

Gauge/gravity duality, Munich, July 30, 2013

Casimir effect and the quark–anti-quark potential

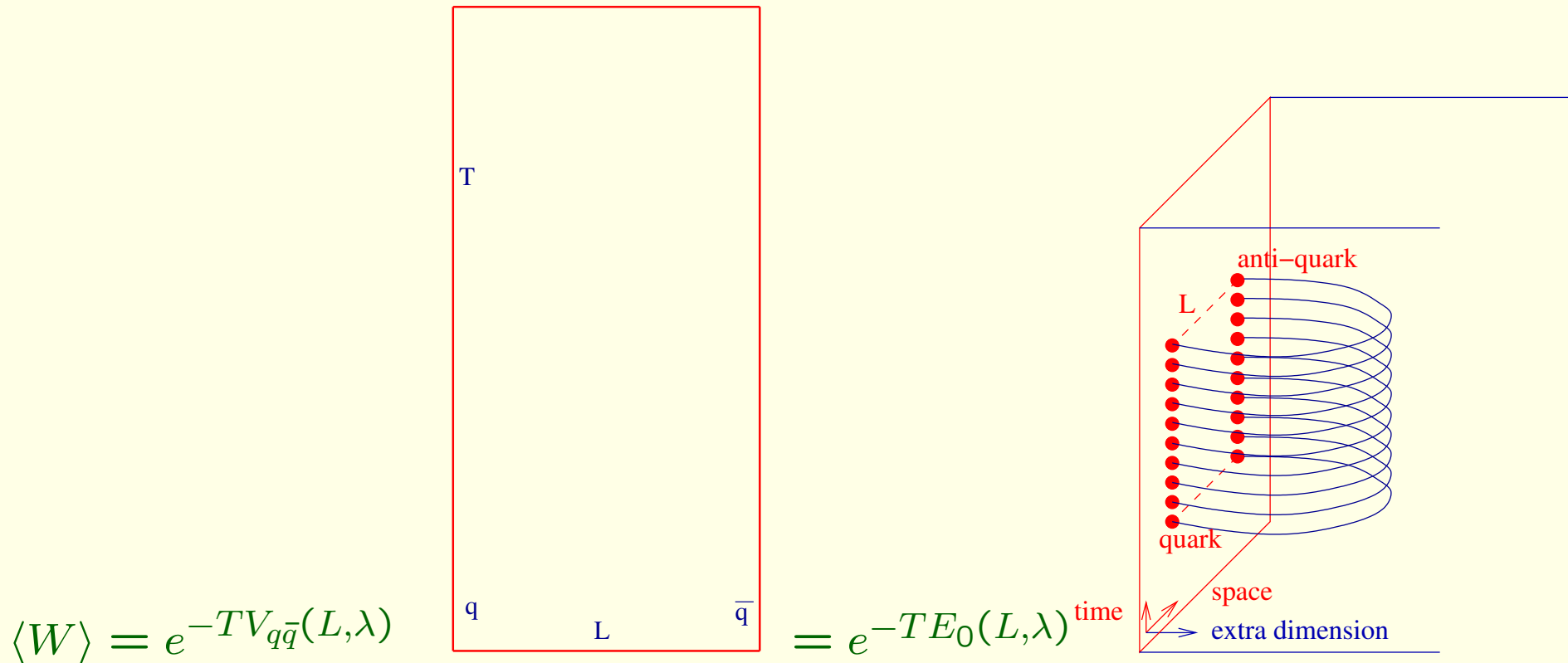
Zoltán Bajnok,

*MTA-Lendület Holographic QFT Group, Wigner Research Centre for Physics, Hungary
with L. Palla, G. Takács, J. Balog, A. Hegedűs, G.Zs. Tóth*

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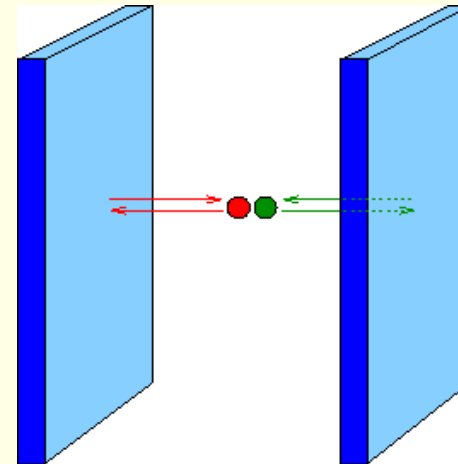
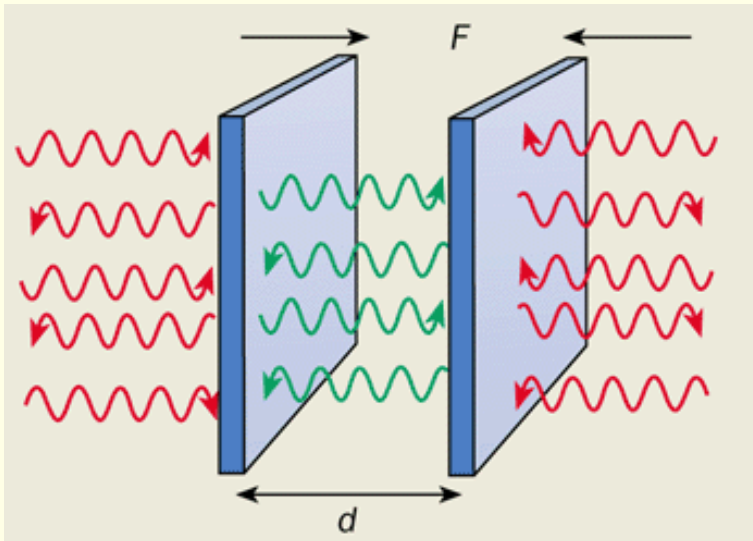
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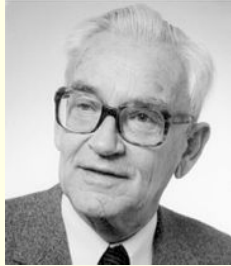
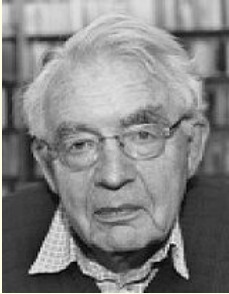
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$$F(L) = \frac{dE_0(L)}{dL} = \frac{d \sum_{k(L)} \omega(k(L))}{2dL}$$

$$E_0(L) = - \int \frac{d\tilde{p}}{2\pi} \log(1 + R(-\tilde{p})R(\tilde{p})e^{-2E(\tilde{p})L})$$

Motivation: Casimir-Polder effect

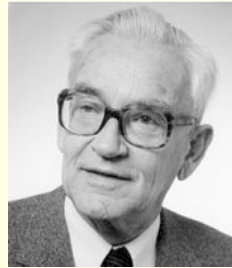


Hendrik Casimir Dirk Polder
colloidal solution: neutral atoms
force not like Van der Waals

$$\frac{F(L)}{A} = -\frac{\hbar c \pi^2}{240 L^4}$$

not a theoretical curiosity!

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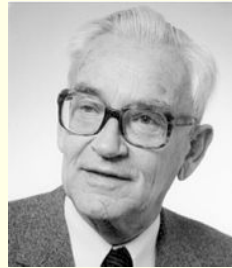
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Gecko legs

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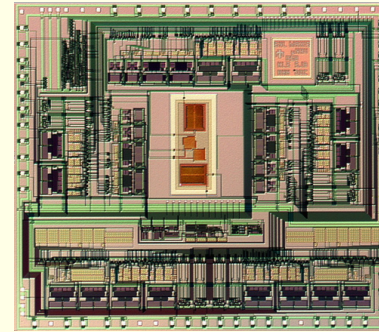
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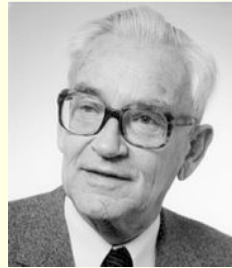


Gecko legs



Airbag trigger chip

Motivation: Casimir-Polder effect



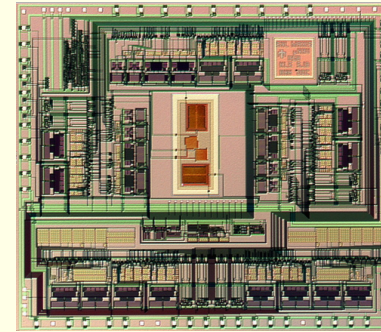
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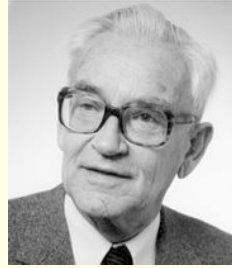
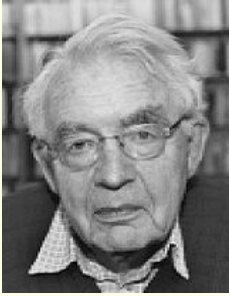


Airbag trigger chip



micromechanical
device: pieces stick
friction, levitation

Motivation: Casimir-Polder effect



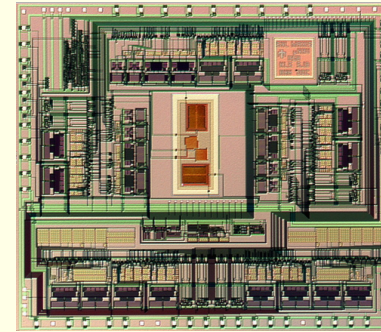
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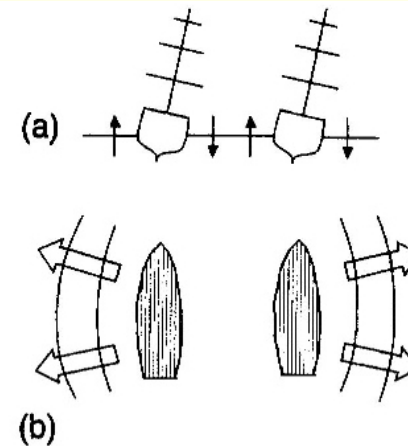
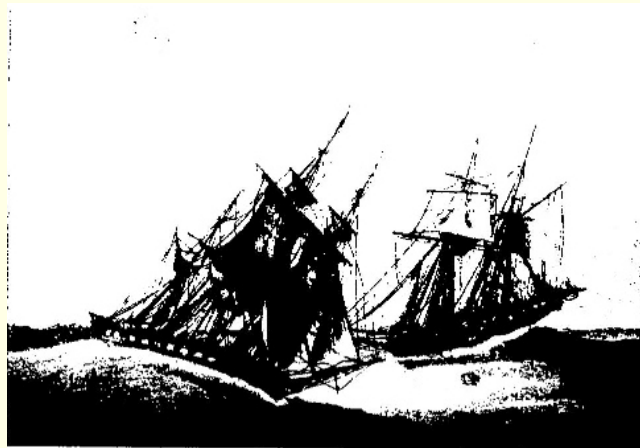


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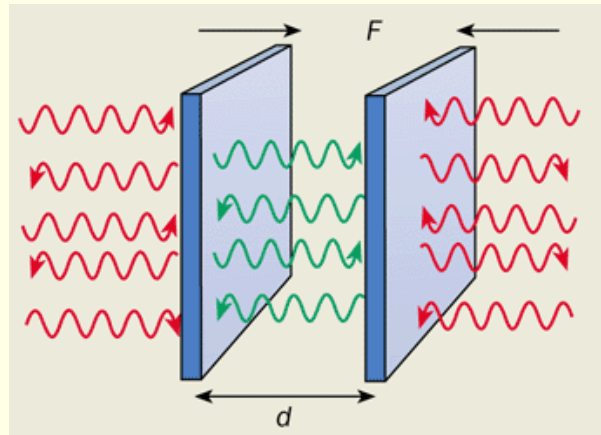
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Maritime analogy:



