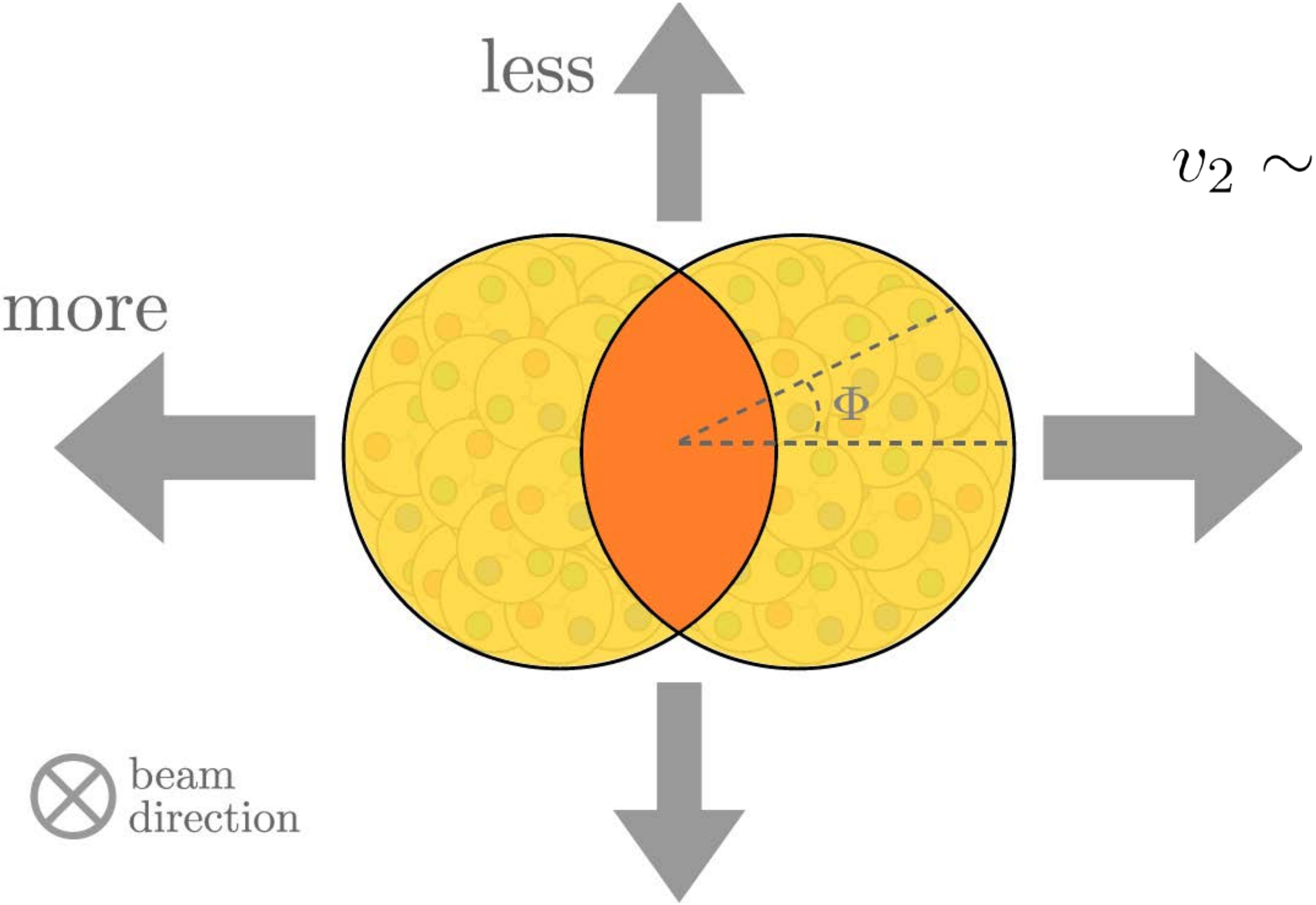
Gauge/Gravity Duality 2013

Munich, 29 July 2013

Vrije Universiteit Brussel

Motivation: heavy ion collisions

Elliptic flow → hydrodynamic regime



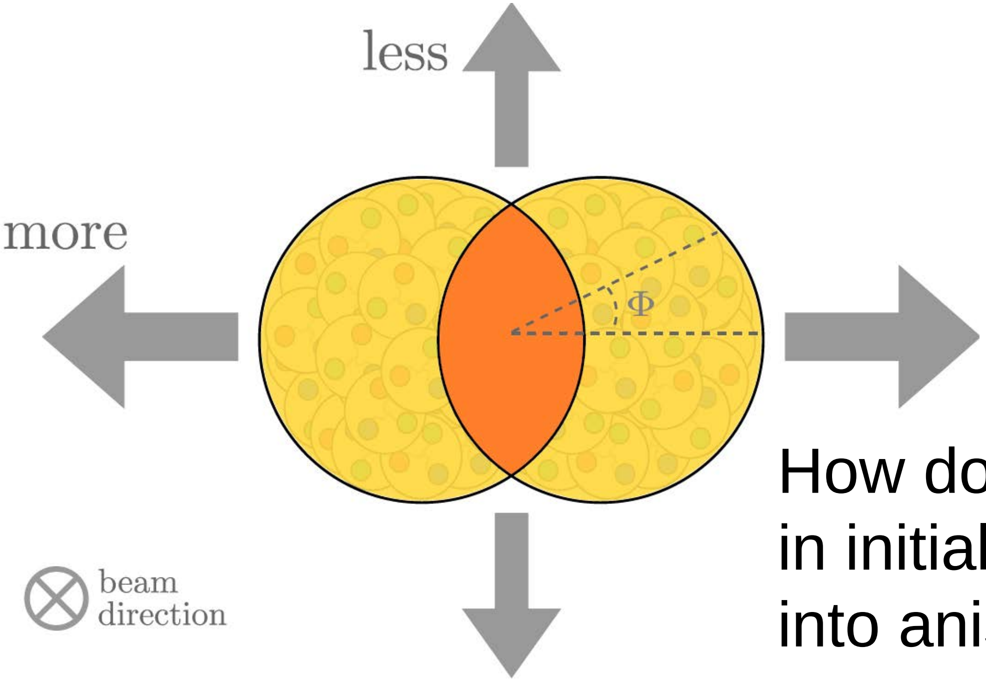
$$v_2 \sim \langle \cos(2\phi) \rangle$$

Properties of the quark-gluon plasma

- Elliptic flow → hydrodynamic regime
- Small viscosity → almost-perfect fluid, strong coupling (cf. jet quenching)
- Rapid thermalization
- Discovery of higher flow coefficients v_3, v_4, v_5, v_6

Higher flow coefficients

Azimuthal particle distribution $$\sim 1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_n)]$$