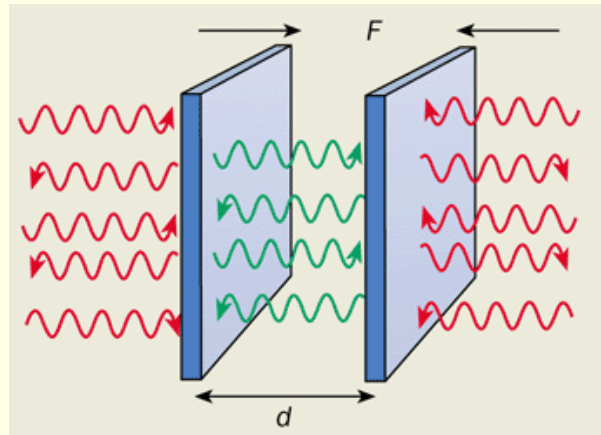
Aim: understand/describe planar Casimir effect

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Usual explanation: energy of the vacuum: $E_0(L) = \frac{1}{2} \sum_{k(L)} \omega(k(L)) \propto \infty$

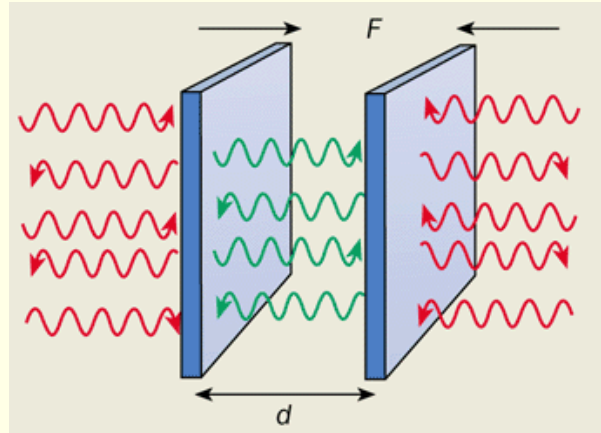
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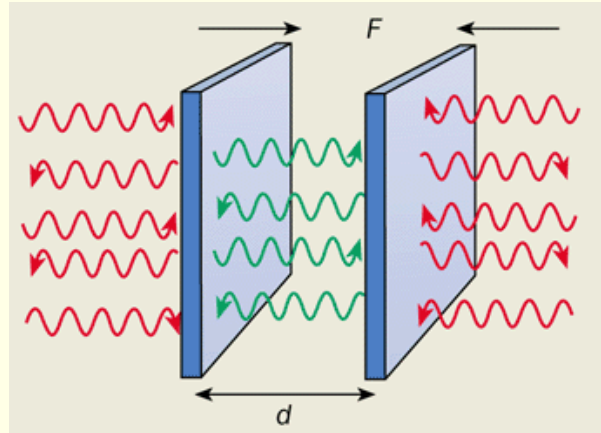
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Lifshitz formula: QED, Parallel dielectric slabs $(\epsilon_1, 1, \epsilon_2)$

$$\begin{aligned} \frac{\Delta E_0(L)}{A} = & \int_0^\infty \frac{d^2 q}{8\pi^2} d\zeta \log \left[1 - \frac{\epsilon_1 \sqrt{\omega^2 - q^2} - \sqrt{\epsilon_1 \omega^2 - q^2}}{\epsilon_1 \sqrt{\omega^2 - q^2} + \sqrt{\epsilon_1 \omega^2 - q^2}} \frac{\epsilon_2 \sqrt{\omega^2 - q^2} - \sqrt{\epsilon_2 \omega^2 - q^2}}{\epsilon_2 \sqrt{\omega^2 - q^2} + \sqrt{\epsilon_2 \omega^2 - q^2}} e^{-2L\sqrt{q^2 + \zeta^2}} \right] \\ & + \int_0^\infty \frac{d^2 q}{8\pi^2} d\zeta \log \left[1 - \frac{\sqrt{\omega^2 - q^2} - \sqrt{\epsilon_1 \omega^2 - q^2}}{\sqrt{\omega^2 - q^2} + \sqrt{\epsilon_1 \omega^2 - q^2}} \frac{\sqrt{\omega^2 - q^2} - \sqrt{\epsilon_2 \omega^2 - q^2}}{\sqrt{\omega^2 - q^2} + \sqrt{\epsilon_2 \omega^2 - q^2}} e^{-2L\sqrt{q^2 + \zeta^2}} \right] \end{aligned}$$

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L

Physics can be understood in 1+1 D QFT

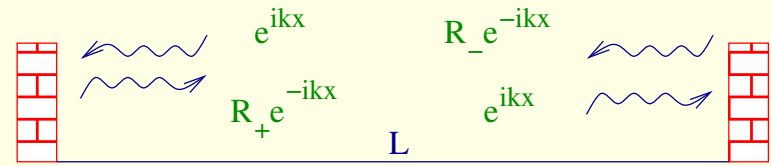


integrability helps to solve the problem even exactly

A simple calculation

Free bulk + interacting boundaries (QED)

$$E_0(L) = \frac{1}{2} \sum_{k(L)} \omega(k(L))$$

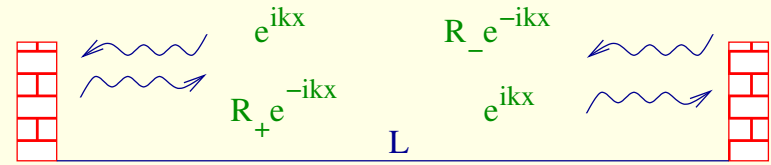

$$Q(k) = e^{2ikL} R_-(k) R_+(-k) - 1 = 0$$

A simple calculation

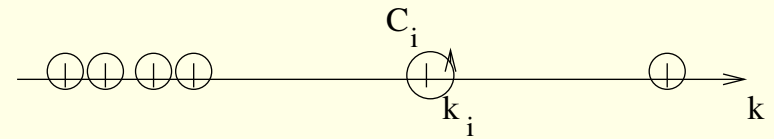
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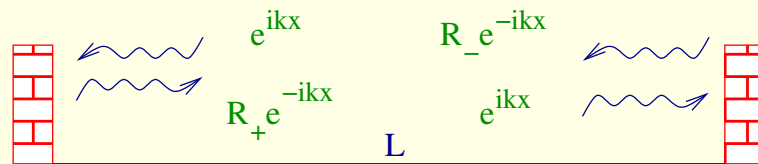
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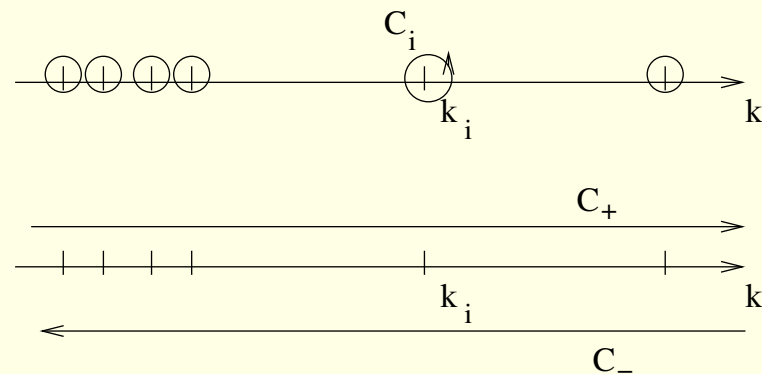
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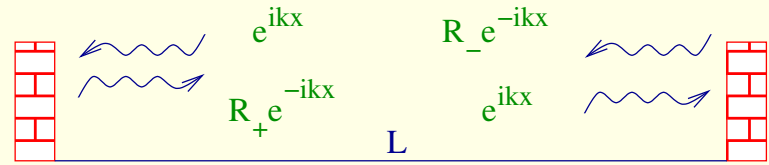
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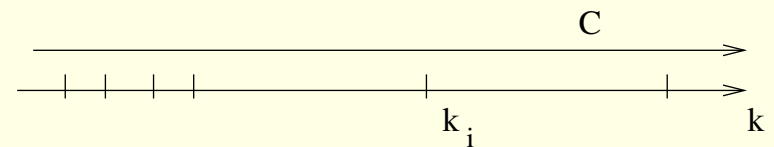
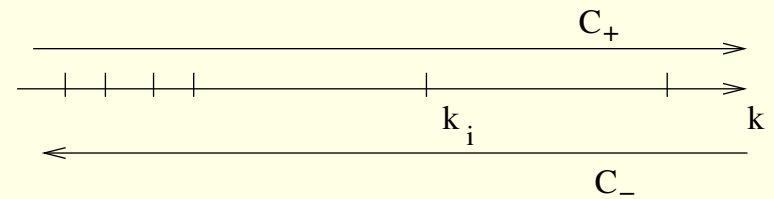
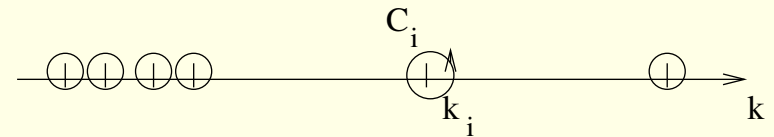
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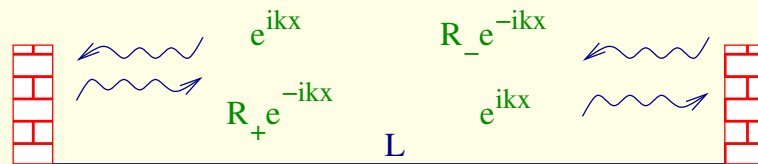
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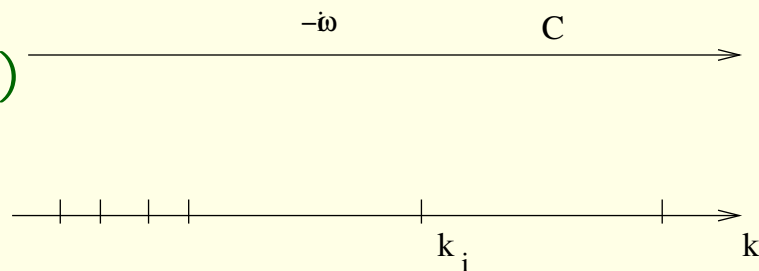
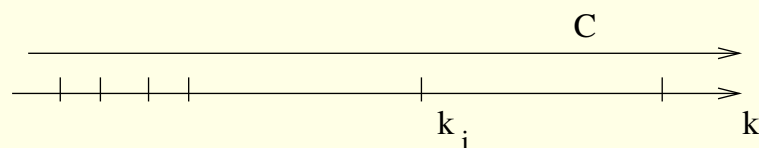
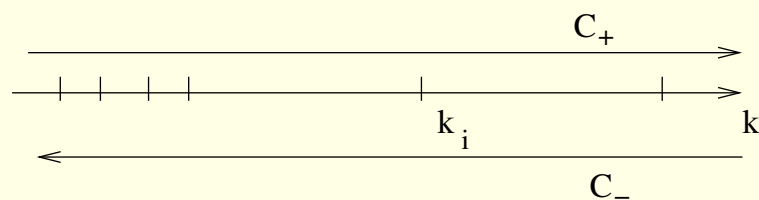
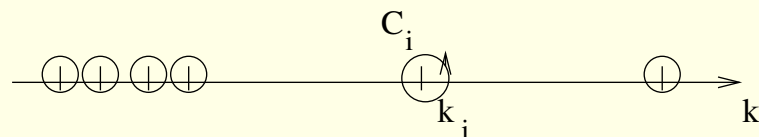
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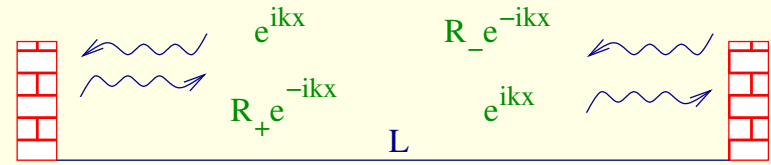
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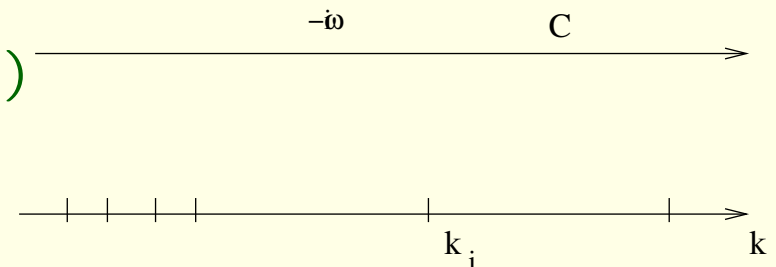
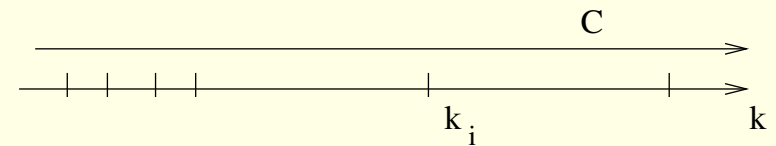
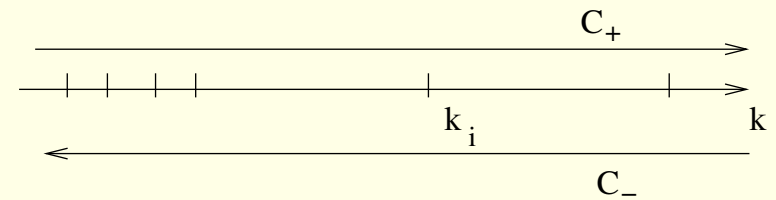
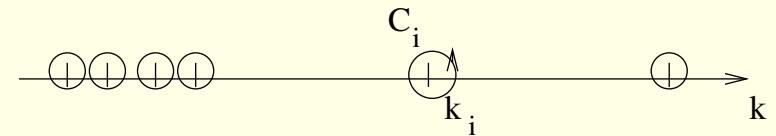
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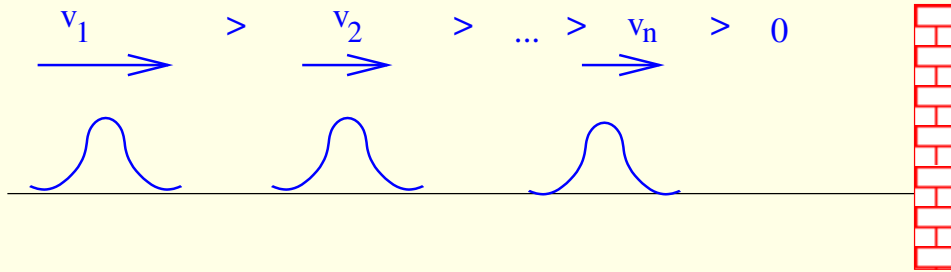