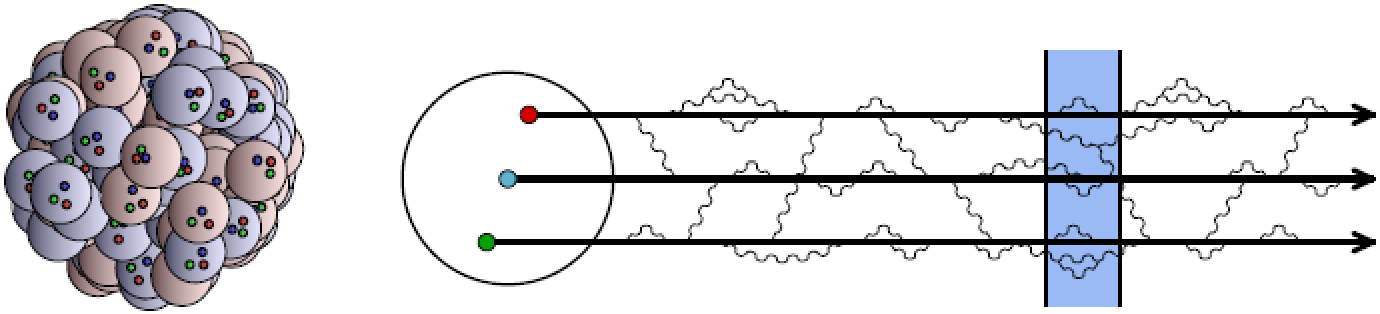


v_{odd} are nonzero (and large!) due to event-by-event fluctuations

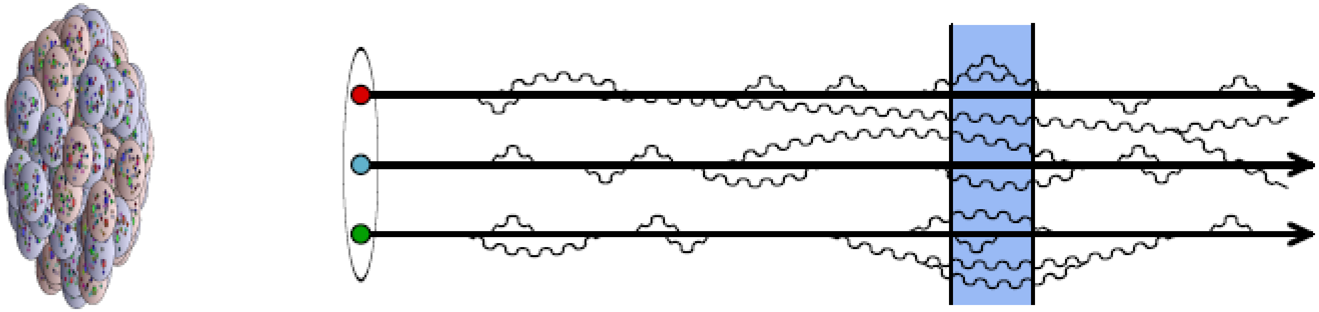
How do fluctuations (inhomogeneities) in initial energy deposition translate into anisotropies?

High energy nuclei

Low E nucleus:



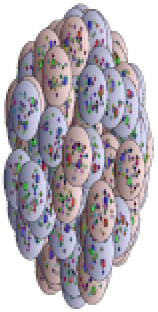
High E nucleus:

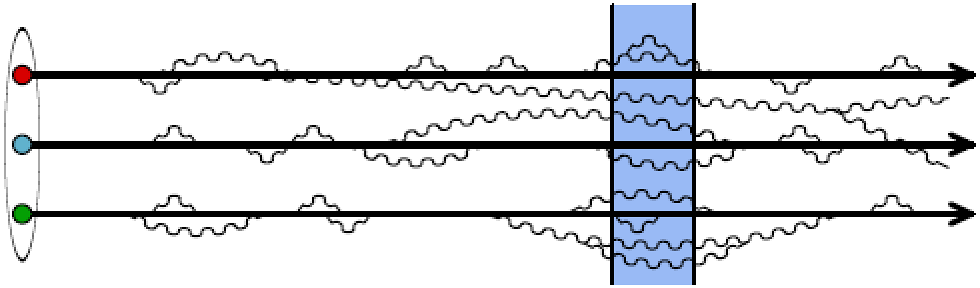


Figures from F. Gelis, 1211.3327

Color Glass Condensate

High E nucleus:





- Time dilation → many long-lived gluonic fluctuations.
- High parton density → non-perturbative despite weak coupling.
- Nonlinear processes are assumed to limit growth of gluon density: "saturation scale" $Q_s \gg \Lambda_{\rm QCD}$, related to gluon density per unit area.
- This enables a weakly coupled description in terms of classical gauge fields: Color Glass Condensate.

Figure from F. Gelis, 1211.3327

Color Glass Condensate and heavy ion collisions

The Color Glass Condensate model can be used to compute (statistics of) the initial deposition of energy after a heavy ion collision.

[Müller, Schäfer]

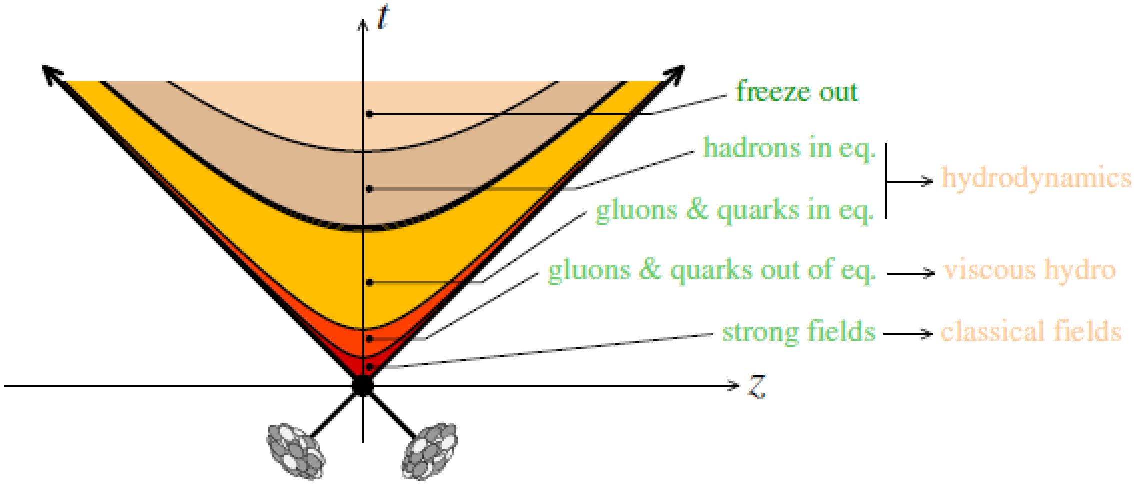


Figure from F. Gelis, 1211.3327

This provides initial conditions for subsequent thermalization process, often modeled by free streaming followed by viscous hydrodynamics.