interacting but integrable: similar formula

Integrable boundary field theory: Bootstrap

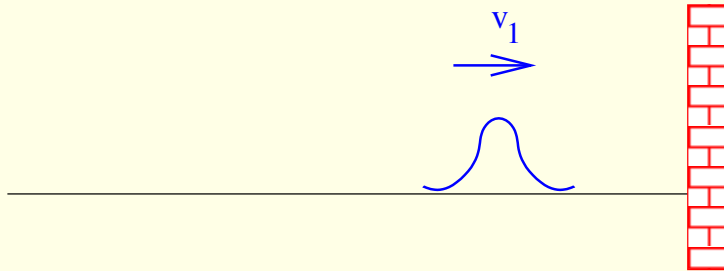
Integrable boundary field theory: Bootstrap

Boundary multiparticle state: with n particles



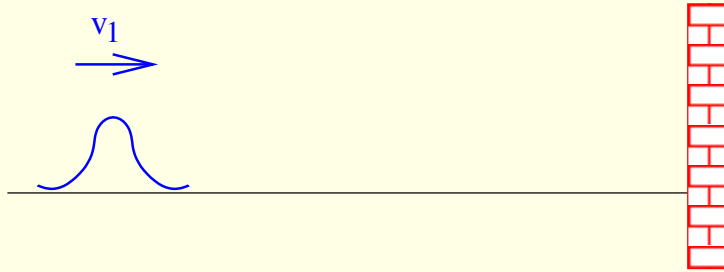
Integrable boundary field theory: Bootstrap

Boundary one particle state:



Integrable boundary field theory: Bootstrap

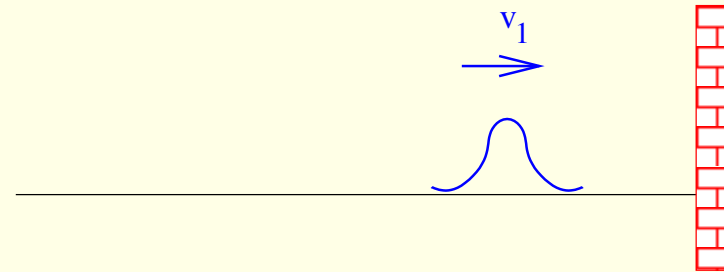
Boundary one particle in state: $t \rightarrow -\infty$



Integrable boundary field theory: Bootstrap

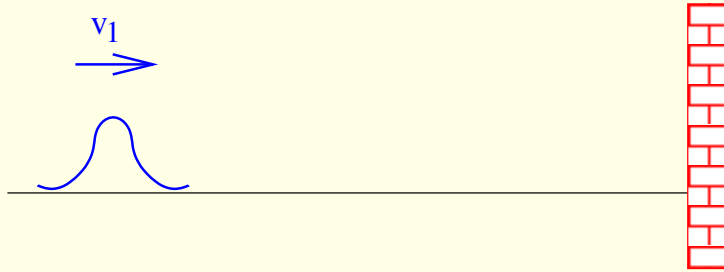
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times develop

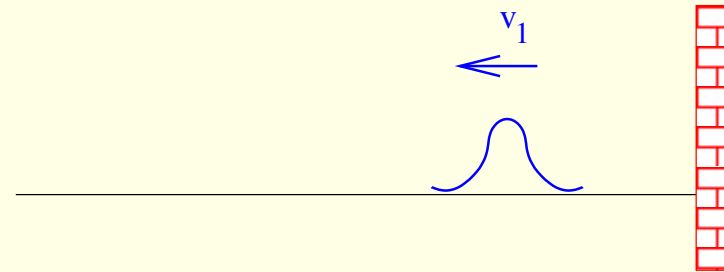


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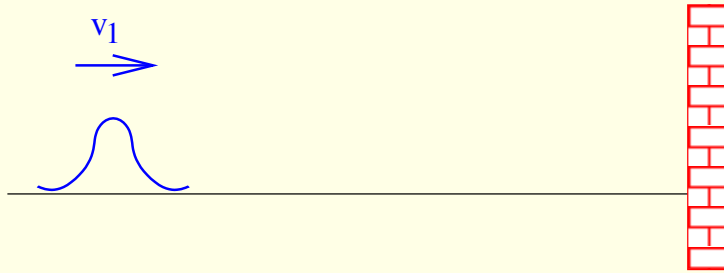


times develop further



Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \rightarrow -\infty$



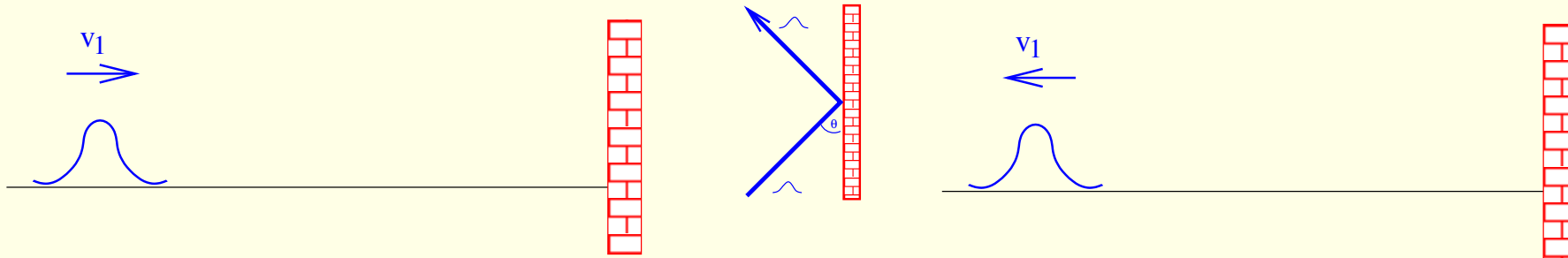
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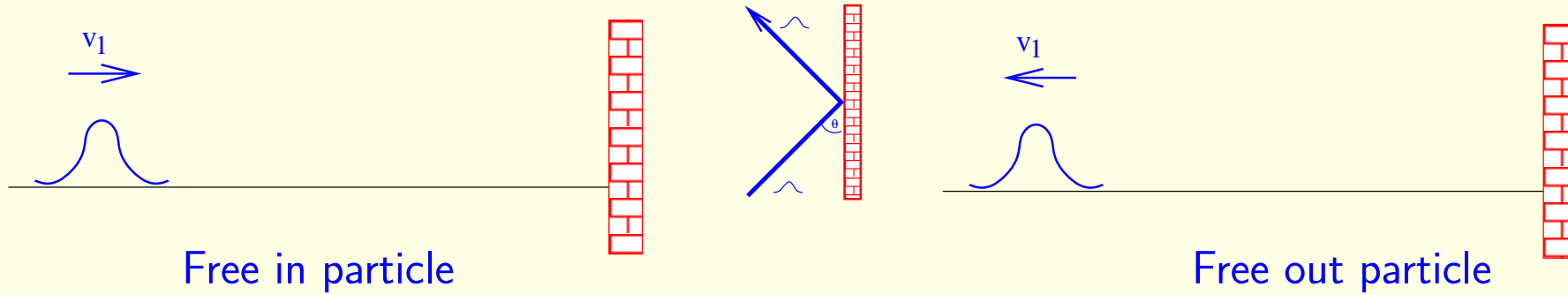
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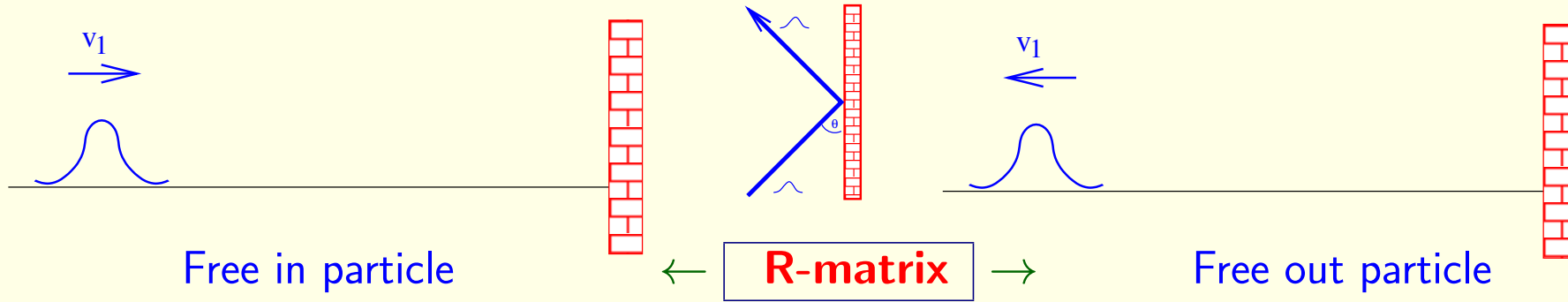
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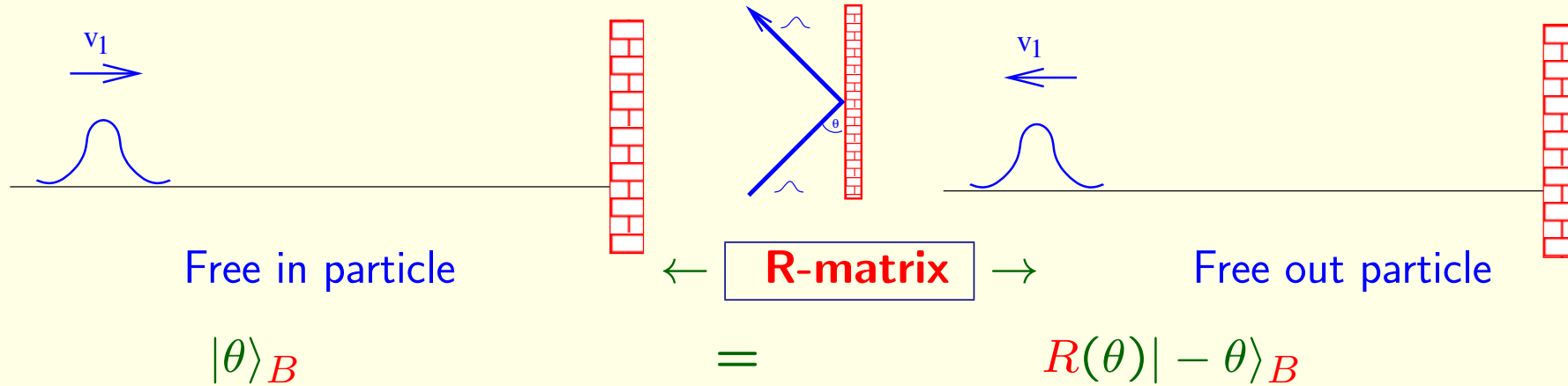
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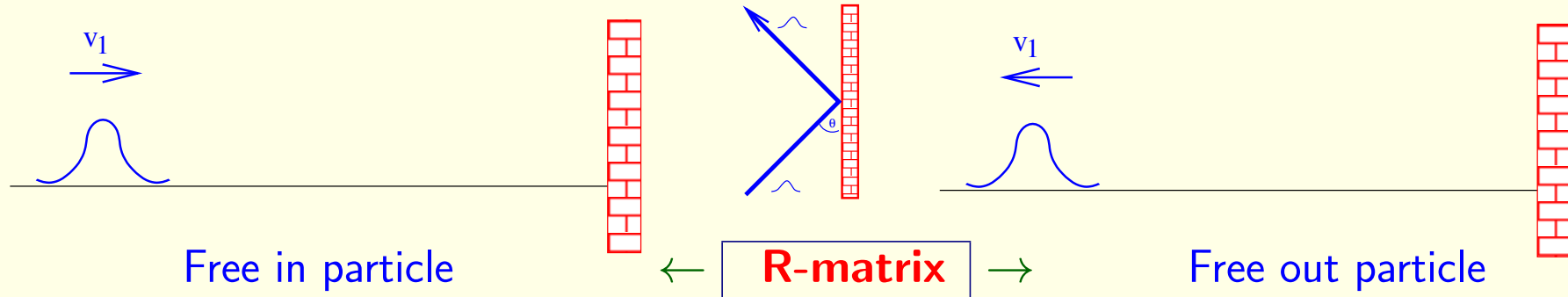
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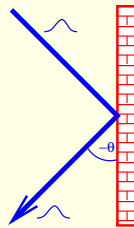
$$|\theta\rangle_B$$