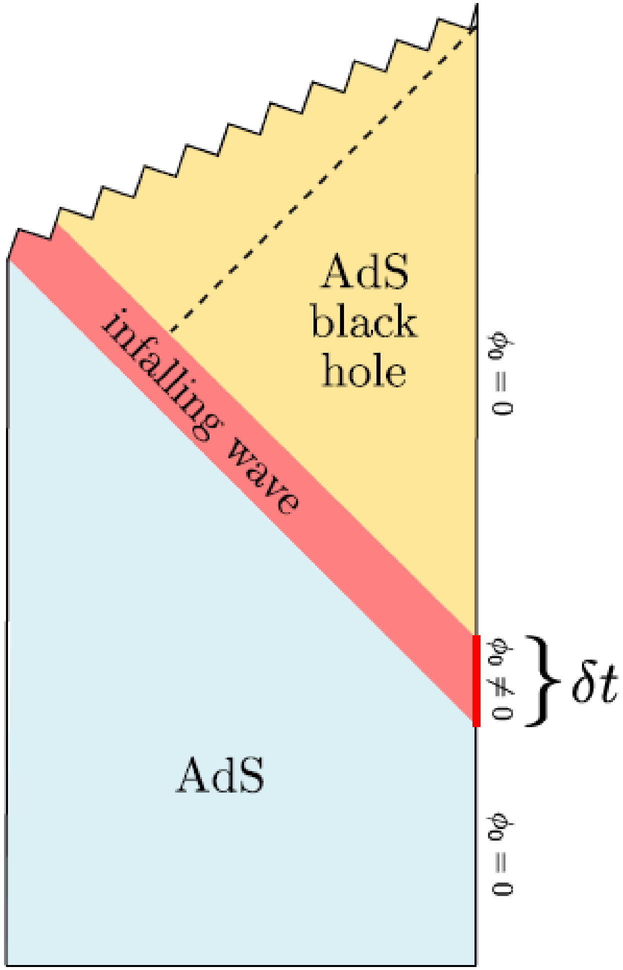
Is this model justified?

Using AdS/CFT

Real-time dynamics hard in lattice QCD → try AdS/CFT

N=4 SYM, large N, large λ	⟺ Gravity in AdS
Finite temperature (T>0)	Black hole
Fluid dynamics	Low-energy, long-wavelength perturbations of black holes
Thermalization	Black brane formation

Weak-field black hole formation in AdS



Massless bulk scalar ϕ

Homogeneous source $\phi_0(v) = \epsilon\tilde{\phi}_0(v)$
at boundary → shell (v=t at boundary)

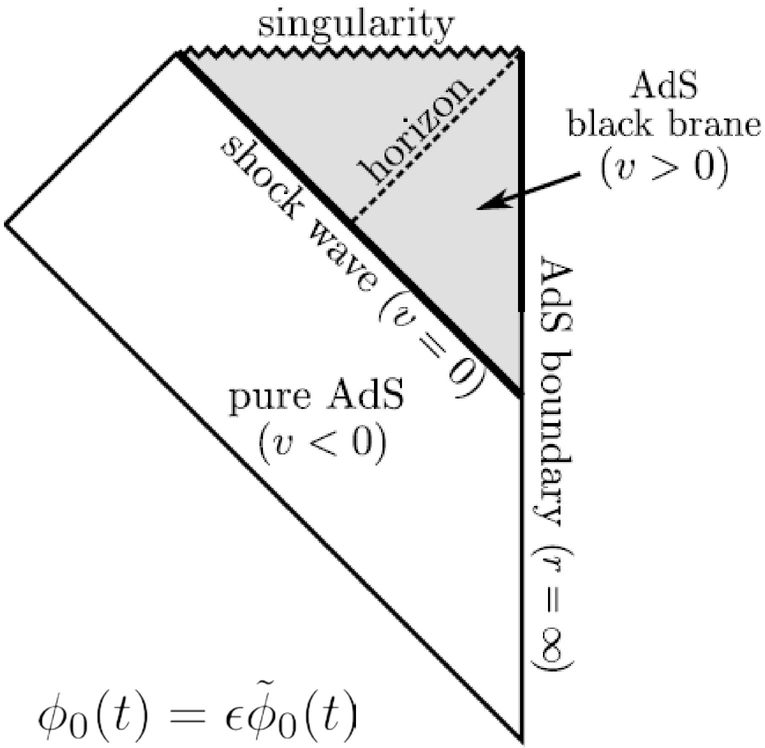
Results in black brane formation
(in Poincaré coordinates)

[BM] construct metric and scalar field
outside event horizon perturbatively in ϵ

[Bhattacharyya, Minwalla 2009]

AdS$_{d+1}$-Vaidya metric

Metric: $$ds^2 = 2dr\,dv - \left(r^2 - \frac{M(v)}{r^{d-2}}\right) dv^2 + r^2 d\vec{x}^2 + \mathcal{O}(\epsilon^4)$$



$$\phi_0(t) = \epsilon\tilde{\phi}_0(t)$$

with $$M(v) \sim \frac{\epsilon^2}{(\delta t)^d}$$

constant for $v > \delta t$ in odd d

and $$T \sim \frac{\epsilon^{2/d}}{\delta t}$$

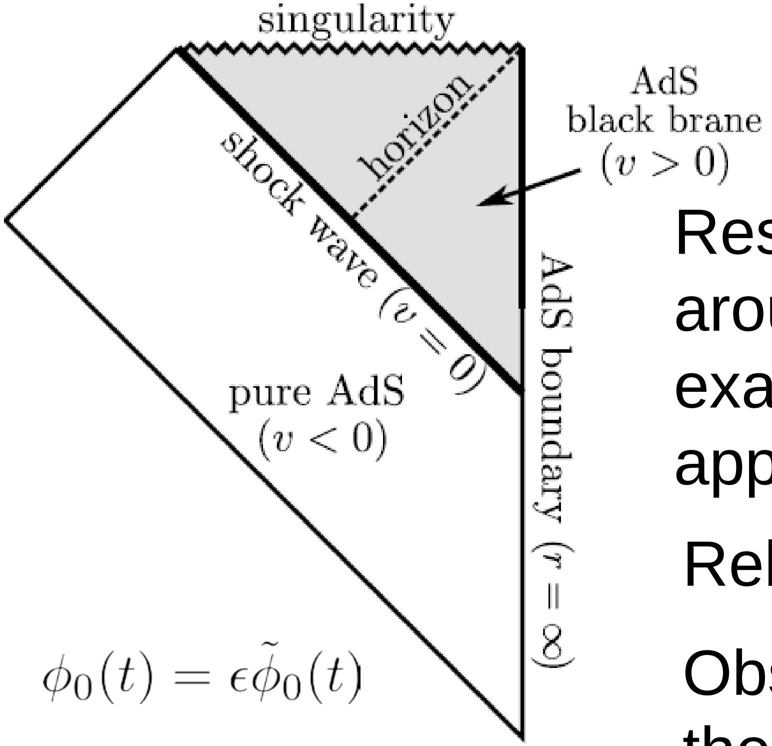
→ injection time short compared to inverse temperature of black brane to be formed

[Bhattacharyya, Minwalla]

Naive vs resummed perturbation theory

Naive perturbation theory in ϵ:

- reliable for $v \ll \frac{1}{T}$
- diverges for $v \gg \frac{1}{T}$



$$\phi_0(t) = \epsilon\tilde{\phi}_0(t)$$

[Bhattacharyya, Minwalla]

Resummed perturbation theory: expand around Vaidya instead of AdS. Work exactly in M(v) and perturbatively in other appearances of ϵ (cf. thermal pert. theory).

Reliable everywhere outside event horizon!

Observables decay exponentially to thermal values: $e^{-\#Tv}$

One-point functions

Holographic renormalization: $\langle T^{\mu\nu} \rangle$ can be read off from near-boundary expansion of $g_{\mu\nu}$

Focus on d=3 (AdS$_4$-Vaidya): metric coincides with black brane right after injection of energy

→ "instantaneous thermalization" of one-point functions

→ non-local observables needed to probe deviations from thermality. Fast thermalization.

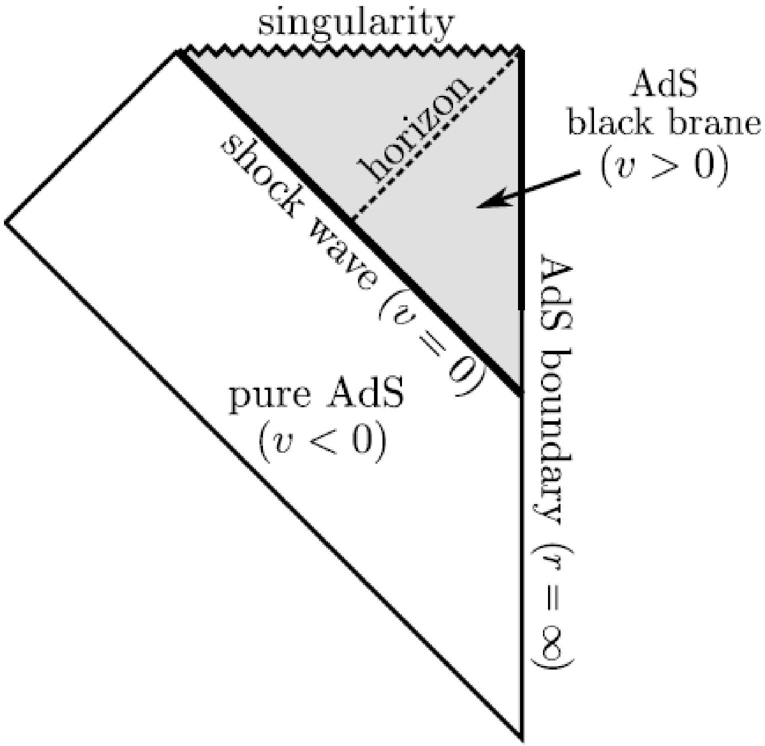
Models for heavy ion collisions

- Vaidya: homogeneous, isotropic injection of energy
- Homogeneous, anisotropic injection of energy
- Boost-invariant models more realistic symmetry
- Colliding shock waves

For stress tensor VEVs in homogeneous models: hydrodynamics appears to agree with holographic results well before local thermal equilibrium is reached.

Inhomogeneous BH formation in AdS$_4$

Let source $\varphi(v,x) = \epsilon\varphi_0(v,x)$ depend on (one of) spatial directions; only nonzero for $0 < v < \delta t$



Work in long-wavelength approximation (gradient expansion): scale of spatial variation $L(x) \gg 1/T(x)$

Regime of validity (naive pert. theory):

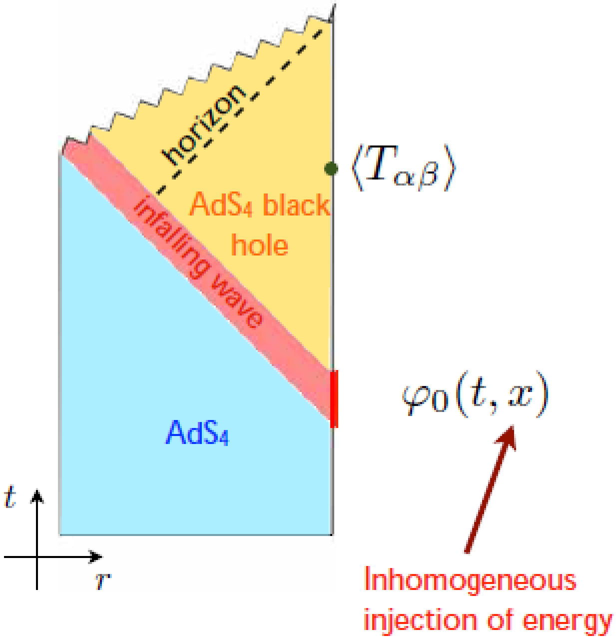
$$t \ll \frac{1}{T(x)} \ll L(x)$$

Procedure (inhomogeneous BH formation in AdS)

1) Write metric in EF gauge. Consider scalar field ϕ
2) Write down Einstein-scalar field equations
3) Impose pure AdS initial conditions (v<0), and boundary conditions with φ_0 as the only source
4) Amplitude expansion: $\phi(v,r,x) = \sum_{n=0}^{\infty} \epsilon^n \phi_n(v,r,x)$ and similarly for metric
5) Gradient expansion: $\phi_n(v,r,x) = \sum_{i=0}^{\infty} \mu^i \phi_{n,i}[\varphi_0(v,\mu x)]$
6) Solve Einstein equations order by order in ϵ and μ
7) Extract boundary stress tensor (holographic renormalization)

A few formulas

$$\phi = \varphi(v,x) + \frac{\dot\varphi(v,x)}{r} + \frac{\varphi''(v,x)}{2r^2} + \frac{\int_{-\infty}^{v} d\tau\, \varphi''''(\tau,x)}{8r^3} + \dots$$



first order in amplitude of source and up to fourth order in spatial gradients

Holographic renormalization:

$$\langle T_{\alpha\beta}\rangle \sim g_{(3),\alpha\beta}$$

1/r bulk metric coefficient in FG coordinates

Analysis

- Have analytic expressions for $\langle T^{\mu\nu} \rangle$ up to second order in source amplitude and fourth order in spatial gradients.

- Have compared with free streaming and with hydrodynamics (1st order, 2nd order).

Disclaimers

- Not a realistic model for heavy ion collisions (3d field theory, “longitudinal direction” is missing). Complementary to earlier work.

- Our amplitude and gradient expansions are only reliable for short times and long wavelengths compared to the local inverse temperature.

- No obvious reason that hydrodynamics should apply, but it did work surprisingly well in homogeneous models. Useful to see if inhomogeneities change this.

- Advantage: analytical control.

Results

- Inhomogeneities in energy density and pressures tend to smooth out after energy injection.

- Pressure anisotropies still grow after injection has ended.