=

$$R(\theta)|-\theta\rangle_B$$

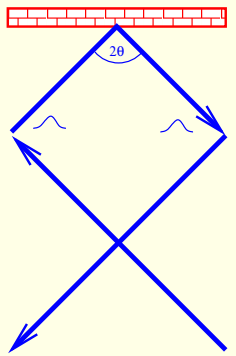
Unitarity

$$R^*(\theta) = R^{-1}(\theta) = R(-\theta)$$



Boundary crossing unitarity

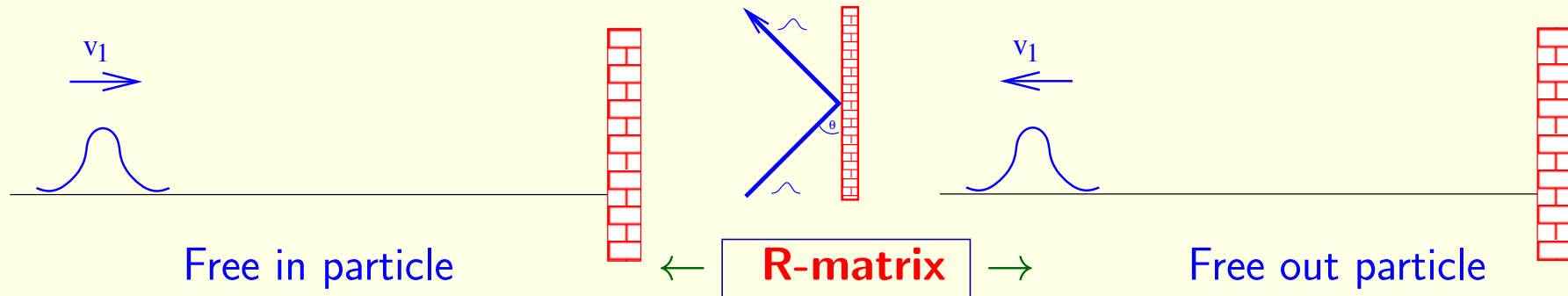
$$R\left(\frac{i\pi}{2} + \theta\right) = S(2\theta)R\left(\frac{i\pi}{2} - \theta\right)$$



Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \rightarrow -\infty$

Boundary one pt out state: $t \rightarrow \infty$



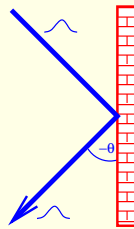
$$|\theta\rangle_B$$

=

$$R(\theta)|-\theta\rangle_B$$

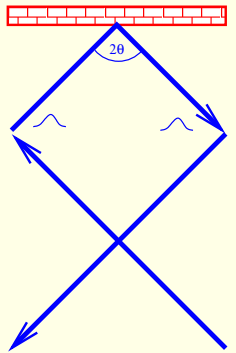
Unitarity

$$R^*(\theta) = R^{-1}(\theta) = R(-\theta)$$



Boundary crossing unitarity

$$R\left(\frac{i\pi}{2} + \theta\right) = S(2\theta)R\left(\frac{i\pi}{2} - \theta\right)$$



sinh-Gordon $S(\theta) = \frac{\sinh \theta - i \sin \pi p}{\sinh \theta + i \sin \pi p} = [-p] = -(-p)(1+p); (p) = \frac{\sinh(\frac{\theta}{2} + \frac{i\pi p}{2})}{\sinh(\frac{\theta}{2} - \frac{i\pi p}{2})}$

reflection factor $R(\theta) = \left(\frac{1}{2}\right) \left(\frac{1+p}{2}\right) \left(1 - \frac{p}{2}\right) \left[\frac{3}{2} - \frac{\eta p}{\pi}\right] \left[\frac{3}{2} - \frac{\Theta p}{\pi}\right]$

Ghoshal-Zamolodchikov '94

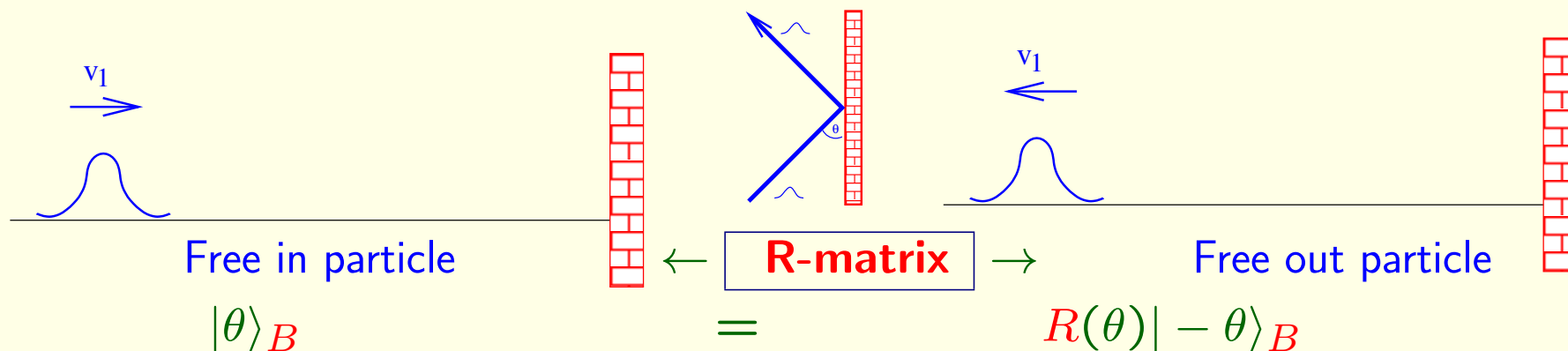
Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) - \delta\{\mu_+^B e^{\frac{b}{2}\phi} + \mu_-^B e^{-\frac{b}{2}\phi}\}$

$$\sqrt{\frac{\mu}{\sin b^2}} \cosh b^2(\eta \pm \Theta) = \mu_{\pm}^B$$

Integrable boundary field theory: Bootstrap

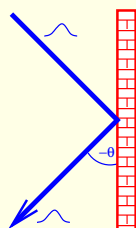
Boundary one particle in state: $t \rightarrow -\infty$

Boundary one pt out state: $t \rightarrow \infty$



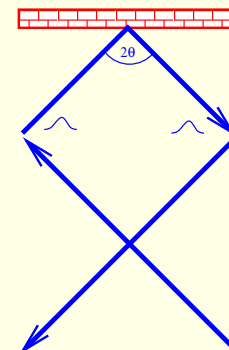
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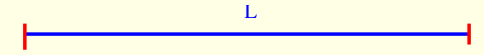
Integrability \rightarrow **factorizability**: $|\theta_1, \theta_2, \dots, \theta_n\rangle_B = \prod_{i < j} S(\theta_i - \theta_j) S(\theta_i + \theta_j) \prod_i R(\theta_i) |-\theta_1, -\theta_2, \dots, -\theta_n\rangle_B$

Boundary thermodynamic Bethe Ansatz

Groundstate energy for large L from IR reflection: $E_0(L) =$

Boundary thermodynamic Bethe Ansatz

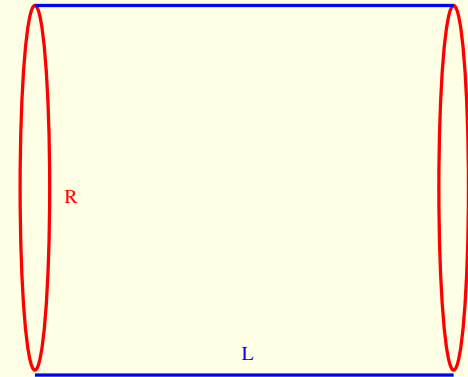
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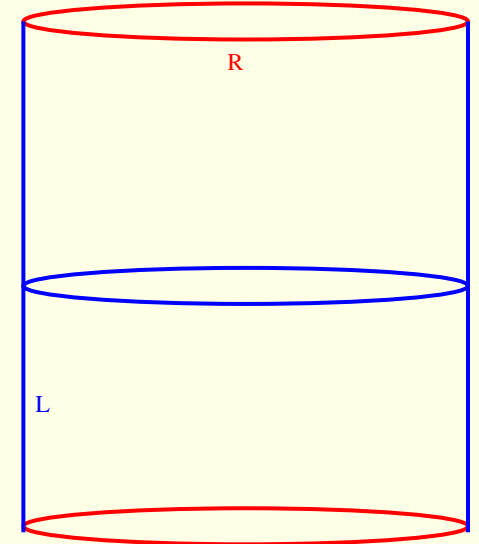
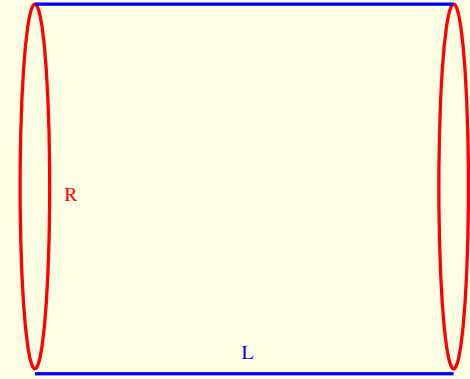
$$-\lim_{R \rightarrow \infty} \frac{1}{R} \log(\text{Tr}(e^{-H^B(L)R})) = -\lim_{R \rightarrow \infty} \frac{1}{R} \log(e^{-E_0(L)R})$$



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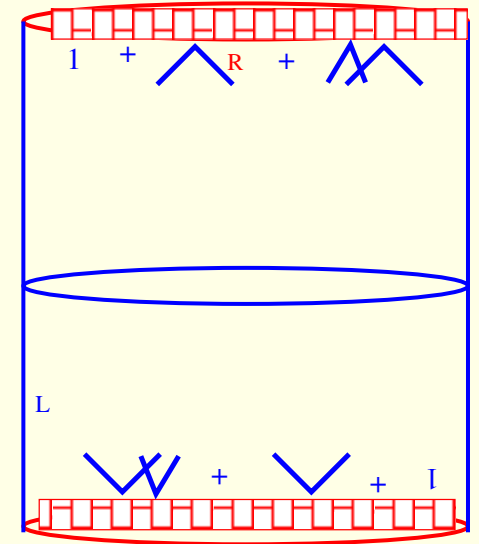
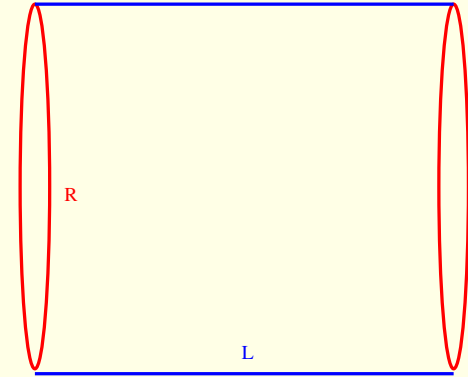


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Boundary state $|B\rangle = \exp \left\{ \int_{-\infty}^{\infty} \frac{d\theta}{4\pi} R \left(\frac{i\pi}{2} - \theta \right) A^+(-\theta) A^+(\theta) \right\} |0\rangle$



Boundary thermodynamic Bethe Ansatz

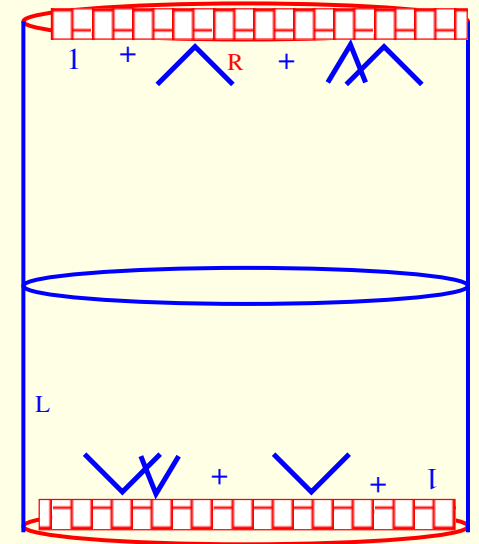
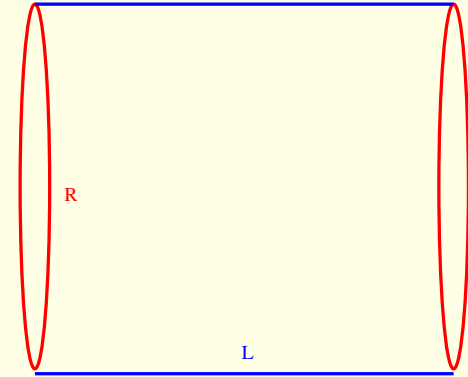
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Dominant contribution for large L : two particle term

$$\langle B | e^{-H(R)L} | B \rangle = 1 + \sum_k R \left(\frac{i\pi}{2} - \theta \right) R \left(\frac{i\pi}{2} + \theta \right) e^{-2m \cosh \theta_k L} + \dots$$



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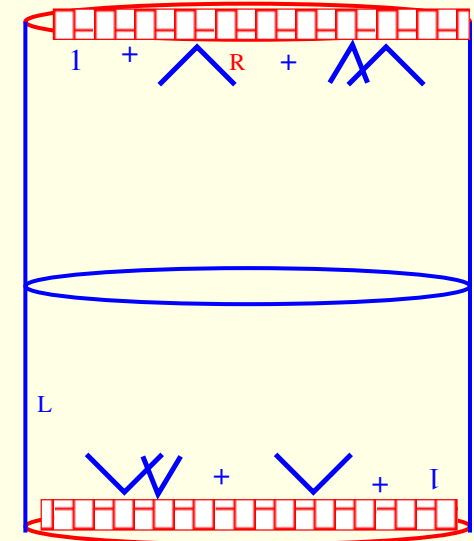
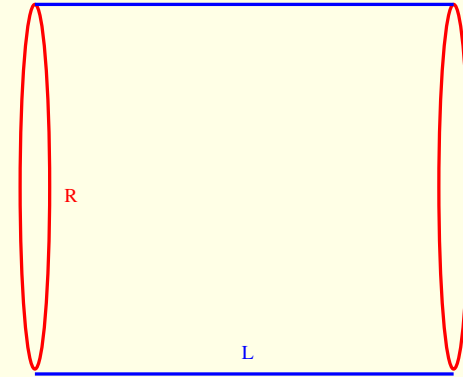
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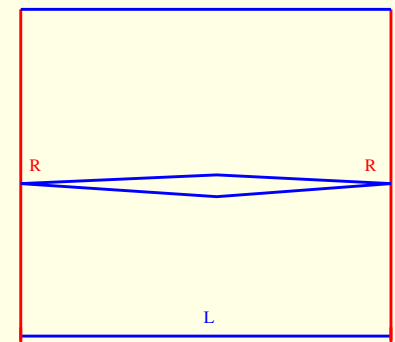
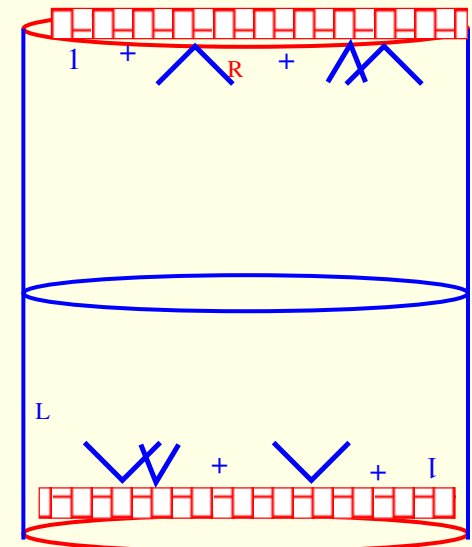
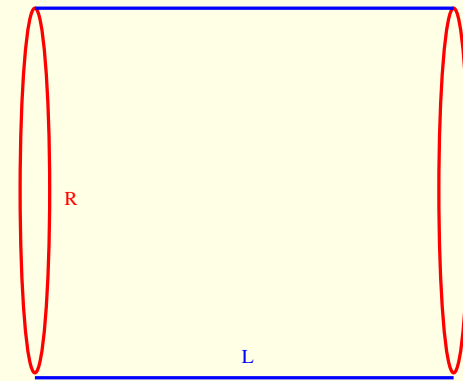
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Ground state energy exactly: $E_0(L) = -m \int \frac{d\theta}{4\pi} \cosh(\theta) \log(1 + e^{-\epsilon(\theta)})$

$$\epsilon(\theta) = 2mL \cosh \theta - \log(R\left(\frac{i\pi}{2} - \theta\right) R\left(\frac{i\pi}{2} + \theta\right))$$

$$- \int \frac{d\theta'}{2\pi} \varphi'(\theta - \theta') \log(1 + e^{-\epsilon(\theta')}) \text{ [LeClair, Mussardo, Saleur, Skorik]}$$



Casimir effect: Boundary finite size effect

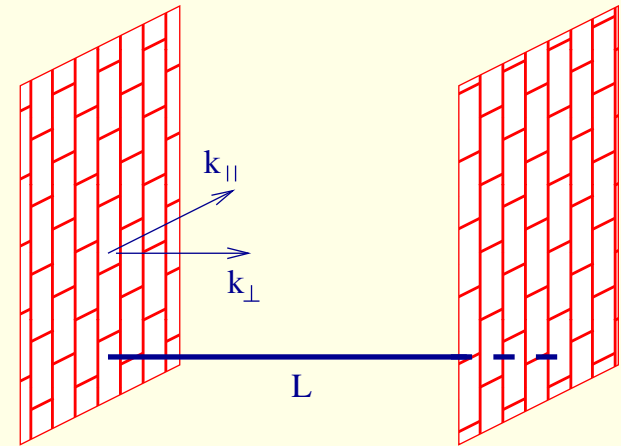
Casimir effect: Boundary finite size effect

Extension to higher dimensions: \vec{k}_{\parallel} label

$$\text{Dispersion } \omega = \sqrt{m^2 + \vec{k}_{\parallel}^2 + k_{\perp}^2} = \sqrt{m_{\text{eff}}^2 + k_{\perp}^2}$$

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Reflection $R(\theta, m_{\text{eff}}(k_{\parallel}))$



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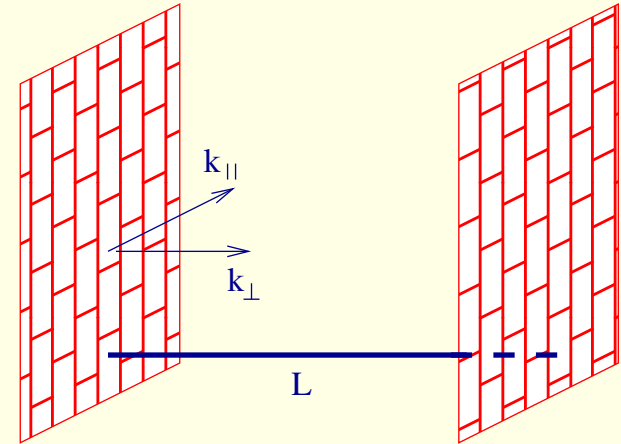
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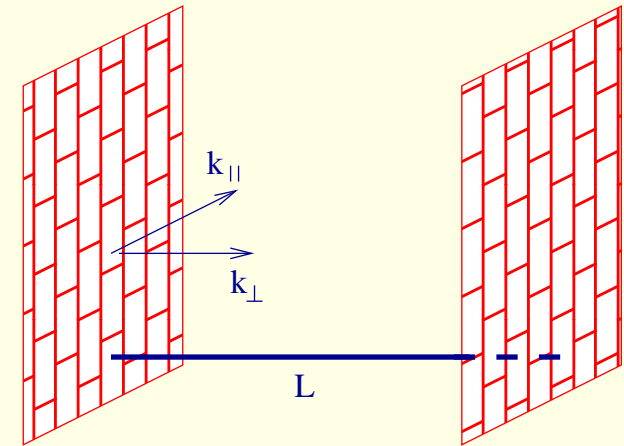
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Ground state energy (for free bulk):

$$E_0(L) = \int \frac{d^{D-1}k_{\parallel}}{(2\pi)^{D-1}} \frac{d\theta}{4\pi} \log(1 + R^1\left(\frac{i\pi}{2} + \theta, m_{\text{eff}}\right) R^2\left(\frac{i\pi}{2} - \theta, m_{\text{eff}}\right) e^{-2m_{\text{eff}} \cosh \theta L})$$