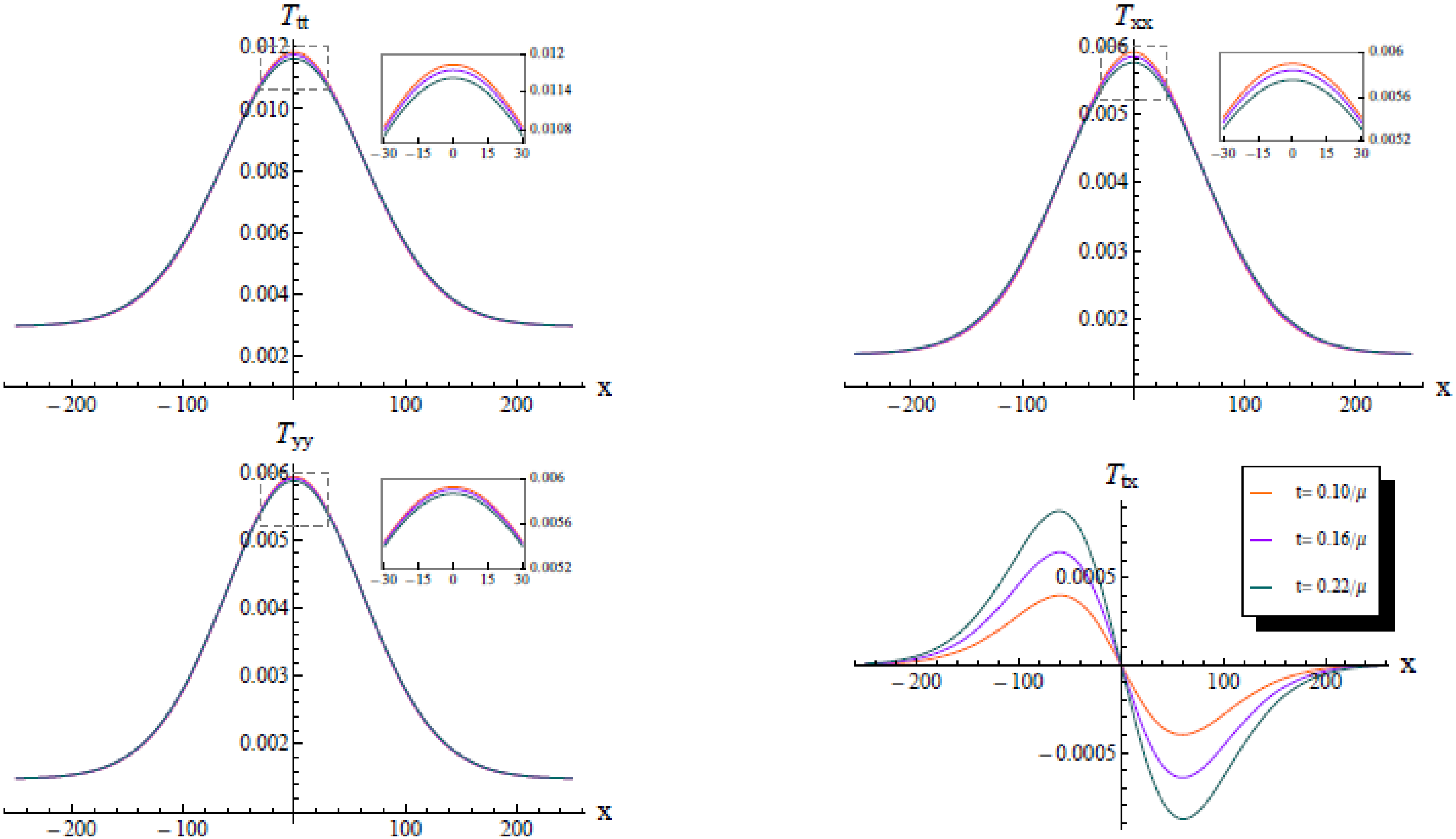
cf [Chesler, Yaffe]

- Qualitative and quantitative agreement with free streaming.

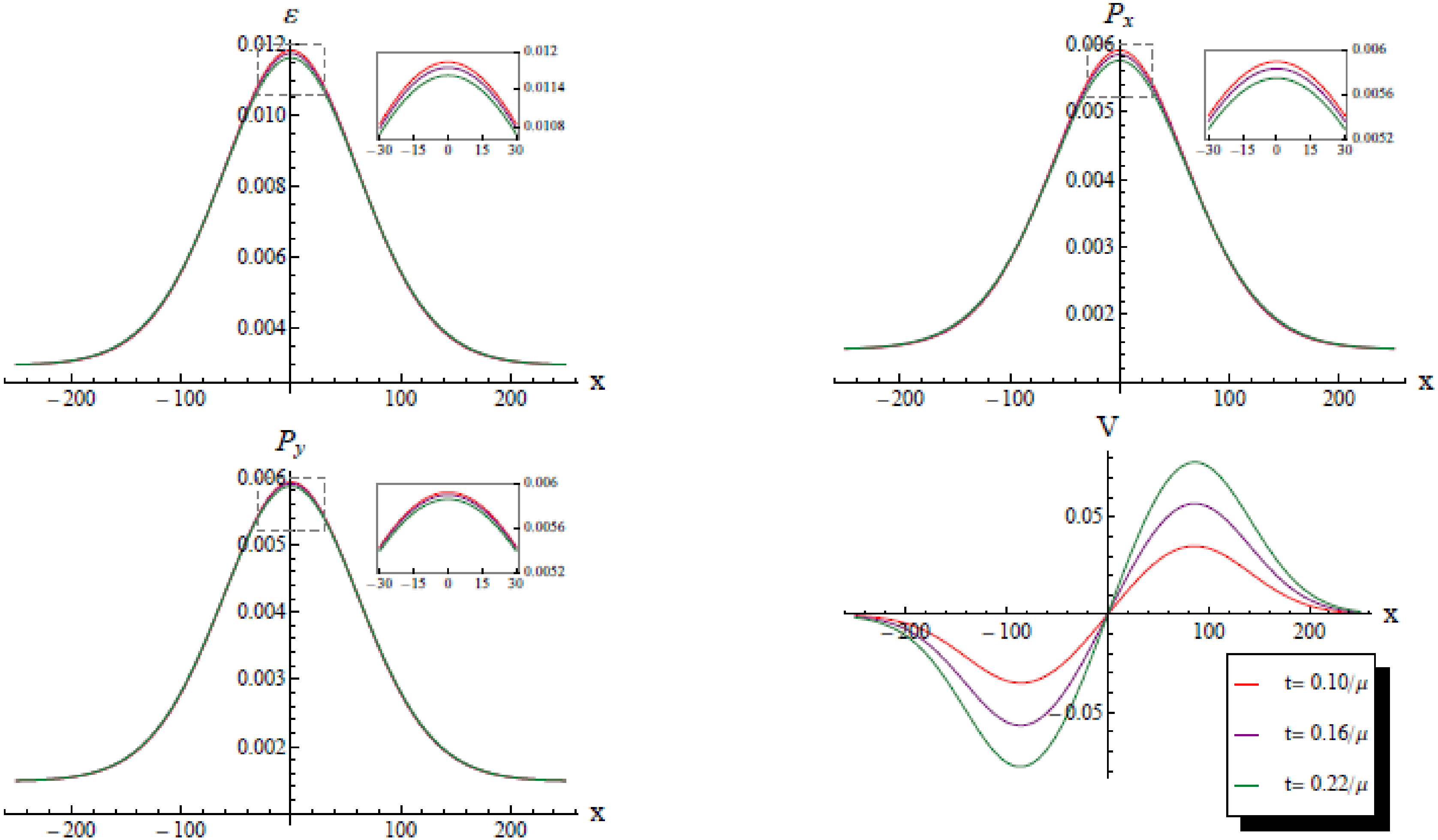
- Significant quantitative deviations from first order hydro (“effective shear viscosity” is smaller than in hydro).