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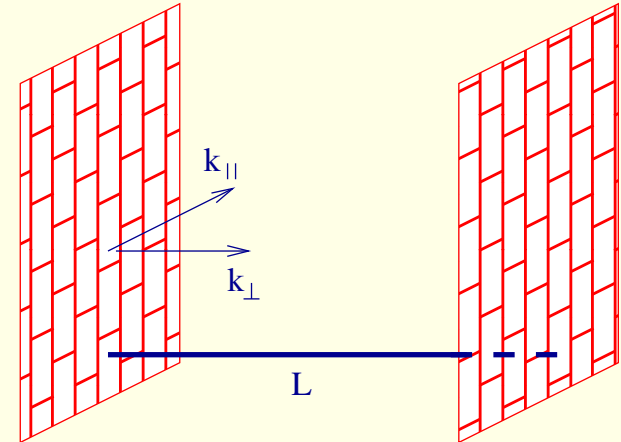
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QED: Parallel dielectric slabs ($\epsilon_1, 1, \epsilon_2$)

reflections $E_{\parallel, \perp}, B_{\parallel, \perp} \longrightarrow R_{\parallel, \perp}$ look it up in Jackson:

$$R_{\perp}(\omega, k_{\parallel} = q) = \frac{\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}} \quad R_{\parallel}(\omega, k_{\parallel} = q) = \frac{\epsilon\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\epsilon\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}}$$



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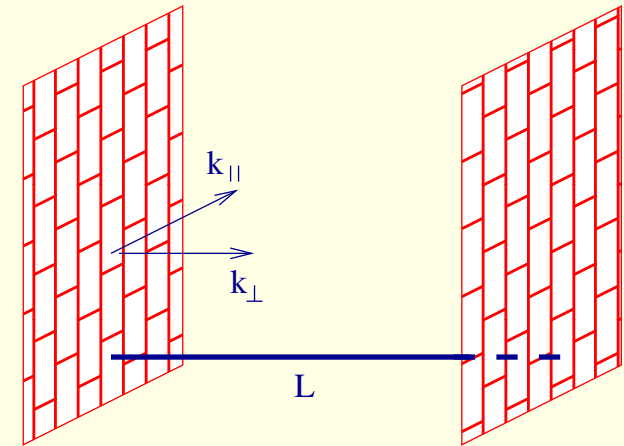
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Lifshitz
formula

Conclusion about Casimir

Conclusion about Casimir

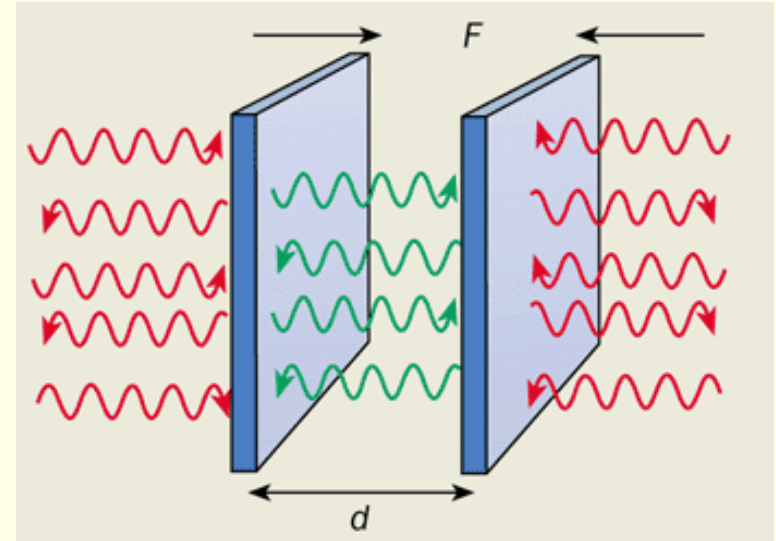
Usual derivation:

summing up zero frequencies

$$E_0(L) = \frac{1}{2} \sum_{k(L)} \omega(k(L)) \propto \infty$$

Complicated finite volume problem

+ divergencies



Conclusion about Casimir

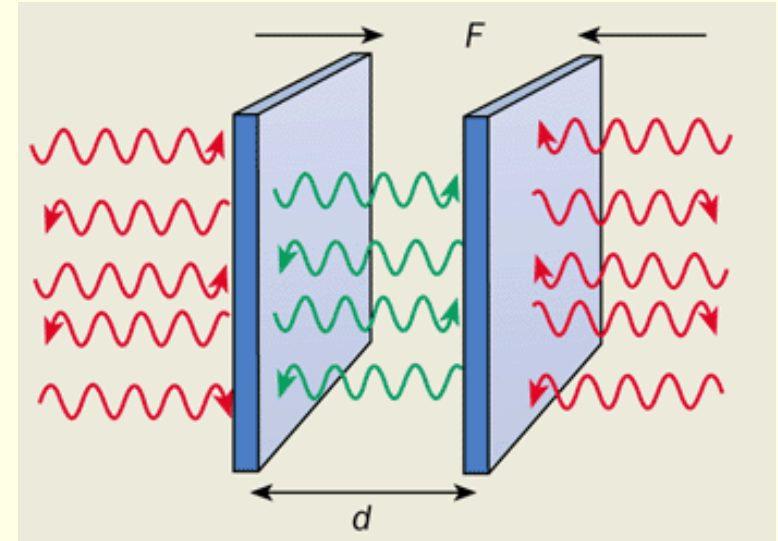
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as a boundary finite size effect

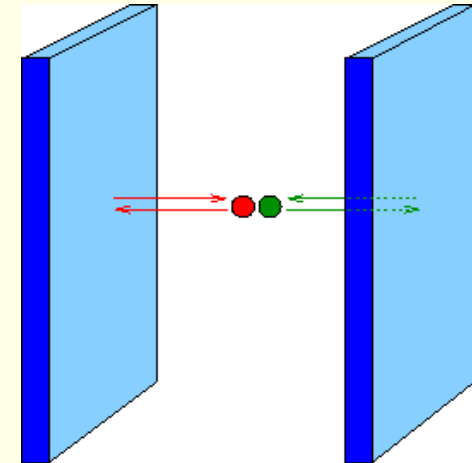
$$E_0(L) = - \int \frac{d\tilde{p}}{2\pi} \log(1 + R(-\tilde{p})R(\tilde{p})e^{-2\tilde{\epsilon}(\tilde{p})L})$$

Reflection factor of the IR degrees of freedom:

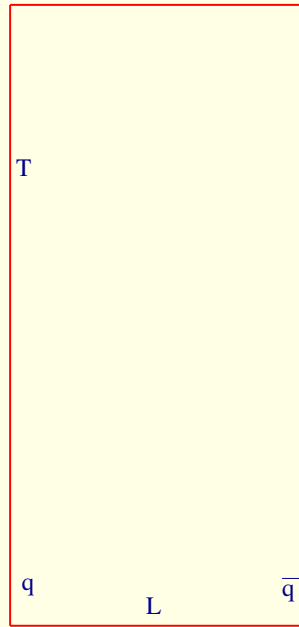
semi infinite settings,

easier to calculate,

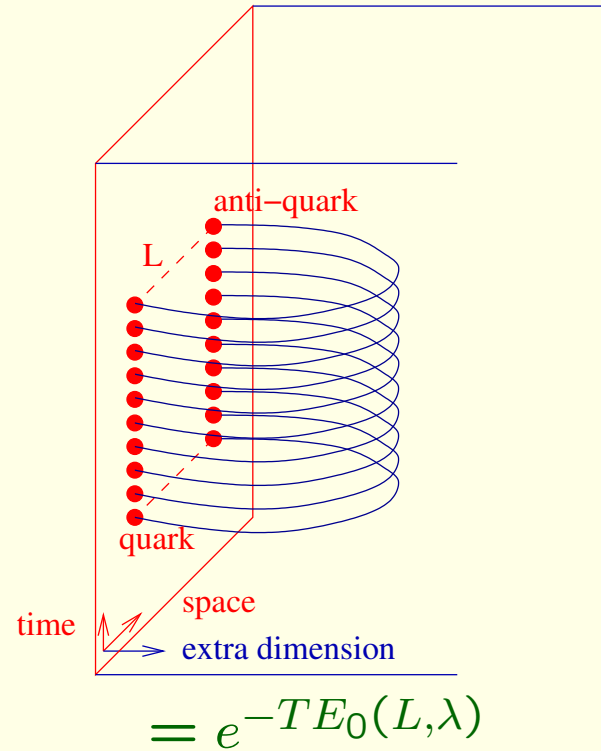
no divergences



Main problem: $q - \bar{q}$ potential in $\mathcal{N} = 4$ SYM



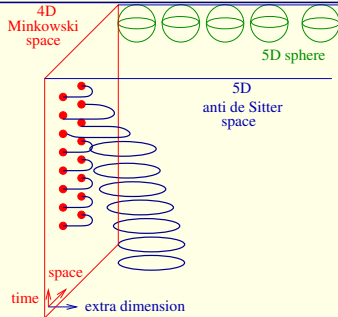
$$\langle W \rangle = e^{-\frac{T}{L} V_{q\bar{q}}(\lambda)}$$



$$= e^{-T E_0(L, \lambda)}$$

AdS/CFT correspondence (Maldacena 1997)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

\equiv

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$A_\mu \begin{matrix} \nearrow \Psi_{1,2,3,4} \\ \searrow \bar{\Psi}_{1,2,3,4} \end{matrix} \quad \begin{matrix} \searrow \\ \nearrow \end{matrix} \Phi_{1,2,3,4,5,6}$$

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

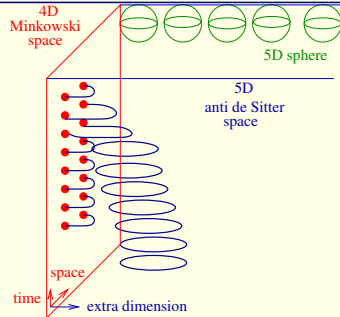
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \text{Tr}(\Phi^L), \det()$

AdS/CFT correspondence (Maldacena 1997)

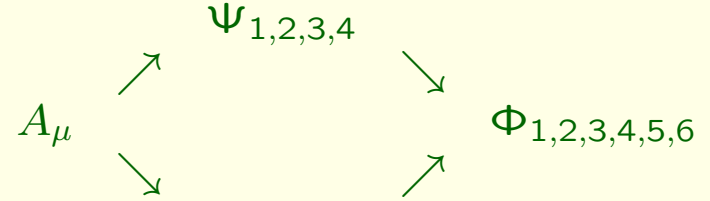
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$\mathcal{N} = 4$ D=4 $SU(N)$ SYM



$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$$\beta = 0 \text{ superconformal } \frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$$

$$\text{gaugeinvariants: } \mathcal{O} = \text{Tr}(\Phi^L), \det(\dots)$$

Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \rightarrow 0$
2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong \leftrightarrow weak



$\lambda = g_{YM}^2 N$, $N \rightarrow \infty$ planar limit

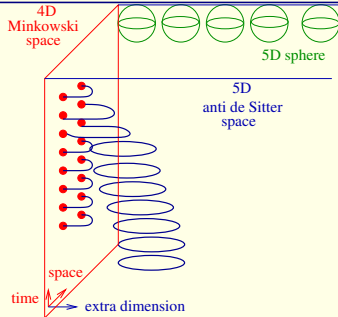
$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

AdS/CFT correspondence (Maldacena 1997)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\begin{array}{ccc} & \Psi_{1,2,3,4} & \\ & \nearrow & \searrow \\ A_\mu & & \Phi_{1,2,3,4,5,6} \\ & \searrow & \nearrow \\ & \bar{\Psi}_{1,2,3,4} & \end{array}$$

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

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2D integrable QFT

AdS/CFT integrability: $q - \bar{q}$ potential

quark-antiquark potential

$V(L) = \frac{-\lambda}{4\pi L} + \dots$

≡

Wilson loop:

$$\langle \mathcal{P} e^{\oint_C A_\mu dx^\mu + \vec{\Phi} \vec{n} \cdot \dot{x} ds} \rangle \propto e^{-\frac{T}{L} V_{q\bar{q}}(\lambda, \theta)}$$

strong coupling

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{L} \left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots \right)$$