- Second order hydro improves the matching near the end of the early-time window we can reliably probe. Not clear if agreement persists beyond this window.

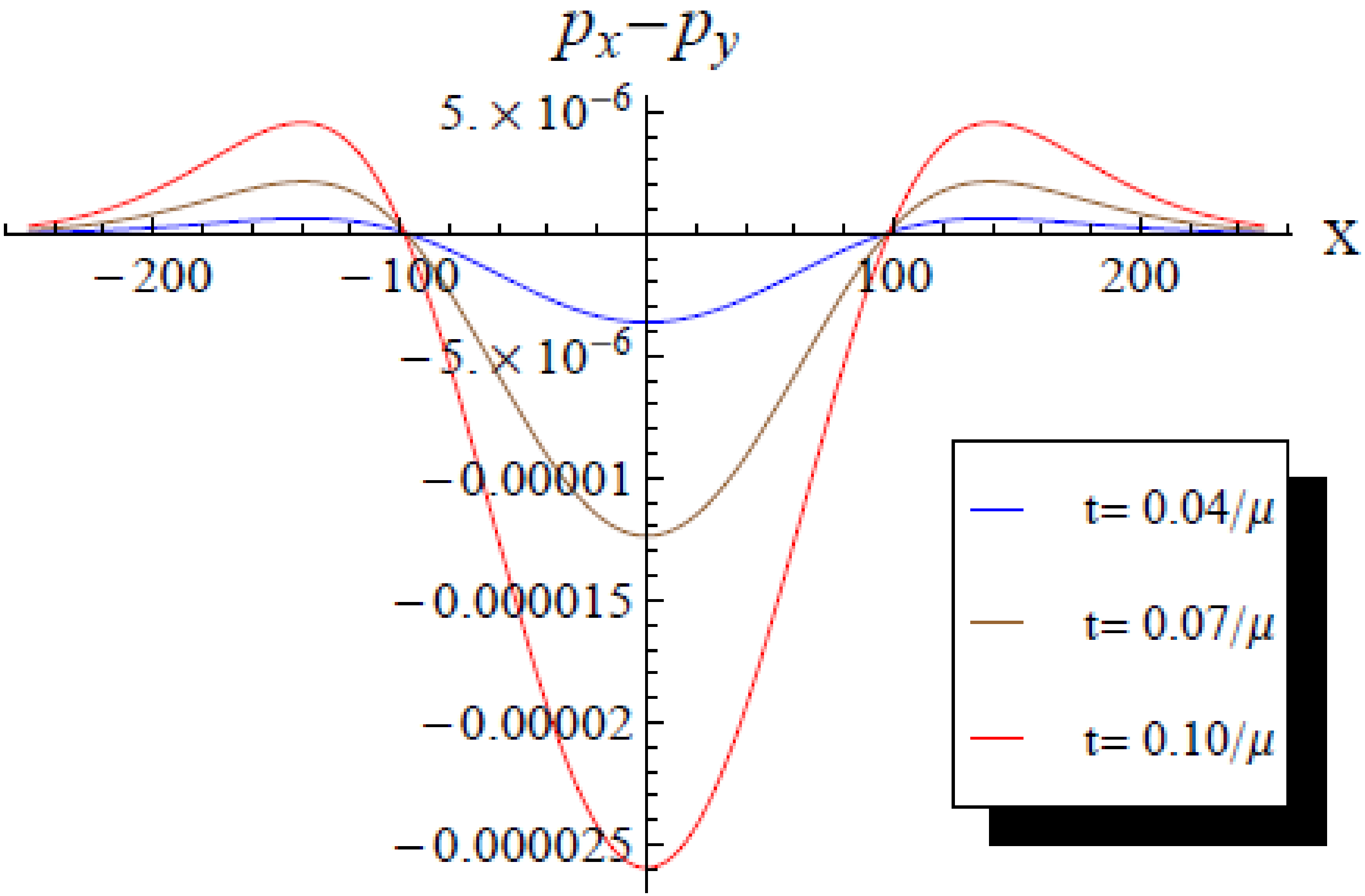
Stress tensor components (AdS/CFT)



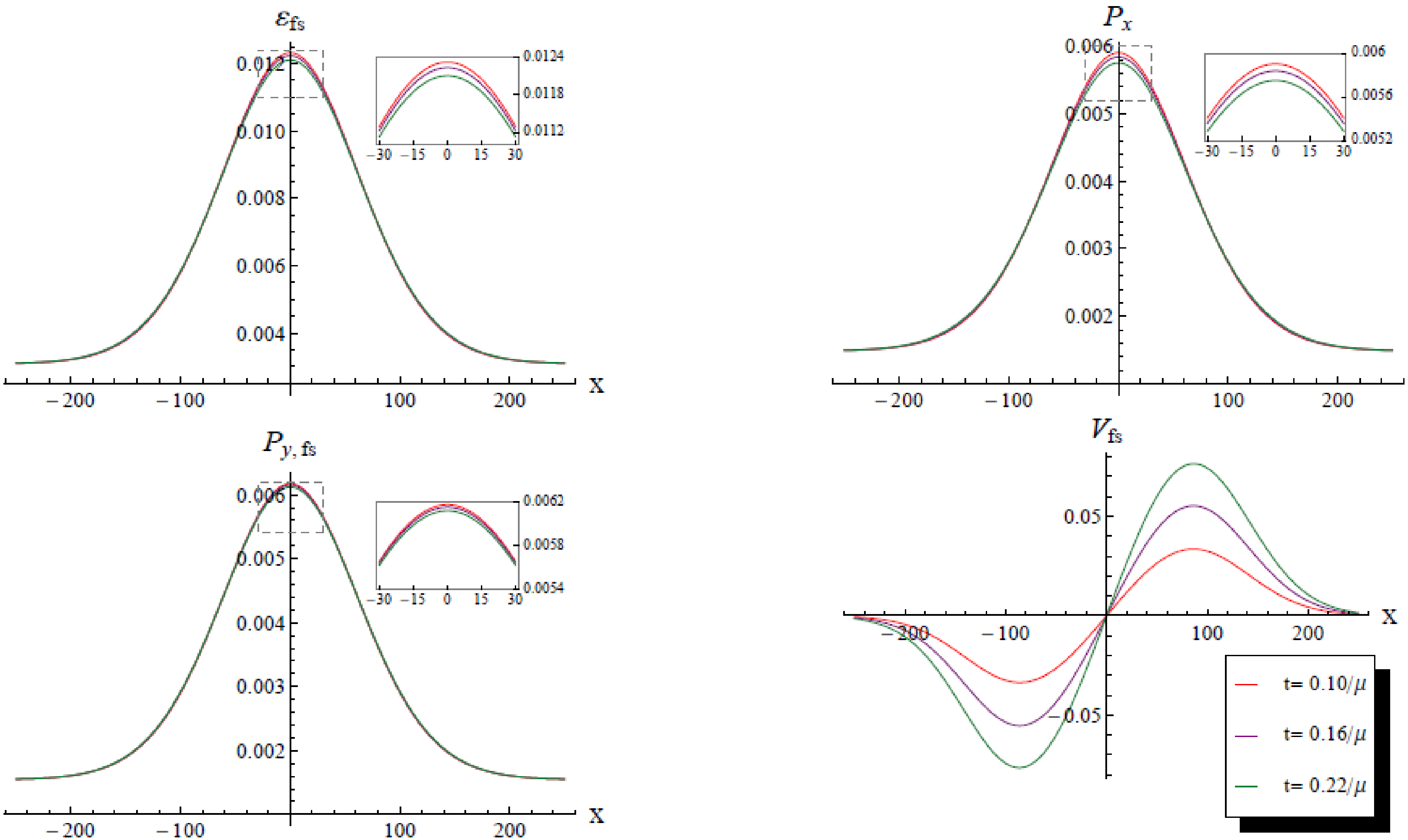
Stress tensor in local rest frame (AdS/CFT)



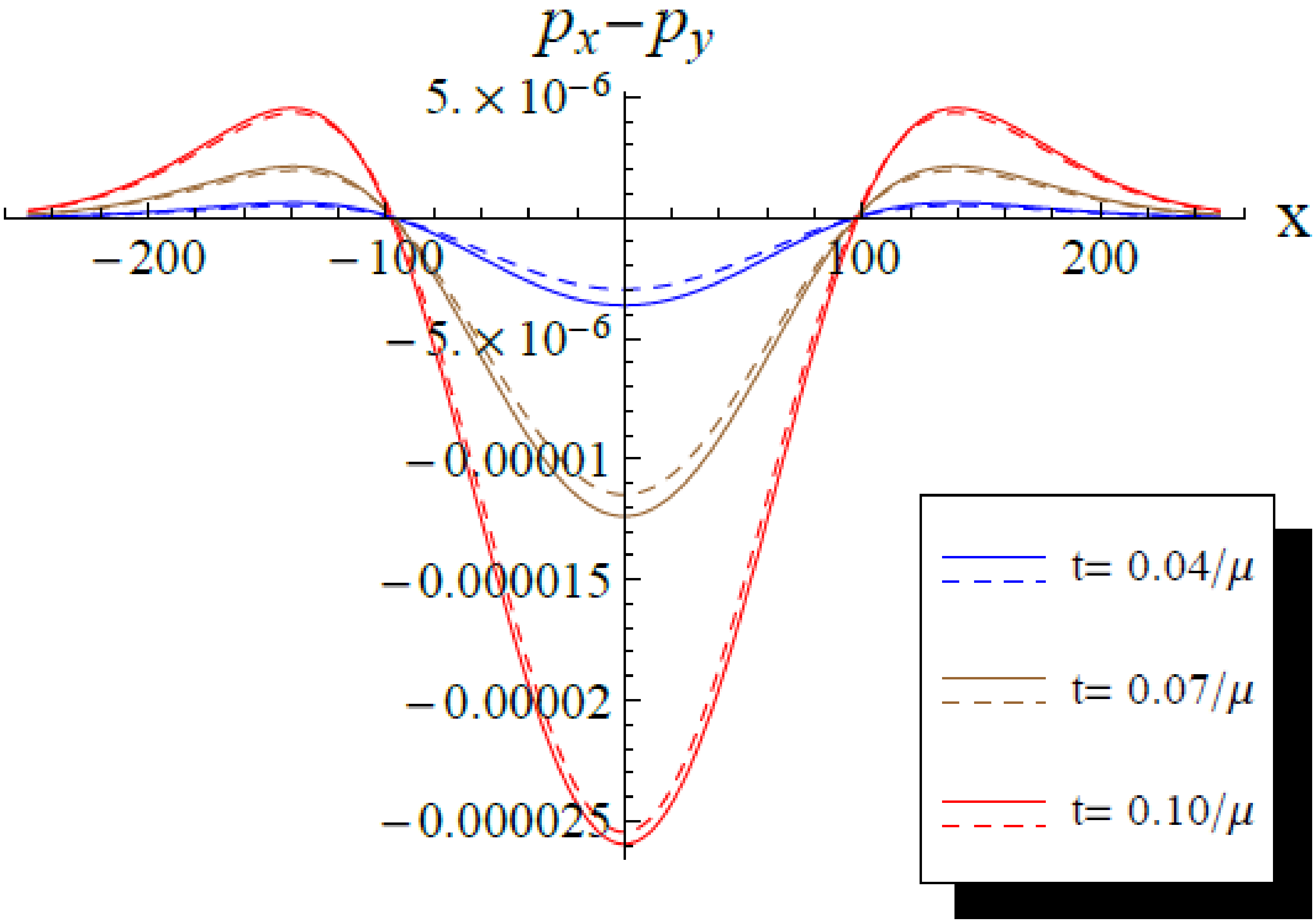
Pressure anisotropy (AdS/CFT)