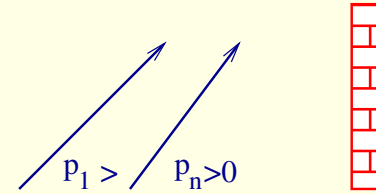
minimal surface + fluctuations

Integrable system on the strip

$$E_0(L) = \int \frac{d\tilde{k}}{2\pi} \log(1 - R_-(\tilde{k})R_+(-\tilde{k})e^{-2\tilde{\epsilon}(\tilde{k})L})$$

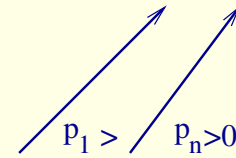
R-matrix bootstrap program

Boundary asymptotic states $|p_1, p_2, \dots, p_n\rangle_{in/out}$
form a representation of global symmetry:



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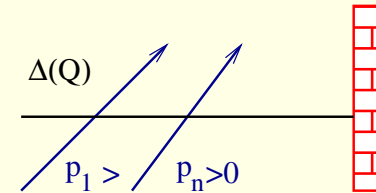
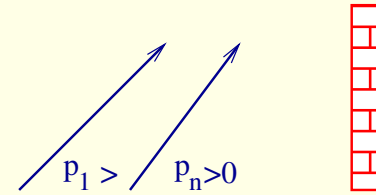
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R-matrix bootstrap program

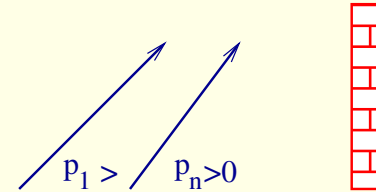
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Reflection matrix R : $|out\rangle \rightarrow |in\rangle$
commutes with sym. $[R, \Delta(Q)] = 0$

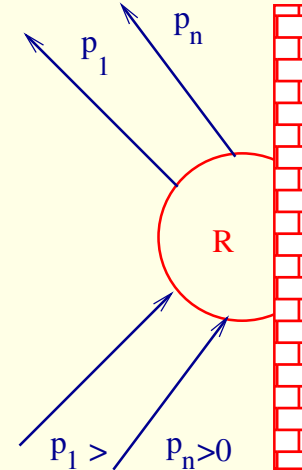
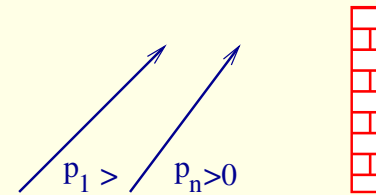
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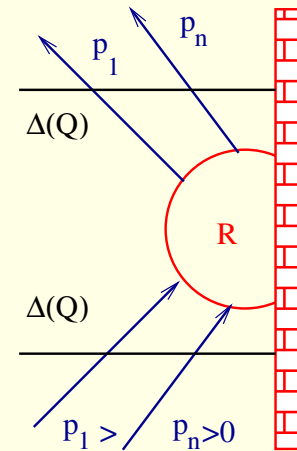
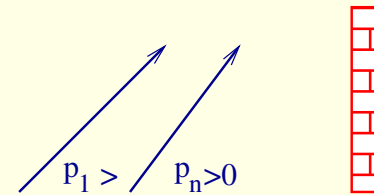
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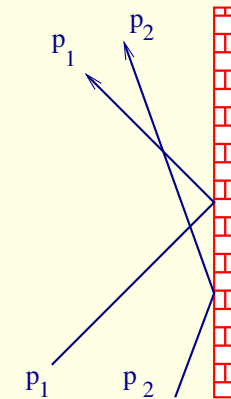
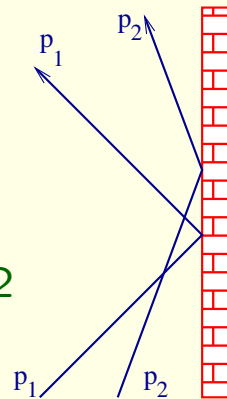
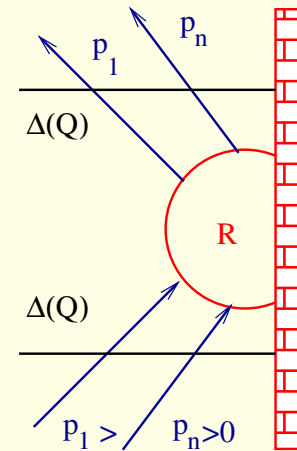
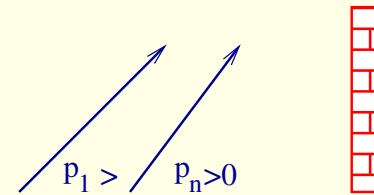
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Higher spin conserved charge

factorization + Bdry Yang-Baxter equation

$$R_{12} = S_{12}R_1\bar{S}_{21}R_2 = R_2\bar{S}_{21}R_1S_{12}$$

R-matrix = scalar . Matrix



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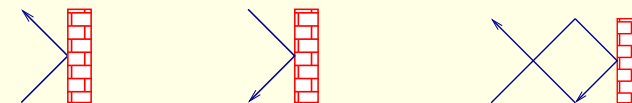
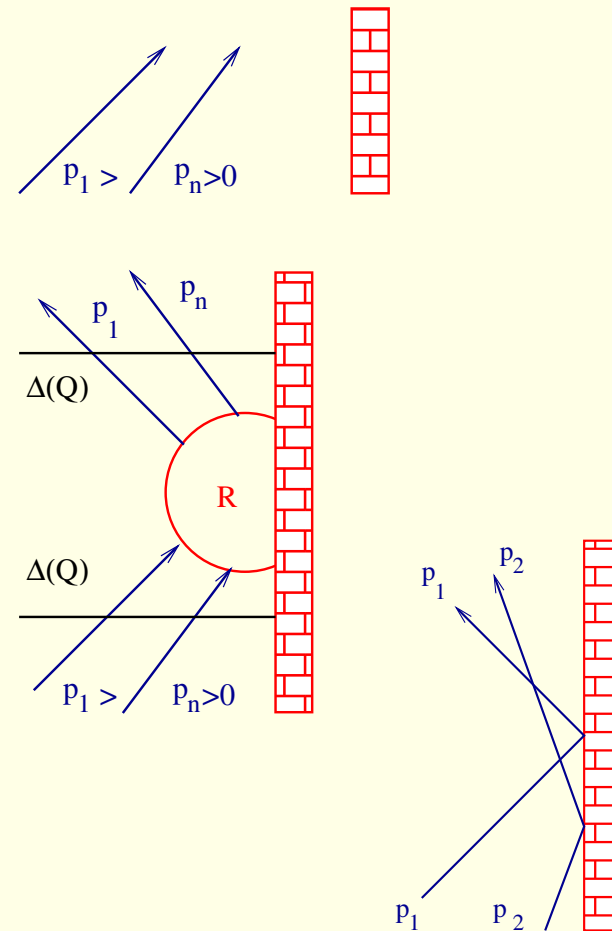
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R-matrix = scalar . Matrix

$$\text{Unitarity } R(p)R(-p) = Id$$

$$\text{Boundary crossing symmetry } R(p) = S(p, -p)R(\bar{p})$$

Maximal analyticity: all poles have physical origin \rightarrow boundstates, anomalous thresholds



R-matrix bootstrap program: AdS

Nondiagonal scattering: $R\text{-matrix} = \text{scalar} \cdot \text{Matrix}$

R-matrix bootstrap program: AdS

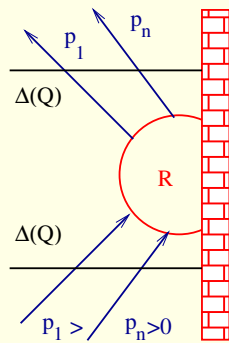
Nondiagonal scattering: $R\text{-matrix} = \text{scalar} \cdot \text{Matrix}$

R-matrix: [Correa-Maldacena-Sever, Drukker]

global symmetry $PSU(2|2)_{diag}$

$$Q = 1 \text{ reps } \begin{pmatrix} b_1 \\ b_2 \\ f_3 \\ f_4 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \\ f_3 \\ f_4 \end{pmatrix}$$

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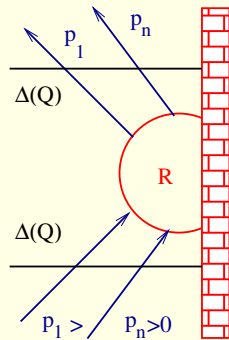
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$$R_{\alpha\dot{\alpha}}^{\beta\dot{\beta}}(p) = S_{\alpha\dot{\alpha}}^{\beta\dot{\beta}}(p, -p) R_0(p)$$

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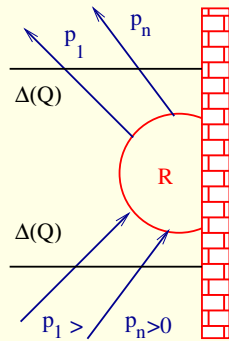
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Unitarity

$$R(z)R(-z) = 1$$

Crossing symmetry

$$R(z) = S(z, -z)R(\omega_2 - z)$$

$$R_0(p) = \frac{\sigma_B(p)}{\sigma(p, -p)}$$

$$\sigma_B = e^{i\chi(x^+) - i\chi(x^-)}$$

boundary dressing phase

$$\chi(x) = \oint \frac{dz}{2\pi} \frac{1}{x-z} \frac{\sinh(2\pi g(z+z^{-1}))}{2\pi g(z+z^{-1})}$$

R-matrix bootstrap program: AdS

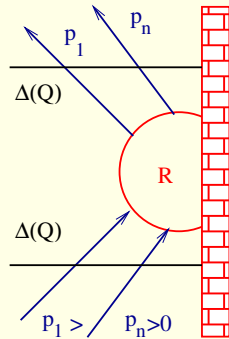
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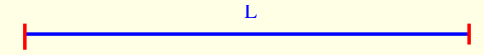
Maximal analyticity: **no boundstates**

Boundary thermodynamic Bethe Ansatz

Groundstate energy for large L from IR reflection: $E_0(L) =$

Boundary thermodynamic Bethe Ansatz

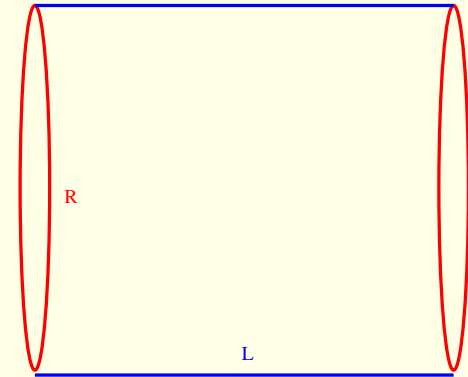
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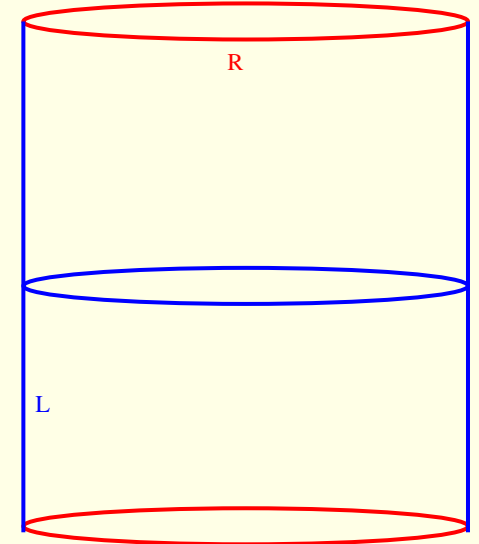
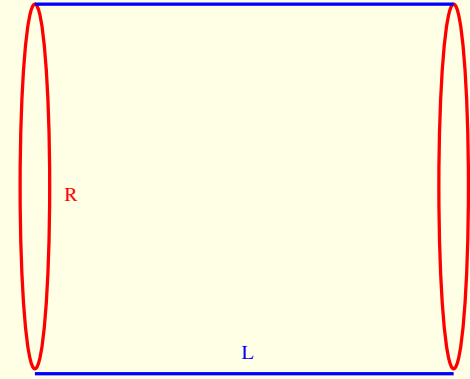
$$-\lim_{R \rightarrow \infty} \frac{1}{R} \log(\text{Tr}(e^{-H^B(L)R})) = -\lim_{R \rightarrow \infty} \frac{1}{R} \log(e^{-E_0(L)R})$$