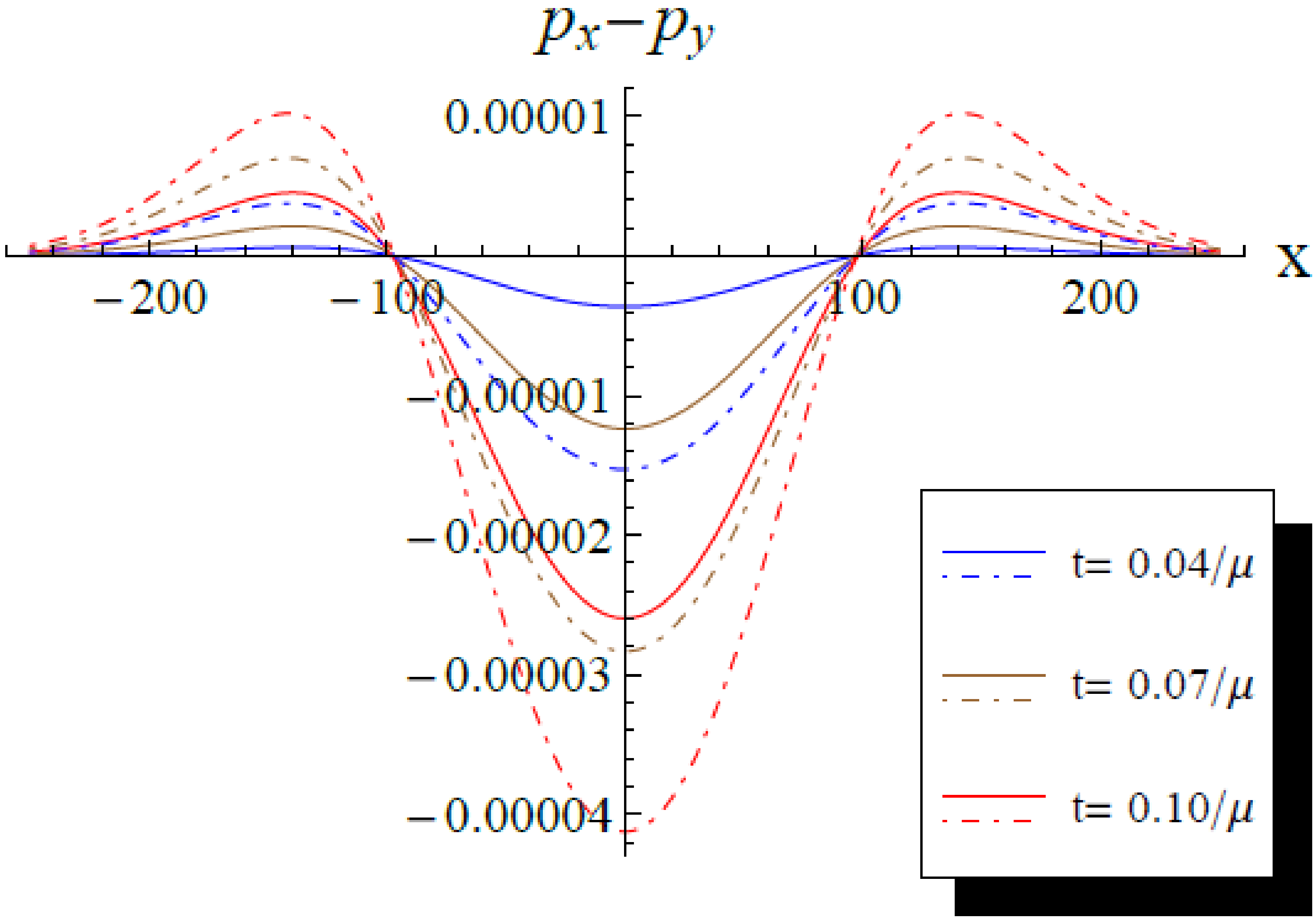
Free-streaming: stress tensor in local rest frame



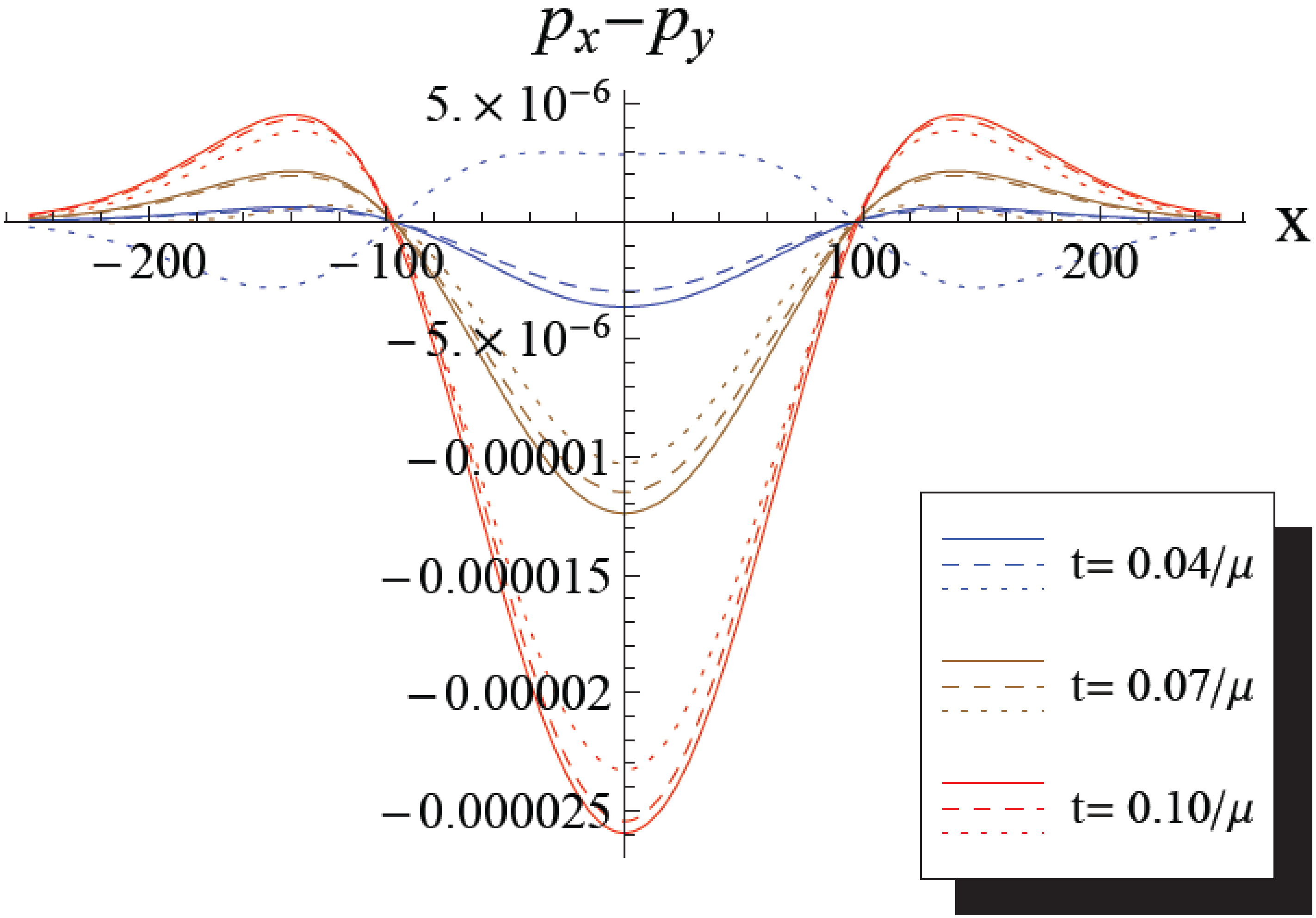
Pressure anisotropy: free-streaming vs AdS/CFT



Pressure anisotropy: 1st order hydro vs AdS/CFT



Pressure anisotropy: 2nd order hydro vs free-streaming vs AdS/CFT



Summary

- Homogeneous holographic thermalization models have led to interesting results (fast isotropization/thermalization, fast applicability of viscous hydrodynamics).

- Recent experimental discovery: higher flow coefficients. Due to event-by-event fluctuations.

- Results on early-time inhomogeneous holographic thermalization: free-streaming → 2nd order hydrodynamics.

- If agreement with 2nd order hydro extends beyond early-time window: would provide justification for standard approach used in simulations.