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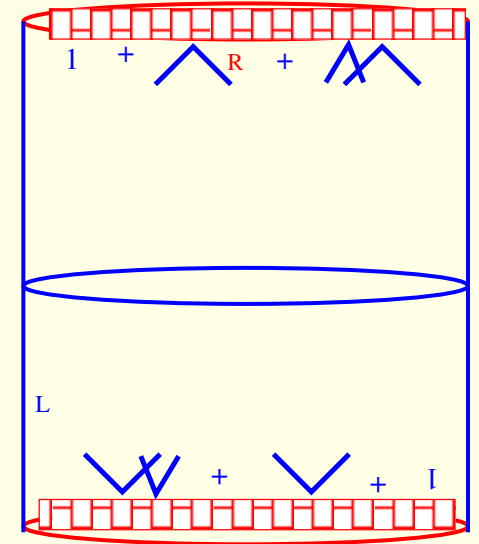
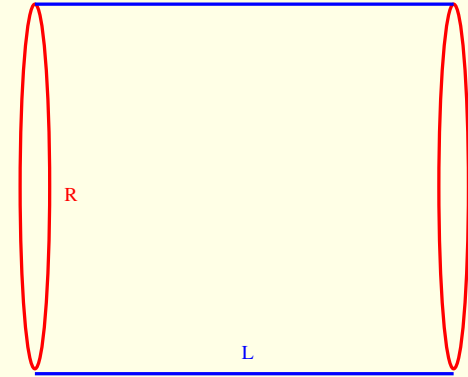


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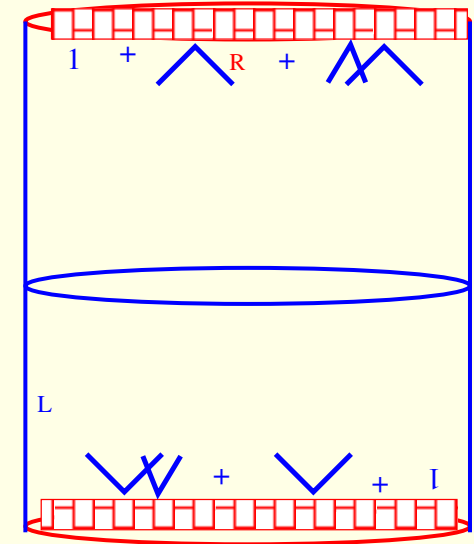
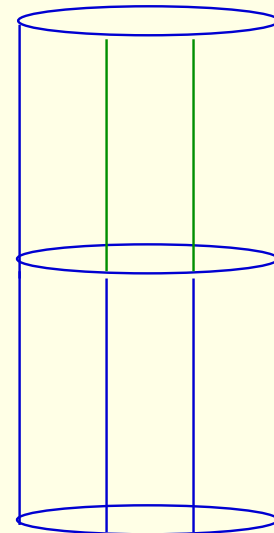
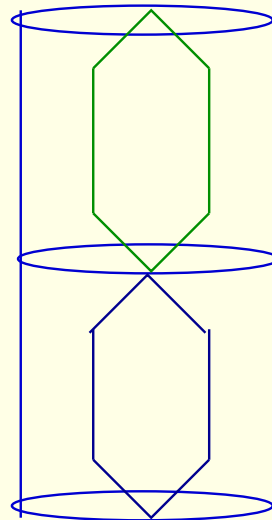
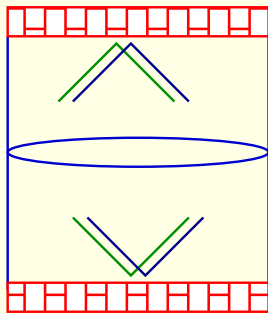
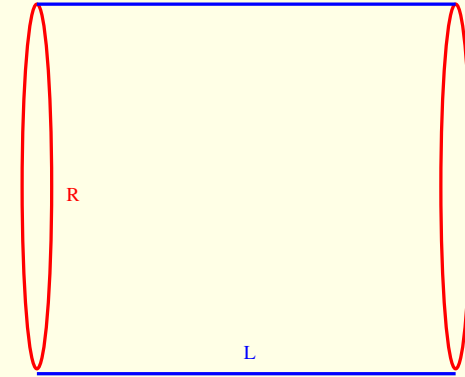


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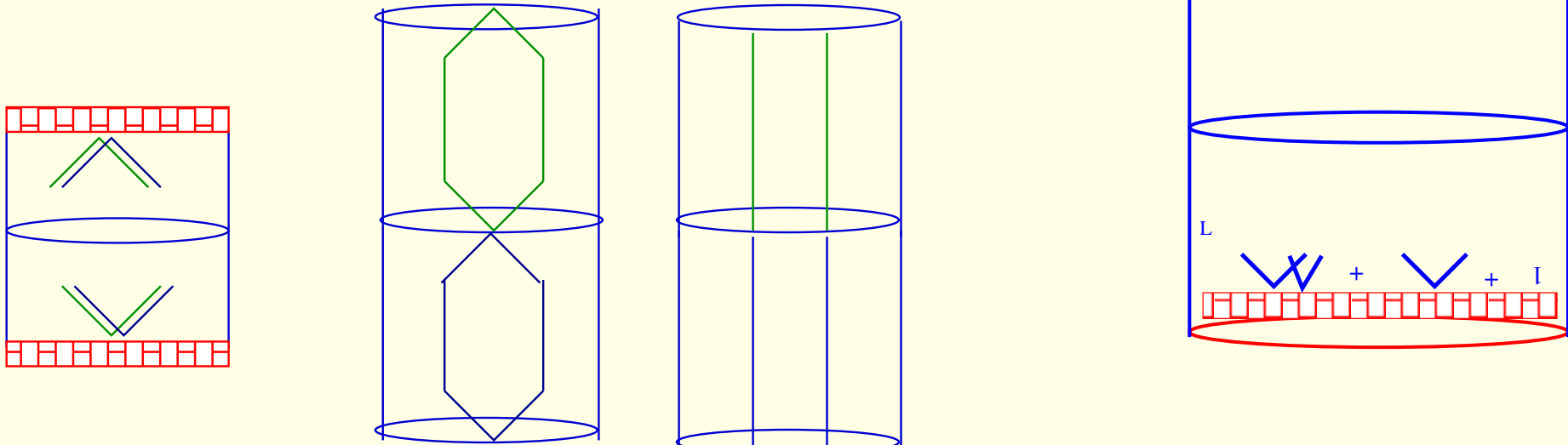
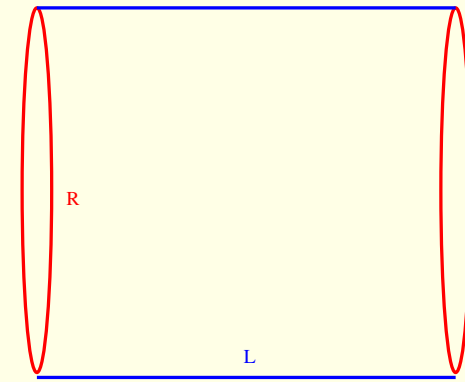
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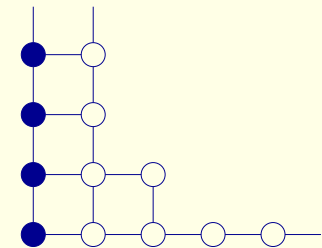
Folding trick:

[Correa, Maldacena, Sever '12][Drukker '12]

Ground state energy exactly:

$$E_0(L) = -\sum_Q \int \frac{d\tilde{p}}{4\pi} \log(1 + e^{-\epsilon_Q(\tilde{p})})$$

$$\epsilon^j(\tilde{p}) = \delta_Q^j(\sigma_Q(\tilde{p}) + 2\tilde{E}_Q(\tilde{p})L) - \int K_i^j(\tilde{p}, \tilde{p}') \log(1 + e^{-\epsilon^i(\tilde{p}')}) d\tilde{p}'$$



Regularized $q - \bar{q}$ BTBA equations

Singular boundary fugacity: $\sigma_Q(0) = \infty$, no-obvious weak coupling expansion
 shifting contours \rightarrow regularization (extra source terms, \sim excited state TBA)

$$\begin{aligned} \log Y_Q &= -2(f + \Psi)Q - R\tilde{\epsilon}_Q + \log \sigma_Q + D^{Q'Q}(iu_Q) + \log(1 + Y_{Q'}) \star_\eta K^{Q'Q} \\ &\quad + [2 \log(1 + Y_{v|1}) \star s \hat{\star} K_{yQ} + 2 \log(1 + Y_{v|Q-1}) \star s - 2 \log \frac{1 - Y_-}{1 - Y_+} \hat{\star} s \star K_{vx}^{1Q} \\ &\quad + \log \frac{1 - \frac{1}{Y_-}}{1 - \frac{1}{Y_+}} \hat{\star} K_Q + \log(1 - \frac{1}{Y_-})(1 - \frac{1}{Y_+}) \hat{\star} K_{yQ}] \end{aligned}$$

$$\log Y_- Y_+ = 2D_{xvs}(iu_Q) - D_Q(iu_Q) - \log(1 + Y_Q) \star_\eta K_Q + 2 \log(1 + Y_Q) \star K_{xv}^{Q1} \star s + 2 \log \frac{1 + Y_{v|1}}{1 + Y_{w|1}}$$

$$\log \frac{Y_+}{Y_-} = D_{Qy}(iu_Q) + \log(1 + Y_Q) \star_\eta K_{Qy}$$

$$\log Y_{v|M} = -D_s(iu_{M+1}) - \log(1 + Y_{M+1}) \star_\eta s + I_{MN} \log(1 + Y_{v|N}) \star s + \delta_{M1} \log \frac{1 - Y_-}{1 - Y_+} \hat{\star} s$$

$$\log Y_{w|M} = I_{MN} \log(1 + Y_{w|N}) \star s + \delta_{M1} \log \frac{1 - \frac{1}{Y_-}}{1 - \frac{1}{Y_+}} \hat{\star} s$$

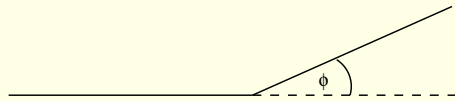
$$f = i(\pi - \phi) \quad ; \quad \Psi = -i(\pi - \phi) \quad ; \quad R = 2L$$

[ZB, Balog, Hegedus, Toth '13]

$q - \bar{q}$ potential: weak coupling expansion

quark-antiquark potential

$$\langle \mathcal{P} e^{\int_C A_\mu dx^\mu + \vec{\Phi} \vec{n}_\theta} Z J \mathcal{P} e^{\int_C A_\mu dx^\mu + \vec{\Phi} \vec{n}_\theta} \rangle$$



$$V_{q\bar{q}}(g, \phi, \theta) = \sum \Gamma_k g^{2k}$$

$$\Gamma = \sum_{n=1}^k \left(\frac{\cos \phi - \cosh \theta}{\sin \phi} \right)^n \gamma_k^{(n)}$$

$$\gamma_1^{(1)} = \frac{\phi}{2} \quad ; \quad \gamma_2^{(1)} = \frac{\phi}{12} (\phi^2 - \pi^2)$$

$$\gamma_2^{(2)}(0) = \gamma_2^{(2)'}(0) = 0$$

$$\gamma_2^{(2)''}(\phi) = \frac{\phi}{2} \cot \phi$$

≡

QMinimal surface

Wilson loop

strong coupling

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{L} \left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots \right)$$

minimal surface+fluctuations